# Emergent and Nonlinear Properties of Macroscopic Quantum Metamaterials

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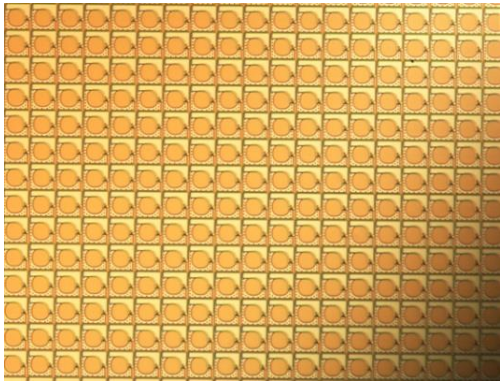
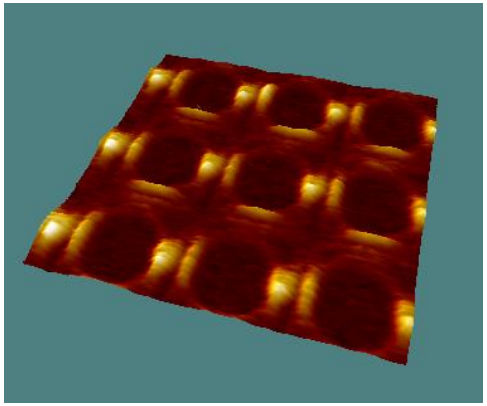




QUANTUM METAMATERIALS & QUANTUM TECHNOLOGY CONFERENCE

19-24 JUNE 2016
Anargyreios and Korgialenios School of Spetses

**Center for Nanophysics and Advanced Materials**

# Outline

**What are Metamaterials?**

**What are SQUIDs?**

**Nonlinear Metamaterials**

**Experiments**

- **Transparency**
- **Coherence**
- **Intermodulation**

**Models of SQUID Dynamics**

**Conclusions**

# What is a Metamaterial?

An artificial structure made up of repeating units of engineered objects having desired properties.

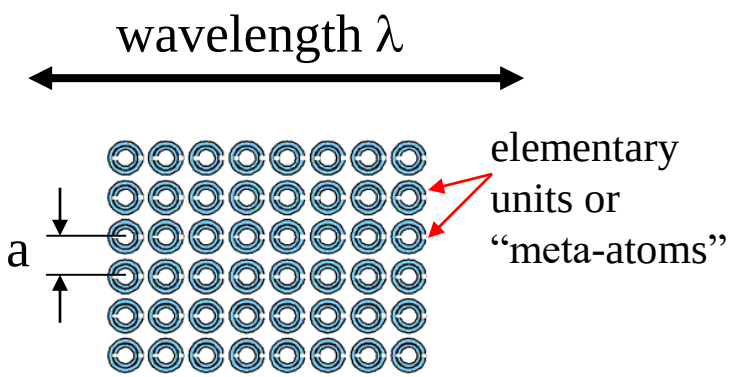
Electromagnetic metamaterials:
The engineered objects (called ‘meta-atoms’) are electrically small (dimension << wavelength) and have ‘designer’ electromagnetic properties.

Large numbers of meta-atoms collected together can create an effective medium with novel coarse-grained properties:

$\varepsilon_{eff}$, $\mu_{eff}$, index of refraction,
enhanced $\chi_{NL}$,
transparency,
novel boundary conditions, etc.

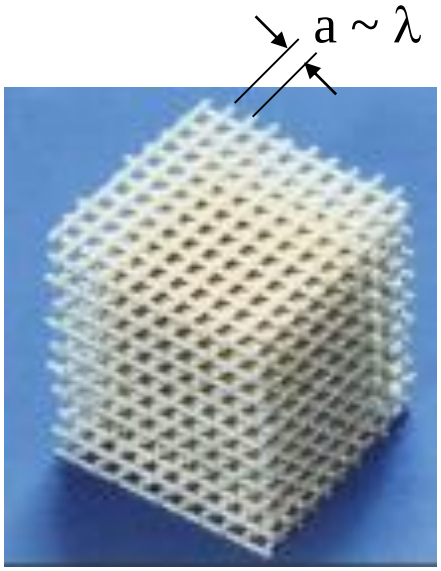
# Metamaterial vs. Photonic Crystal

**Metamaterial**




Create an "effective medium," using engineered "atoms," with macroscopic $\varepsilon_{eff}$, $\mu_{eff}$, $n$ properties

**Photonic Crystal**



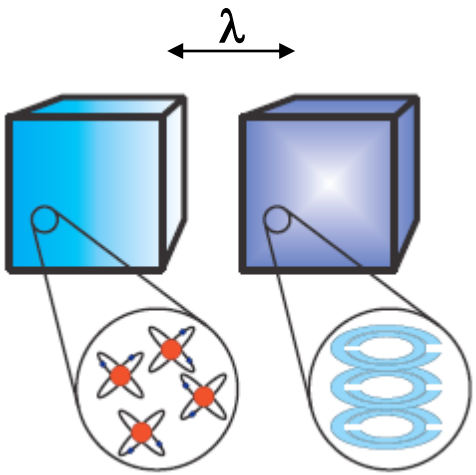

Use constructive and destructive interference to engineer properties of light $\rightarrow$ $\omega(\vec{k})$

- band structure
- band gaps
- defect states
- negative group velocity …

# Why <u>Superconducting</u> Metamaterials?

Many exciting applications of metamaterials:

- Metasurface Light Manipulation
- Super-resolution Imaging
- Cloaking Devices
- etc. …




Pendry (2004)

… have strict <u>REQUIREMENTS</u> on the <u>metamaterials</u>:

- Ultra-Low Losses
- Ability to scale down in size (e.g. $\lambda/10^2$) and texture the "atoms"
- **Nonlinearity with wide and fast tunability of the index of refraction *n***

… and superconductors bring these <u>new capabilities</u> to the metamaterials field:

- **Flux quantization and Josephson effects**
- Quantized energy states and quantum interactions with light
- Strong diamagnetism

S.M.A. "**The Physics and Applications of Superconducting Metamaterials**," J. Opt. **13**, 024001 (2011);
P. Jung, A. V. Ustinov, and S.M.A., "**Progress in Superconducting Metamaterials**," Supercond. Sci. Technol. **27**, 073001 (2014).

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Conclusions

# Macroscopic Quantum Effects In Superconductors

**Superconductor is described by a single-valued Macroscopic Quantum Wavefunction**

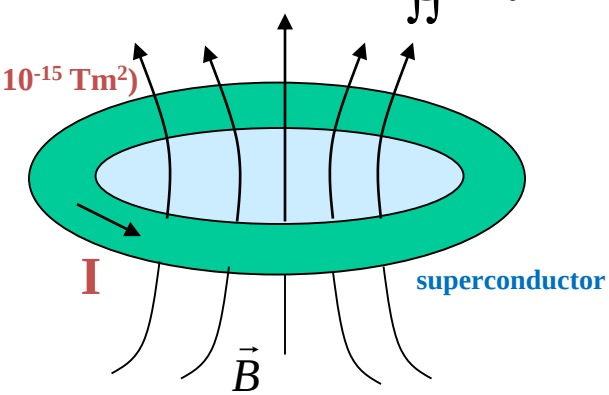
$$\Psi = |\Psi| e^{i\theta}$$

**Consequence:**

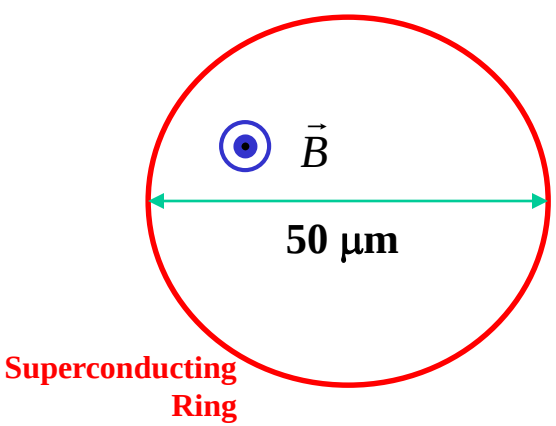
**Flux** $\Phi = \oiint \vec{B} \cdot d\vec{A}$

**Magnetic flux is quantized in units of $\Phi_0 = h/2e$** (= 2.07 x $10^{-15}$ $Tm^2$)

$$\Phi = n\,\Phi_0, \qquad n = \text{integer}$$




**Example of Flux Quantization**



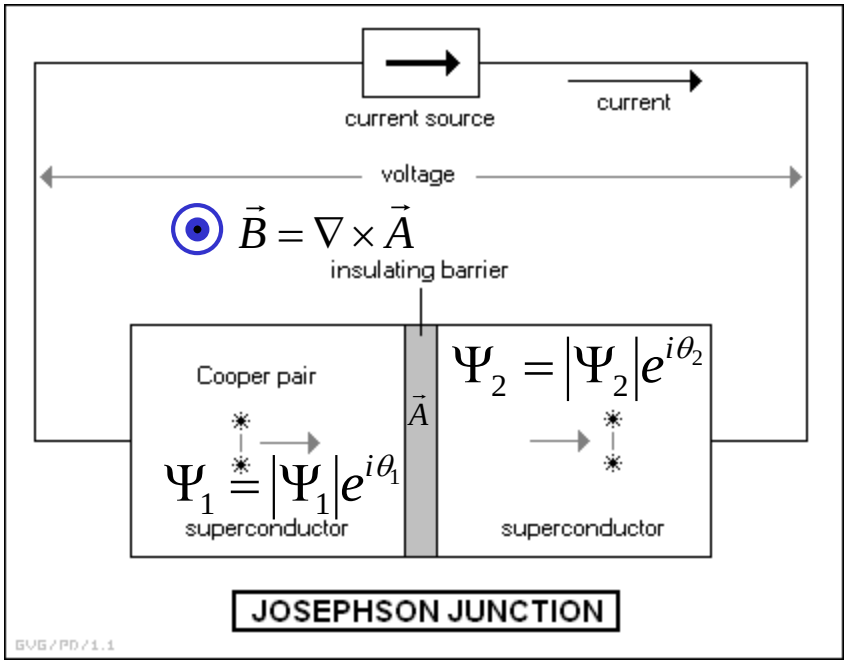

**One flux quantum in this loop requires a field of B = $\Phi_0$/Area = 1 μT**

**Earth's magnetic field $B_{earth}$ ~ 50 μT**

# Macroscopic Quantum Effects

Continued

## Josephson Effects (Tunneling of Cooper Pairs)




$$I = I_c \sin(\delta) \quad \textbf{DC}$$

$$\frac{d\delta}{dt} = \frac{2e}{\hbar} V \quad \textbf{AC}$$

**Circuit representation of a JJ**

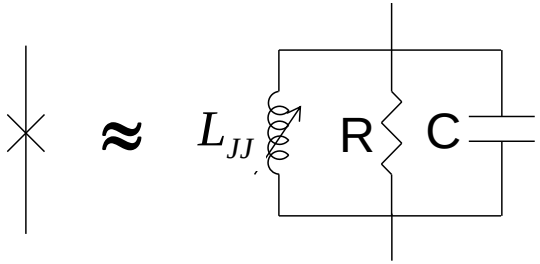
$$\delta = \theta_1 - \theta_2 - \frac{2e}{\hbar} \int_1^2 \vec{A} \bullet d\vec{l}$$

**Gauge-invariant phase difference**

$\delta(t)$ is the solution of a nonlinear diff. Eq.

# Why Josephson / SQUID Metamaterials?

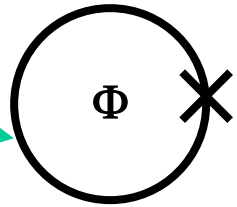
**Josephson Inductance is large, tunable and nonlinear**




Resistively and Capacitively Shunted Junction (RCSJ) Model

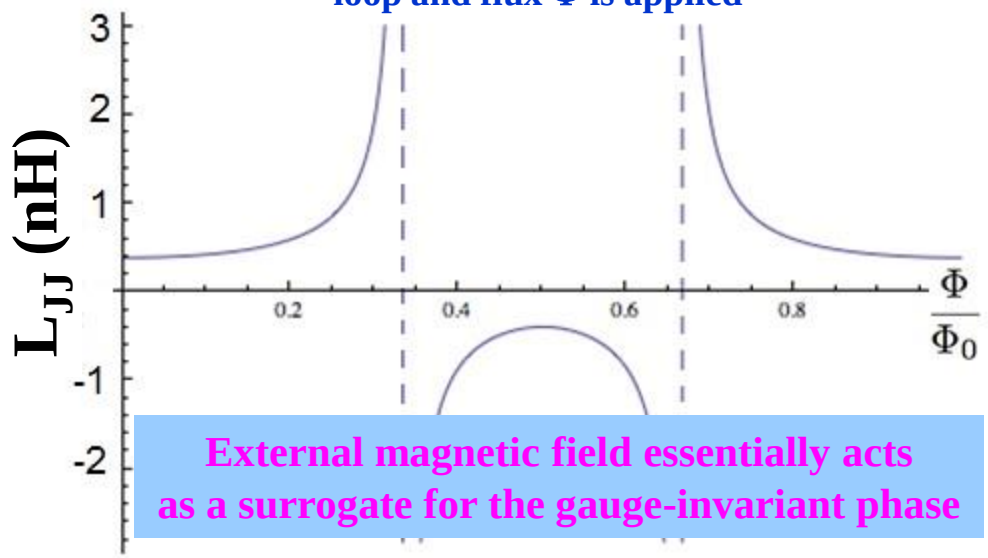
$$L_{JJ} = \frac{\Phi_0}{2\pi I_c \cos(\delta)}$$

Superconducting Loop




Combines the Josephson effects with flux quantization

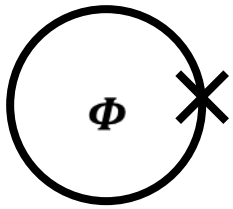
**When the JJ is incorporated into a loop and flux Φ is applied**



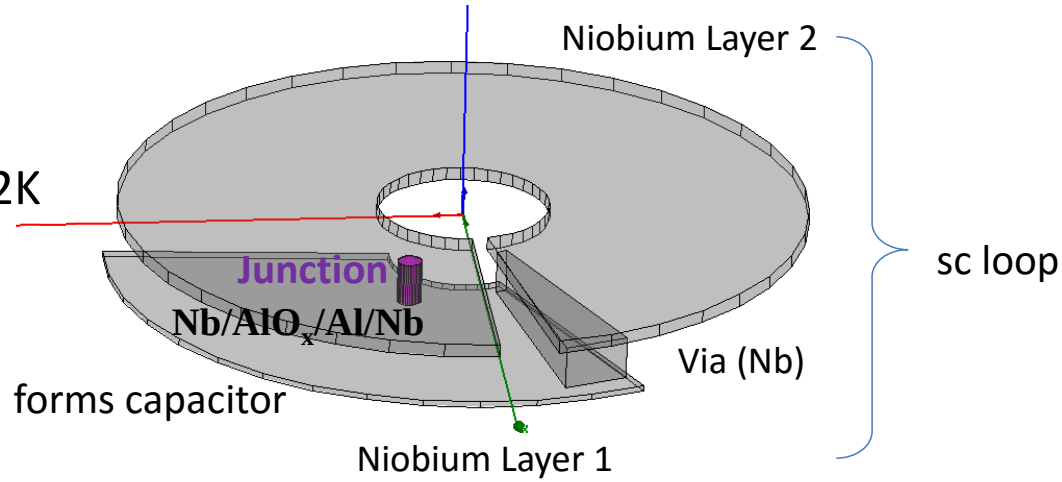

**External magnetic field essentially acts as a surrogate for the gauge-invariant phase**

# rf SQUID Meta-Atoms

rf SQUID




Nb: $T_c$ = 9.2K

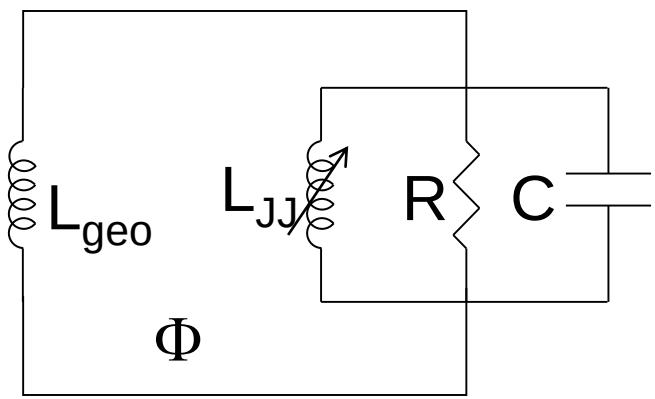



**A 'Macroscopic Quantum' Split-Ring Resonator**

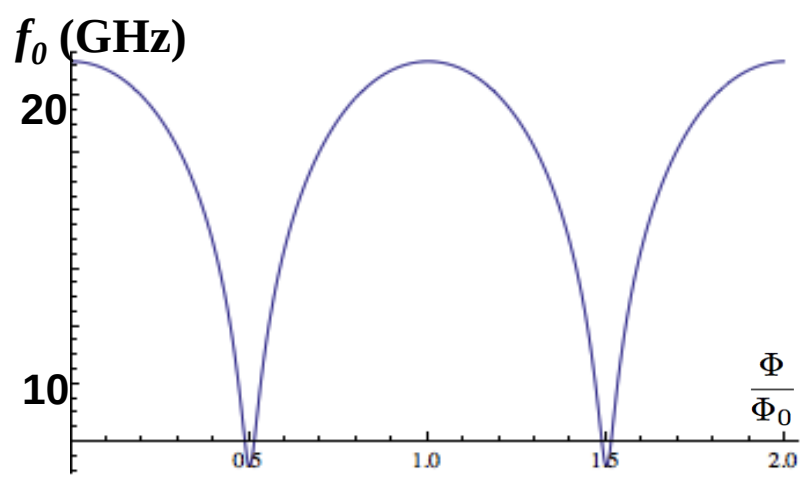
Overlap→ forms capacitor

SQUID = **S**uperconducting **QU**antum **I**nterference **D**evice

A self-resonant meta-atom with very nonlinear properties



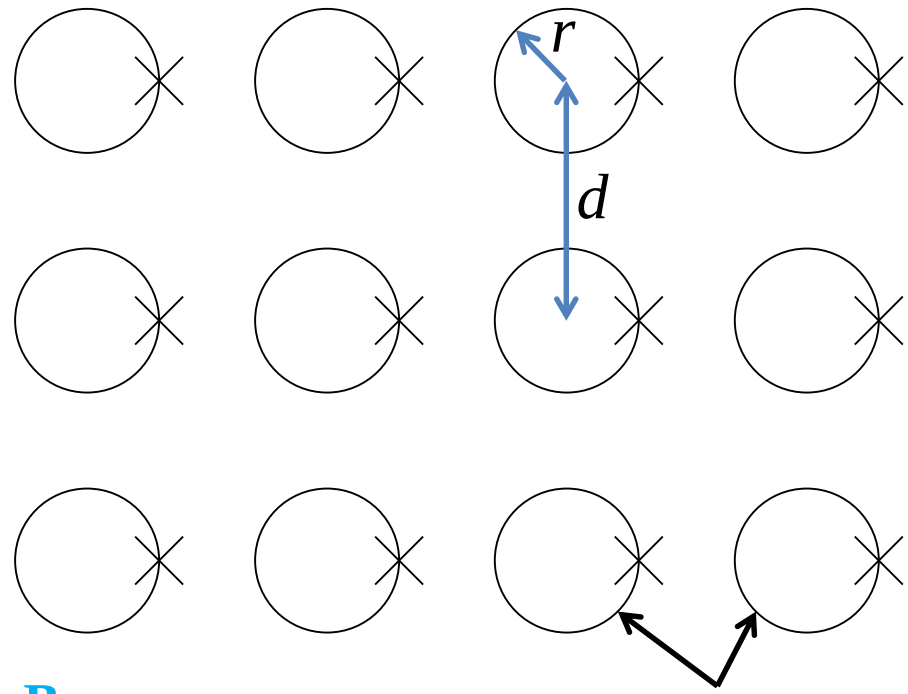

Resonant Frequency of rf SQUID



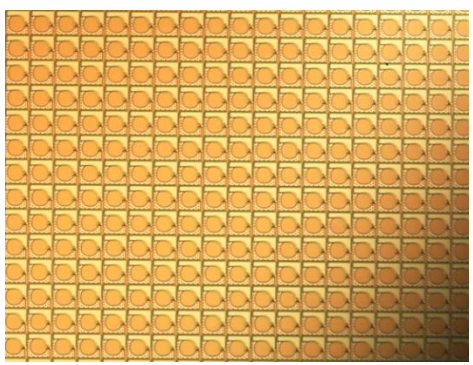
# rf SQUID Superconducting Metamaterial

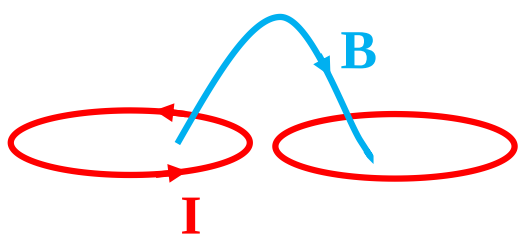
$$\lambda \gg r, d$$

- Low loss
- Small Size
  - $\lambda$ ~ 3 cm (~ 10 GHz)
  - $2r$ = 20 ~ 800 $\mu$m




rf SQUID meta-atoms






The SQUIDs interact by means of dipole – dipole coupling

→ Collective Behavior

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Nonlinear Metamaterials

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- Coherence
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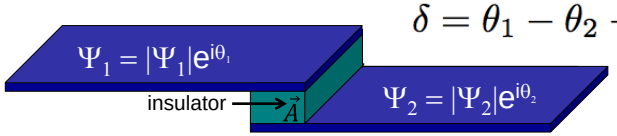
Conclusions

# Modeling RF SQUIDs: Nonlinearity

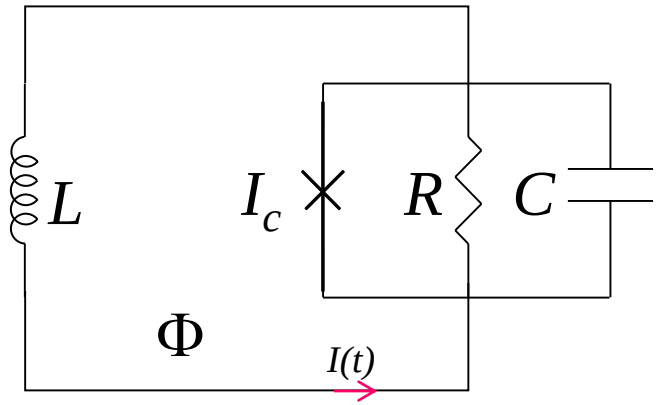
$$\Phi_{applied} + \Phi_{induced} = n\Phi_0$$

$$\Phi_{DC} + \Phi_{rf}\sin\omega t = \frac{\Phi_0\delta}{2\pi} + L\left(I_C \underline{\sin\delta} + \frac{L}{R}\frac{\Phi_0}{2\pi}\frac{d\delta}{dt} + C\frac{\Phi_0}{2\pi}\frac{d^2\delta}{dt^2}\right)$$

**Flux Quantization in the loop**




$$\delta = \theta_1 - \theta_2 - \frac{2e}{\hbar}\int_1^2 \vec{A}\cdot d\vec{l}$$




Flux quantization linearizes the rf SQUID at high excitation levels!

Solve for $\delta(t)$, calculate $L_{JJ}$, $I(t)$, $\mu_r(f)$, S-parameters, etc.

M. Trepanier, *et al*. Phys. Rev. X. 3, 041029 (2013)

# Why Nonlinear Metamaterials?

**Tunability** – Change the properties of the metamaterial after it has been fabricated – improved design flexibility

**Nonlinear response** – the metamaterial 'looks' different when probed at different intensity/power, frequency, direction, etc.

Engineer enhanced coupling to light

Self-induced nonlinear response

Some examples:

Tunable band-pass filter

Power limiters

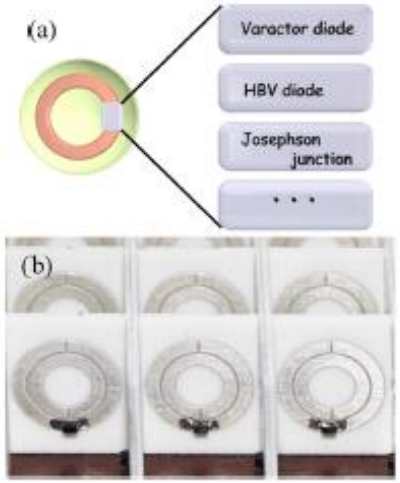
Nonlinear gain media

***Colloquium*: Nonlinear metamaterials**

Mikhail Lapine

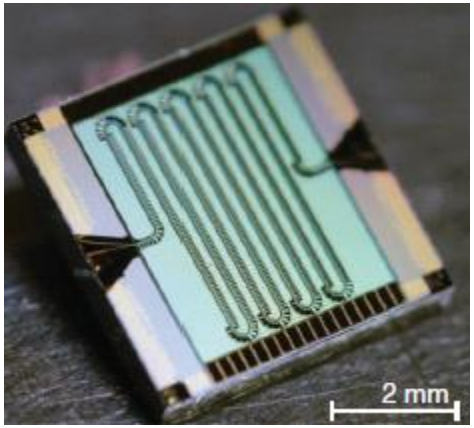
Rev. Mod. Phys., Vol. 86, No. 3, July–September 2014

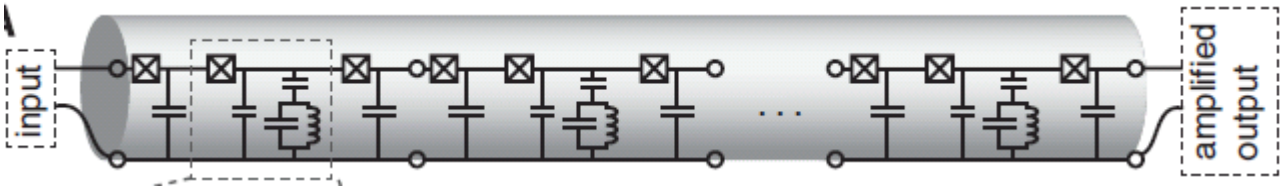
Ilya V. Shadrivov and Yuri S. Kivshar*



# Why Nonlinear Metamaterials?

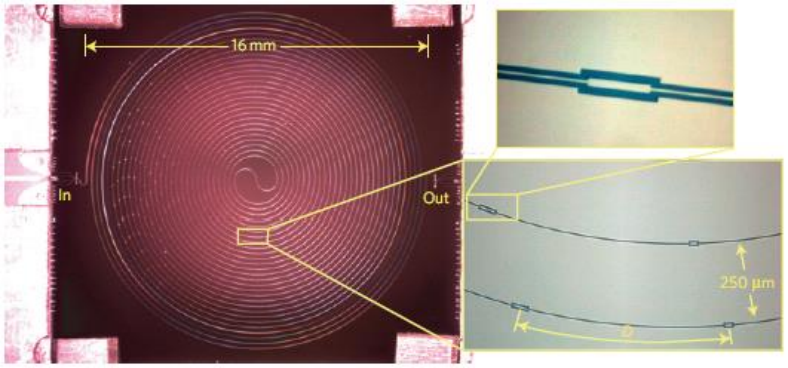
## Quantum-Limited Amplifiers




**A near-quantum-limited Josephson traveling-wave parametric amplifier**

C. Macklin,[1,2*] K. O'Brien,[3] D. Hover,[4] M. E. Schwartz,[1] V. Bolkhovsky,[4] X. Zhang,[3,5]
W. D. Oliver,[4,7] I. Siddiqi[1]




**A wideband, low-noise superconducting amplifier with high dynamic range**

Byeong Ho Eom[1], Peter K. Day[2]*, Henry G. LeDuc[2] and Jonas Zmuidzinas[1,2]

# Outline

What are Metamaterials?

What are SQUIDs?

Nonlinear Metamaterials

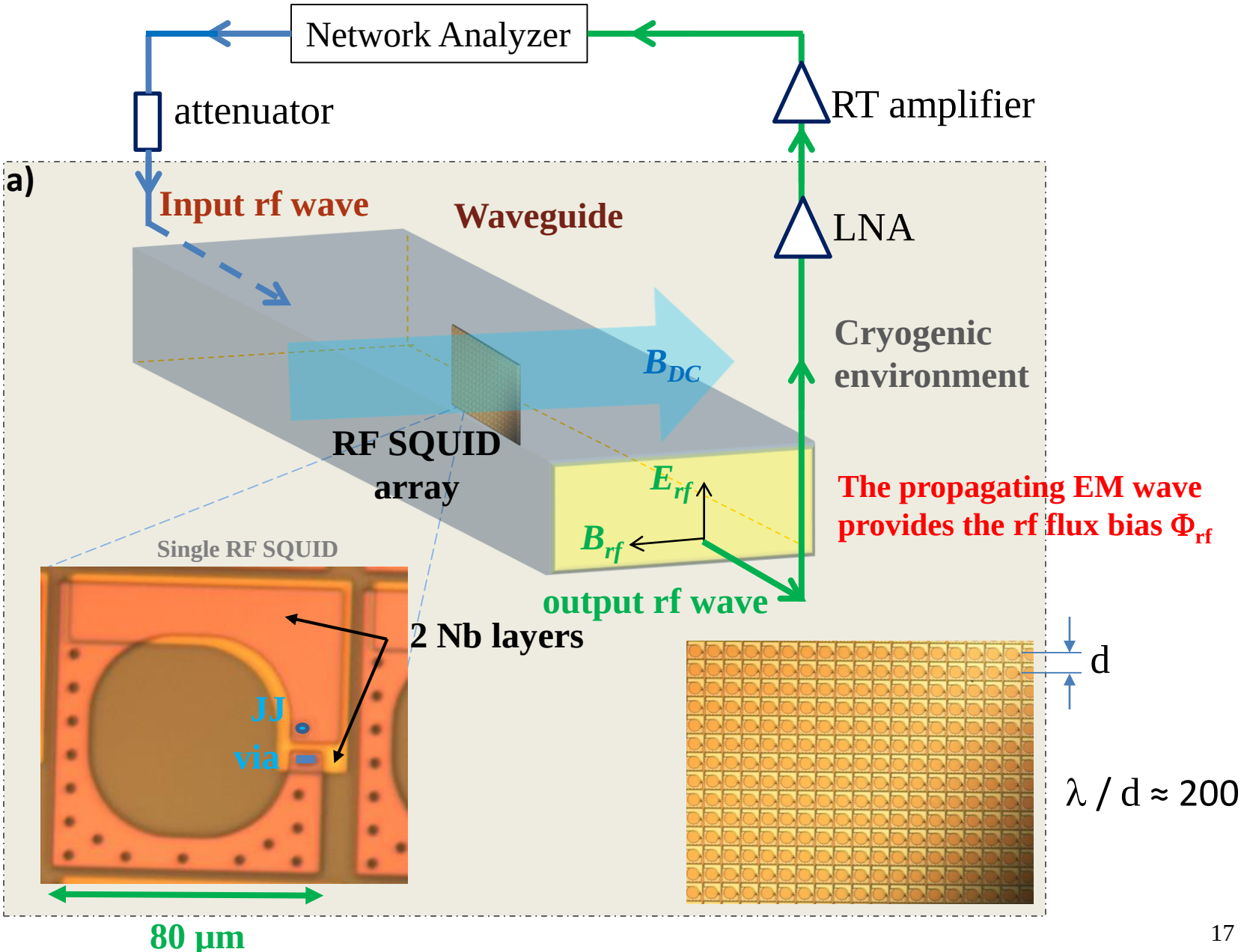
Experiments

- Transparency
- Coherence
- Intermodulation

Models of SQUID Dynamics

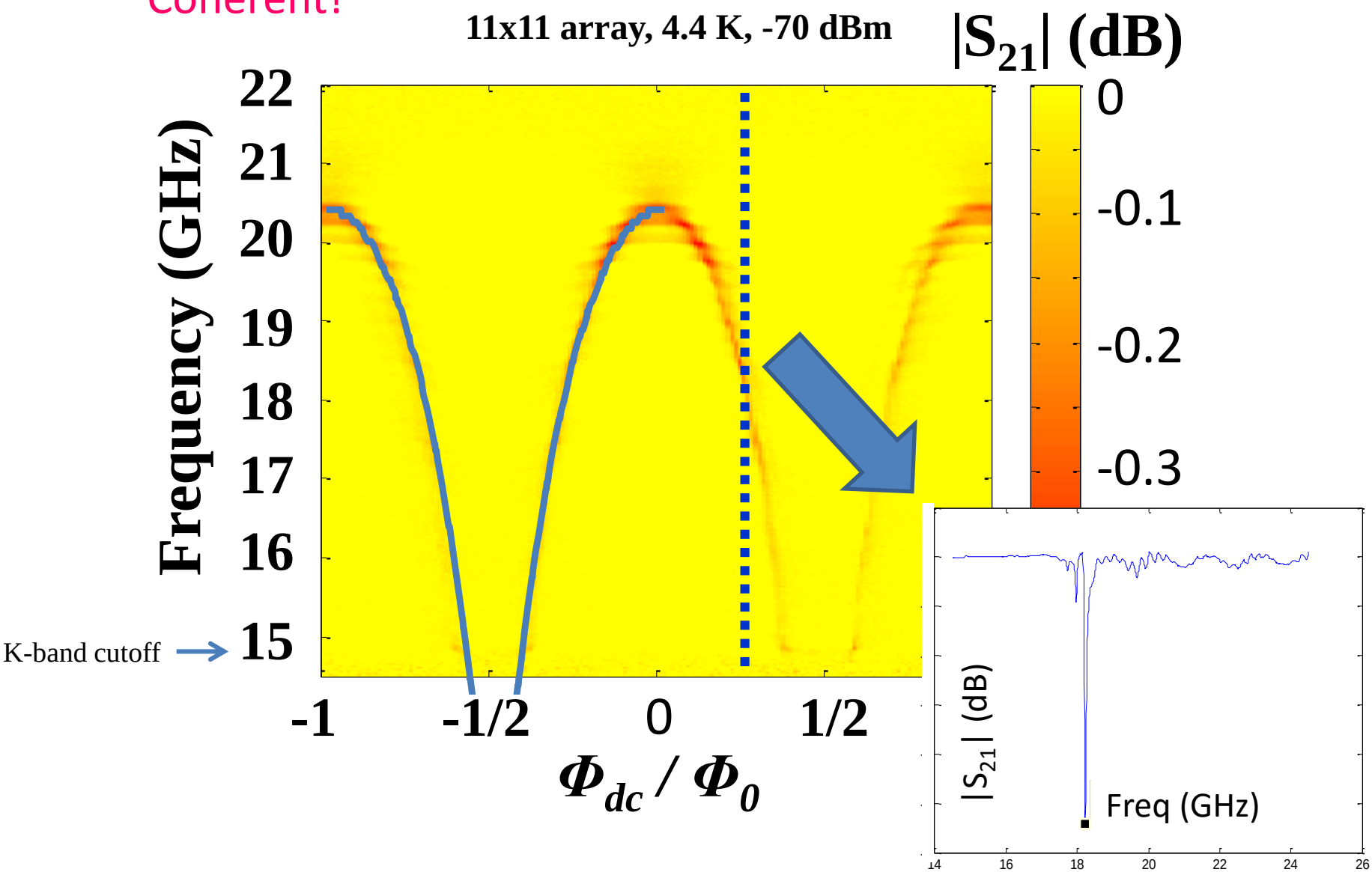
Conclusions

# Measurement of rf SQUID Metamaterial



# DC magnetic flux tuned resonance

Coherent!



# Overview

## Transparency in rf SQUID Metamaterials

Under certain circumstances the metamaterial will ‘disappear’

Due to bi-stability in rf SQUID

Analytical model for the onset of transparency

## Coherence of rf SQUID Metamaterial Collective Response

Disorder prevents all of the SQUIDs from responding coherently

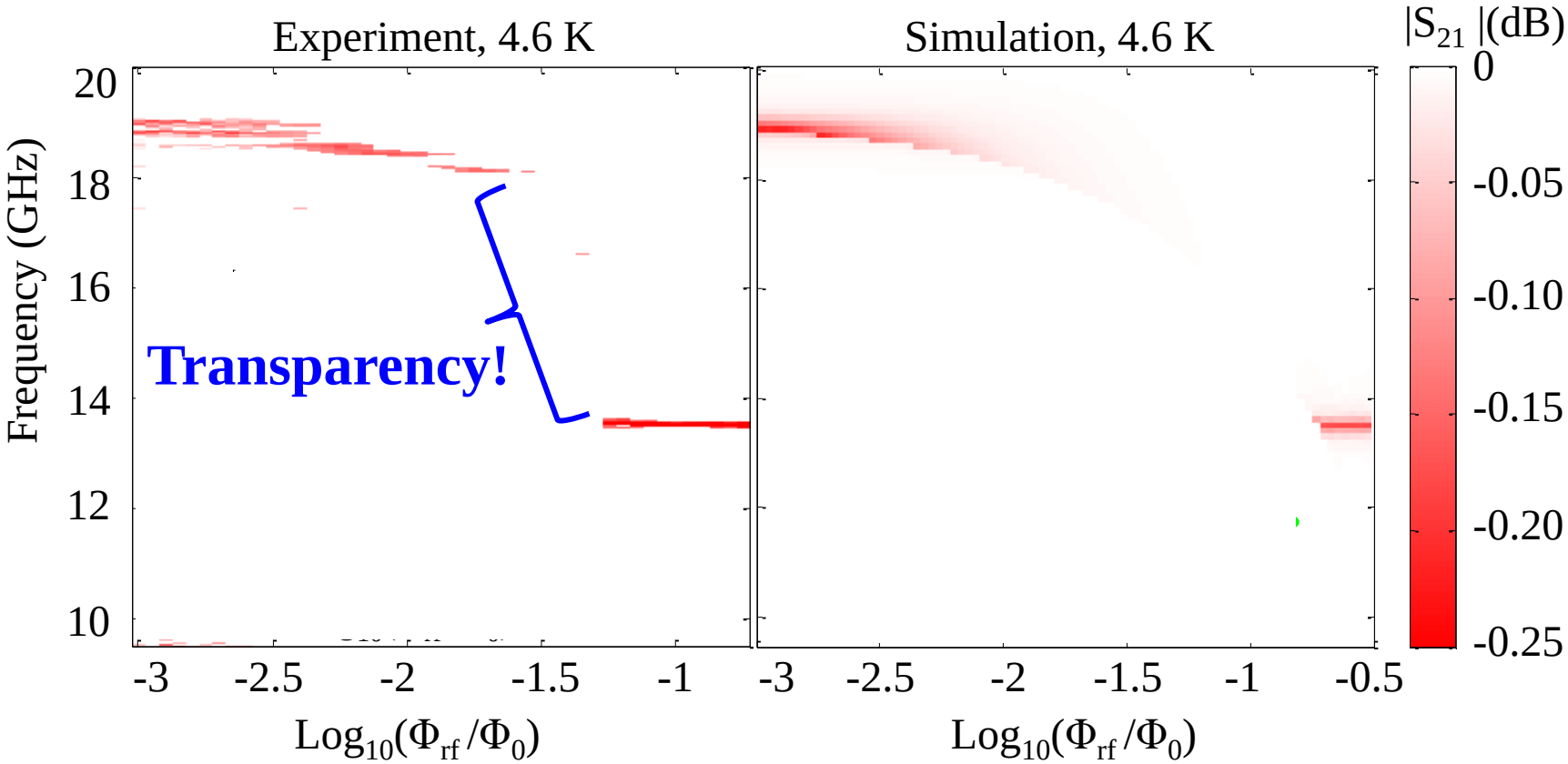
Study ways to overcome disorder to enhance coherence

## Intermodulation in rf SQUID Metamaterials

Characterizing the nonlinear properties

# Transparency in a SQUID Meta-atom

**Data and model agree that the single-SQUID "disappears" over a range of incident power**

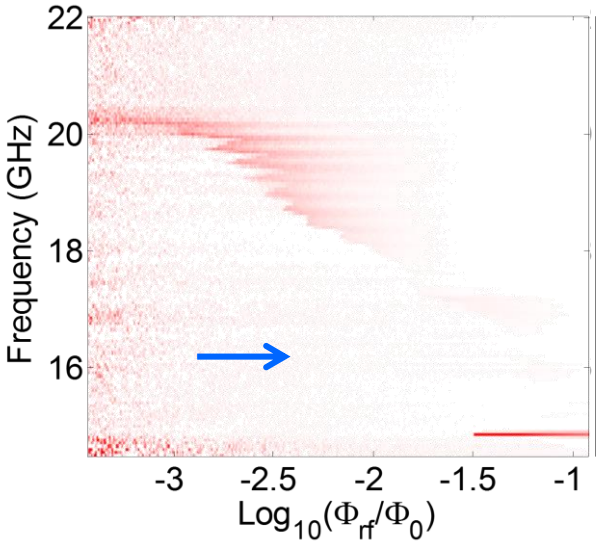



**Broad-band effect**

$$Transparency(\Phi_{rf}/\Phi_0) = 1 - \frac{S_{21}(\Phi_{rf}/\Phi_0)}{S_{21}(\Phi_{rf}/\Phi_0 = 10^{-3})}$$

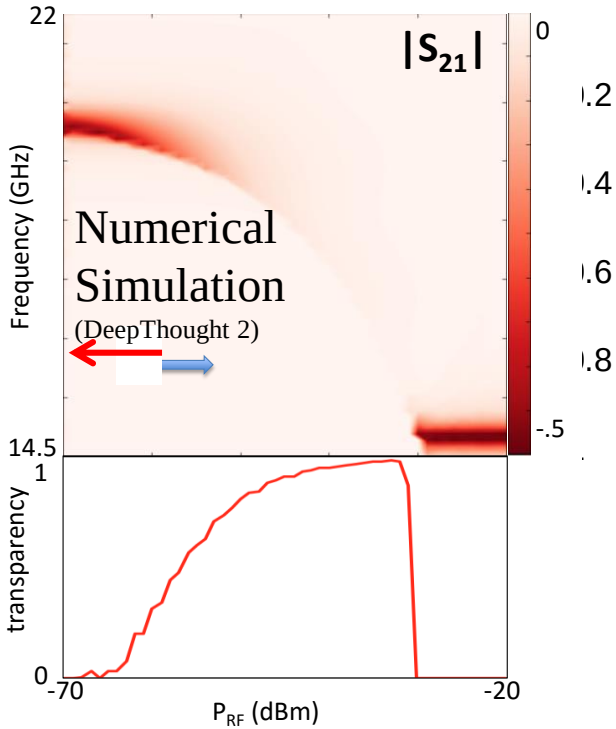
# Bi-Stability of Transparency of an 11x11 SQUID array Metamaterial

Experimental Data

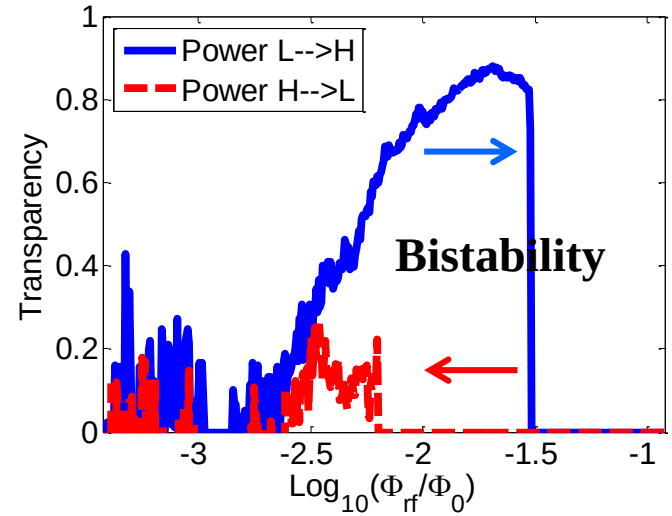


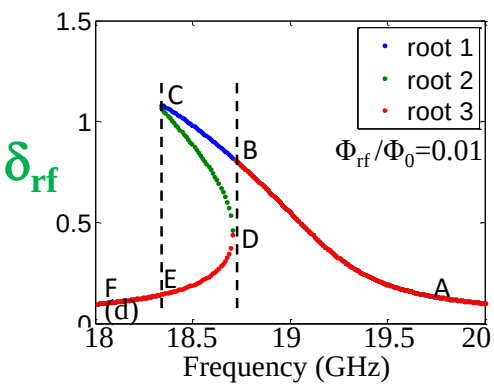

**Transmission**

$|S_{21}\,(dB)|$




T = 4.6 K




D. Zhang, *et al.*, Phys. Rev. X 5, 041045 (2015)

$$\sin\delta \cong \delta - \delta^3/3!$$

Transparency shows up when bistability begins. → Evident from Duffing Oscillator

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Under certain circumstances the metamaterial will ‘disappear’

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Study ways to overcome disorder to enhance coherence

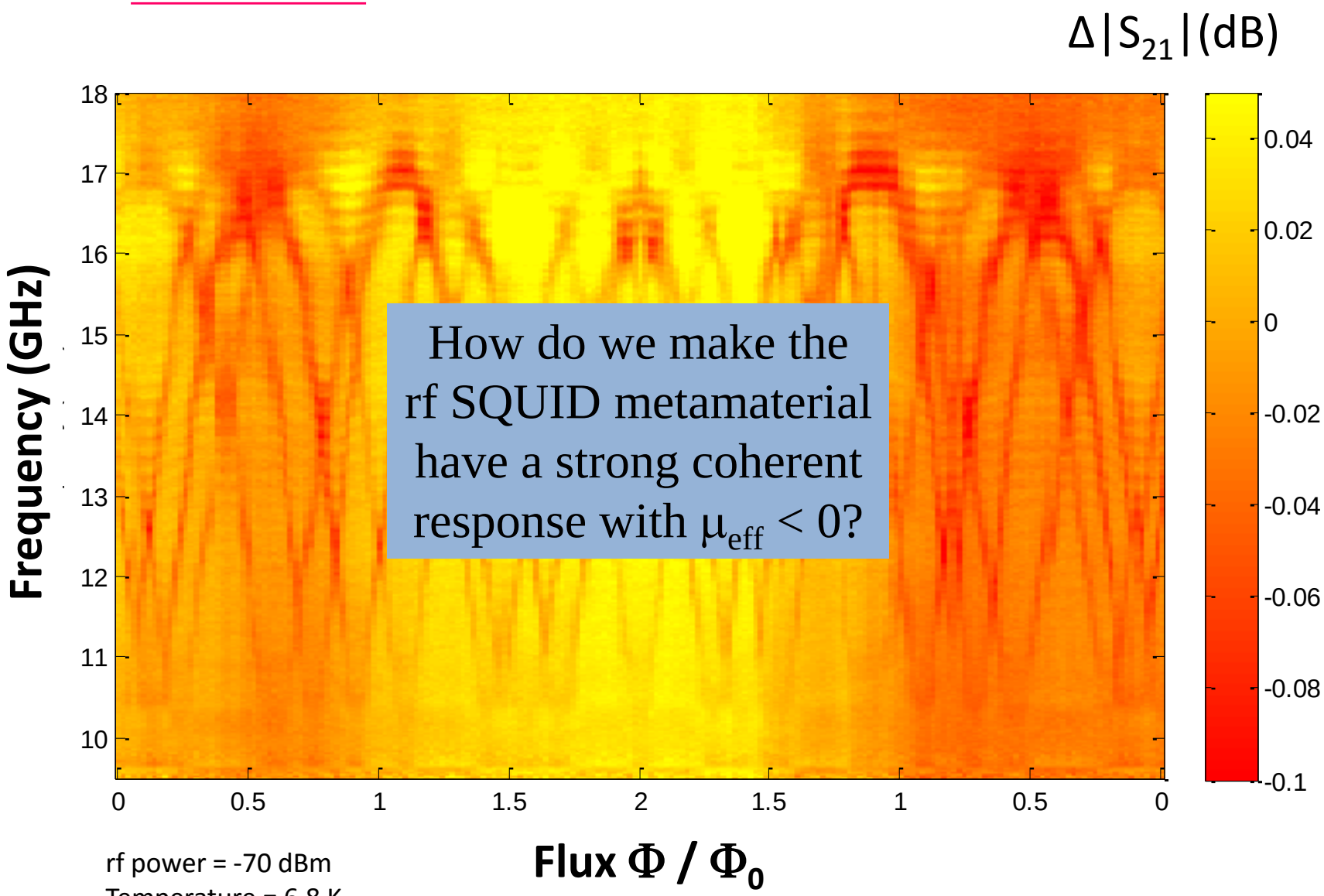
## Intermodulation in rf SQUID Metamaterials

Characterizing the nonlinear properties

# Flux-Tuning of 2x2 SQUID Array

Incoherent!

**Insufficient Magnetic Shielding**




How do we make the rf SQUID metamaterial have a strong coherent response with $\mu_{eff} < 0$?

rf power = -70 dBm
Temperature = 6.8 K

# Solving for $\delta(t)$: SQUID Arrays

## One SQUID

$$\frac{2\pi}{\Phi_0}(\Phi_{dc} + \Phi_{rf}\sin\Omega\tau) = \delta + \beta_{rf}\sin\delta + \gamma\delta' + \delta''$$

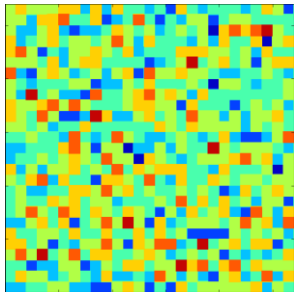
**Once $\delta(t)$ is known all observable quantities can be calculated**

## SQUID Array

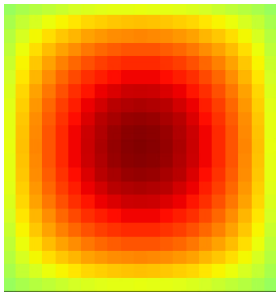
**Nonlinearity**

$$\frac{2\pi}{\Phi_0}(\hat{\Phi}_{dc} + \hat{\Phi}_{rf}\sin\Omega\tau) = \hat{\delta} + \bar{\bar{\kappa}}(\beta_{rf}\sin\hat{\delta} + \gamma\hat{\delta}' + \hat{\delta}'')$$

Snapshot of δ(t) in array



vs.



$$\kappa = \frac{M}{L} \quad \begin{matrix}\text{mutual inductance}\\ \text{self inductance}\end{matrix}$$

$\delta(t)$ solutions are nearly sinusoidal:

$$\delta_j(t) \sim A_j e^{i(\Omega\tau + \phi_j)}$$

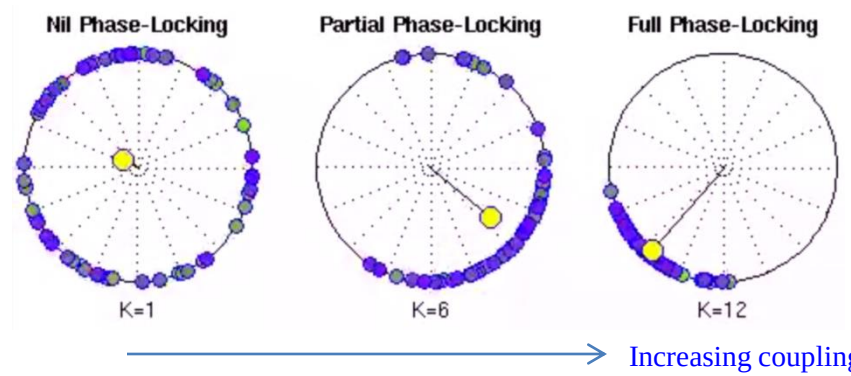
# Quantifying Coherence

- Kuramoto order parameter*

$$re^{i\psi} = \frac{1}{N}\sum_{j}^{N} e^{i\phi_j}$$

Phase of $\delta_j(t)$ oscillation

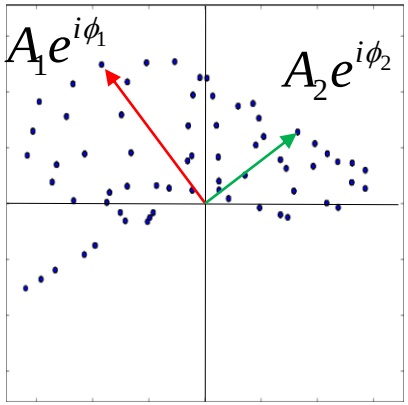
  - $r = 1$: complete coherence
  - $0 < r < 1$: partial coherence
  - $r = 0$: complete incoherence




Wikipedia

**For SQUID metamaterials**:

$$\delta_j(t) \sim A_j e^{i(\Omega\tau + \phi_j)}$$




$$r_A = \left| \frac{\sum_{j}^{N} A_j e^{i\phi_j}}{\sum_{j}^{N} A_j} \right|$$

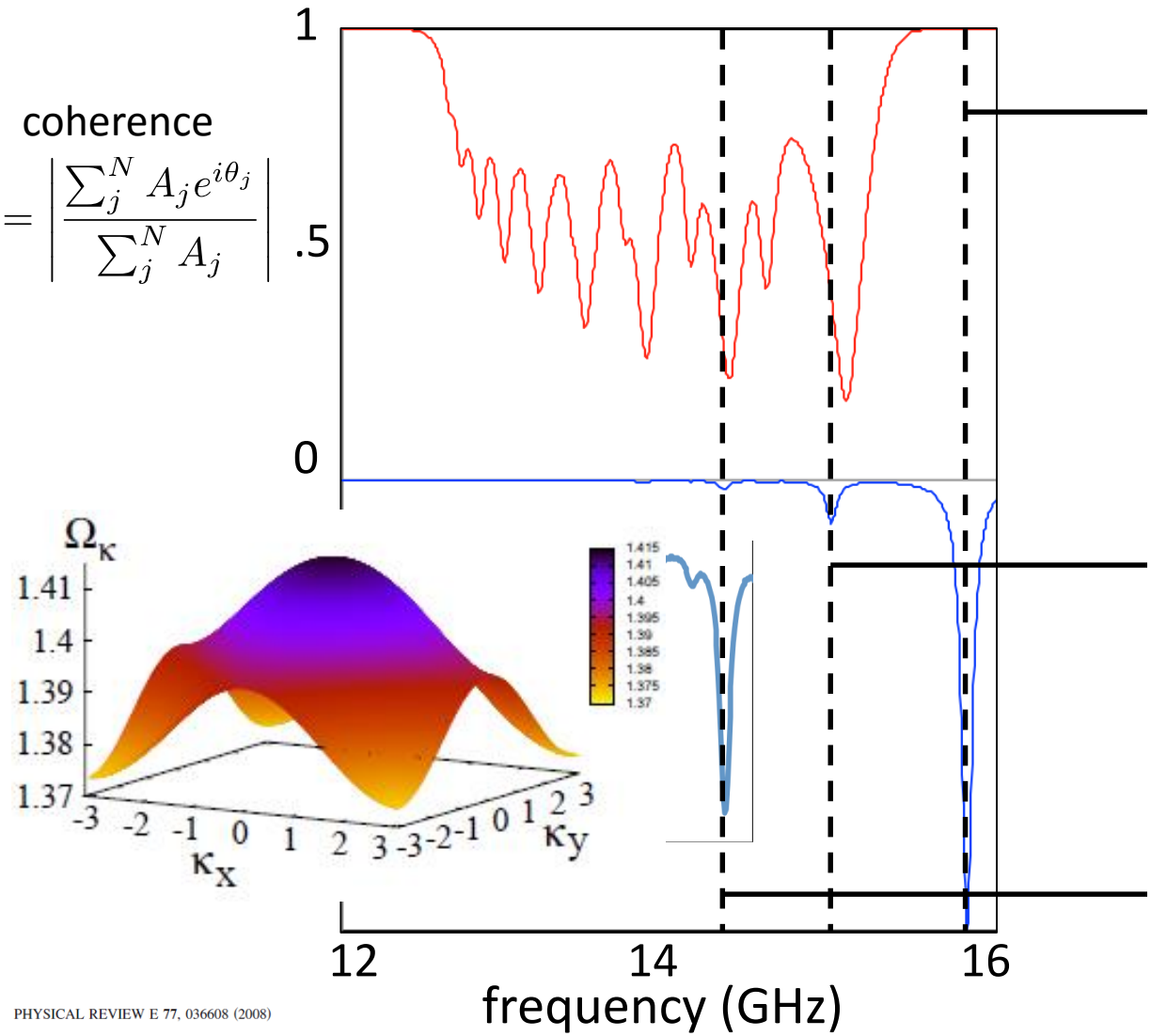
* Y. Kuramoto, *Chemical Oscillations, Waves, and Turbulence*. New York, NY: Springer-Verlag (1984).

# Coupling Introduces Magneto-Inductive Modes

Simulation results for strong coupling (21 x 21 array), low power (linear)

coherence

$$r_A = \left| \frac{\sum_j^N A_j e^{i\theta_j}}{\sum_j^N A_j} \right|$$




$\delta_j(t)$

2.5

0

-2.5

.6

0

-.6

.3

0

-.3

Lazarides, arXiv:1310.5445

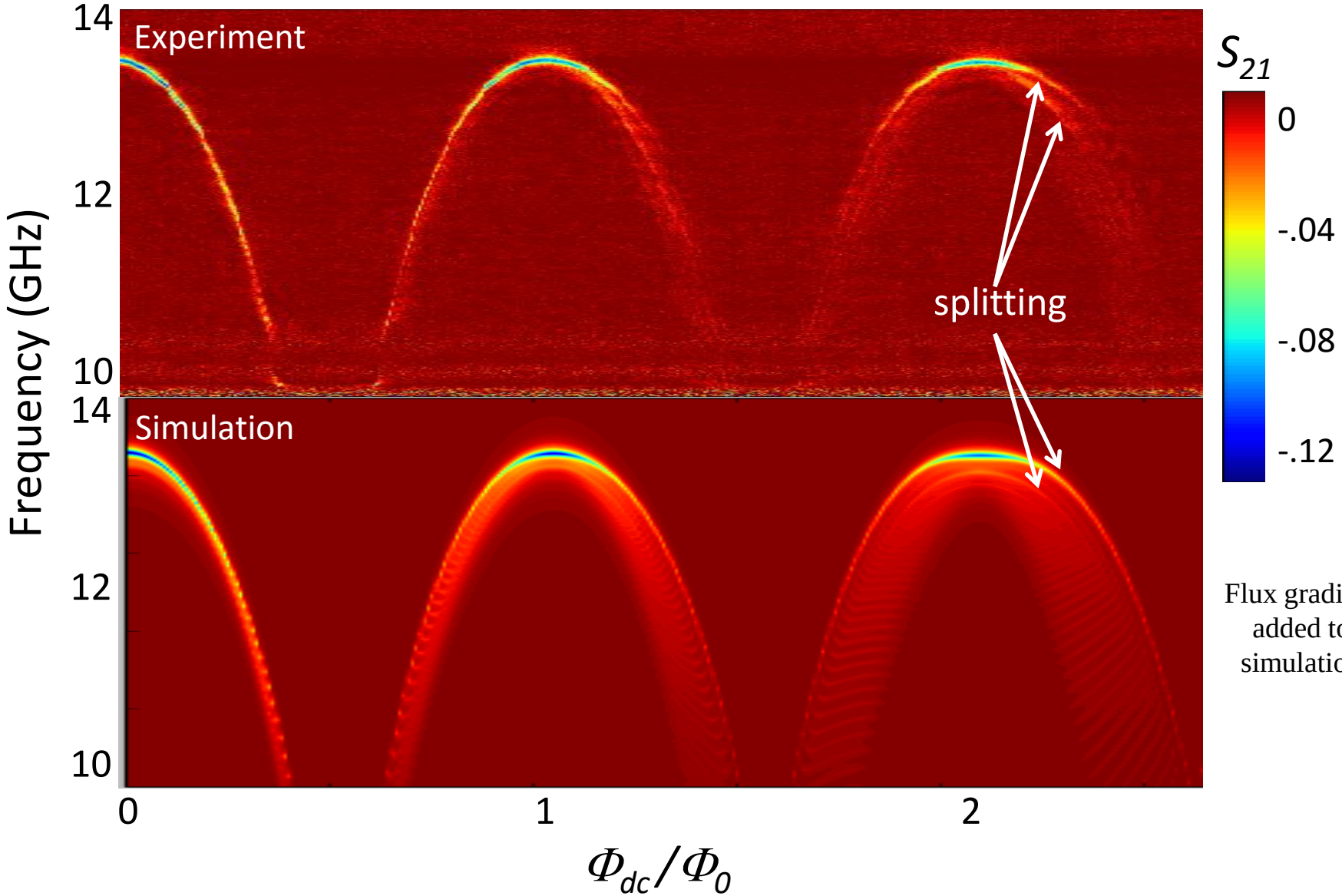
PHYSICAL REVIEW E 77, 036608 (2008)

**Magnetoinductive breathers in metamaterials**

M. Eleftheriou,[1,2] N. Lazarides,[1,3] and G. P. Tsironis[1]

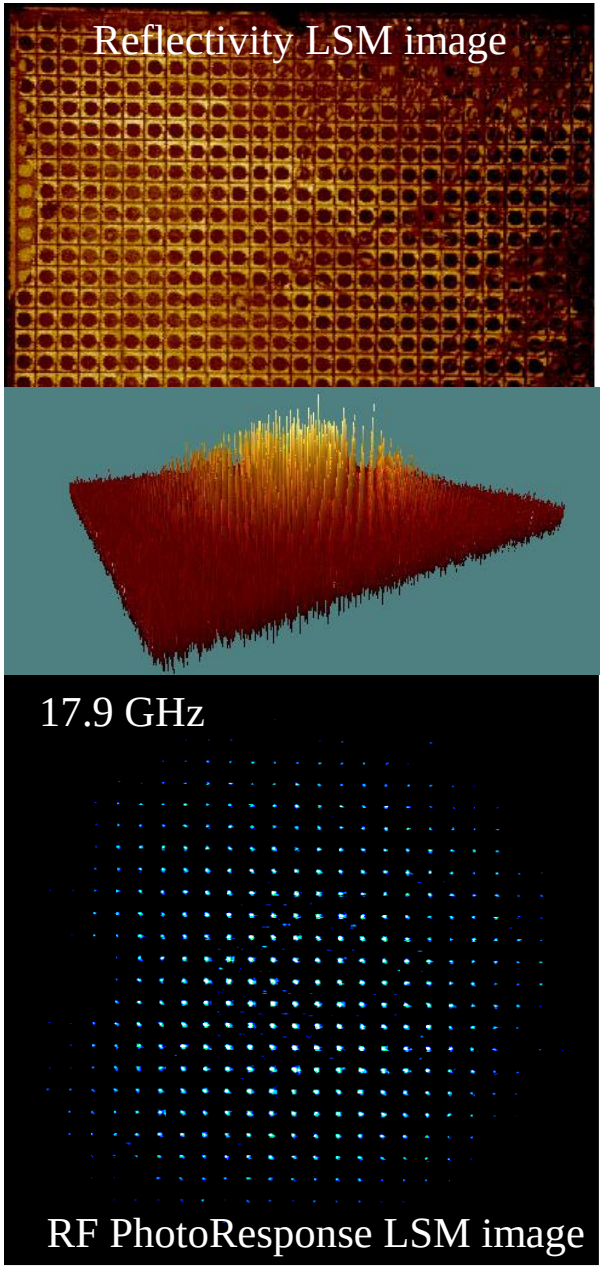
# Experimental Evidence for MI Modes

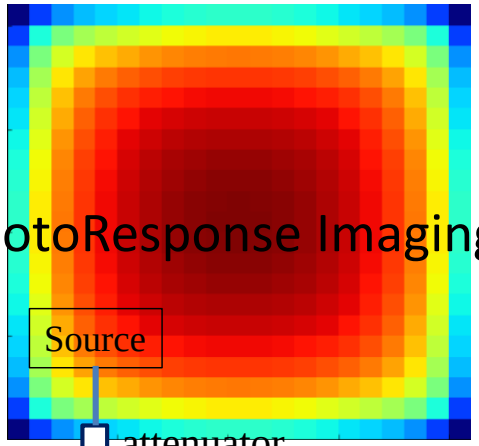
21 x 21 rf SQUID metamaterial (κ = 0.02 [strong coupling], linear response limit)




Flux gradient added to simulation

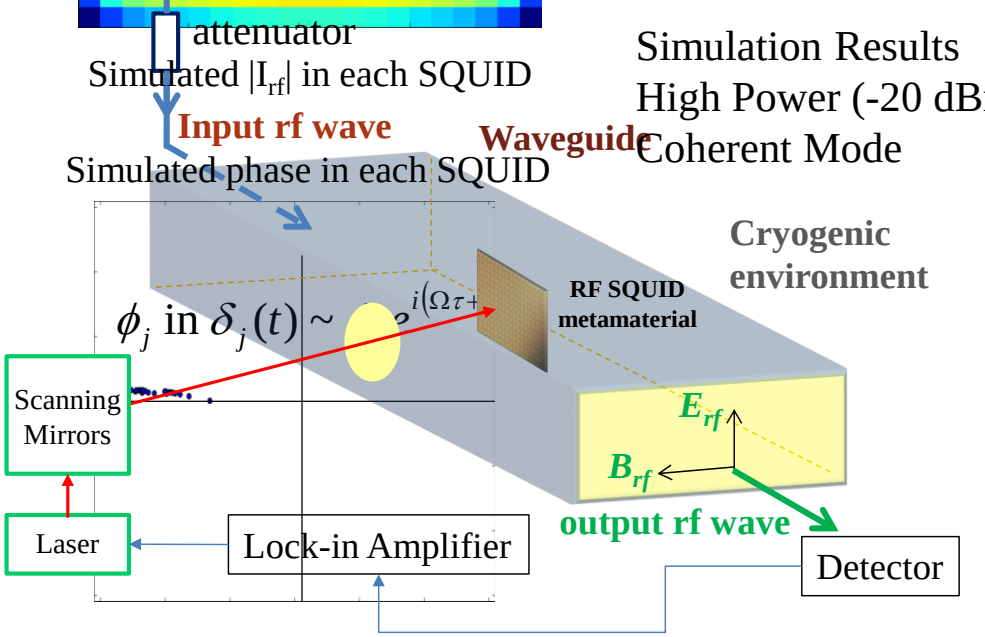
# Imaging rf SQUID Metamaterial High-Power Coherent Mode



27 x 27 rf SQUID metamaterial

PhotoResponse Imaging of rf SQUID Metamaterial




Simulation Results
High Power (-20 dBm)
Coherent Mode

**A. P. Zhuravel**, P. Jung, S. Anlage, A. Ustinov, KIT, Germany

# How to Re-Establish Coherence in the Presence of Disorder?

dc flux gradients are the biggest culprit in experiments

**1. Increased Coupling Between Meta-Atoms**

**2. Increasing rf Flux / Power**

**3. Increasing Temperature**

To decrease the SQUID dc flux sensitivity

By using these strategies to maximize coherence and taking steps to minimize uneven dc flux bias, rf SQUID metamaterials can be tuned coherently over a broad frequency range

# Overview

## Transparency in rf SQUID Metamaterials

Under certain circumstances the metamaterial will ‘disappear’

Due to bi-stability in rf SQUID

Analytical model for the onset of transparency

## Coherence of rf SQUID Metamaterial Collective Response

Disorder prevents all of the SQUIDs from responding coherently

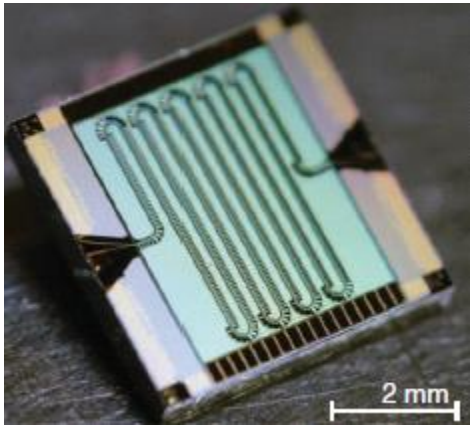
Study ways to overcome disorder to enhance coherence

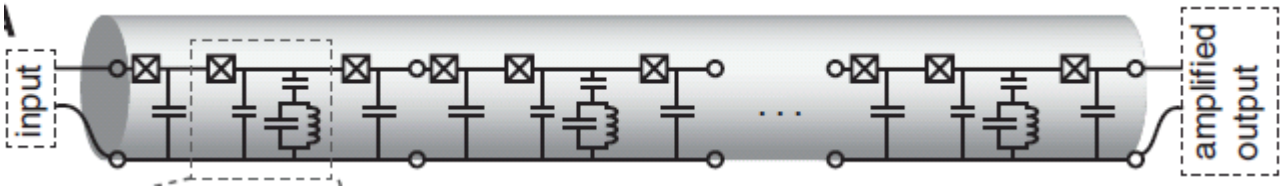
## Intermodulation in rf SQUID Metamaterials

Characterizing the nonlinear properties

# Why Nonlinear Metamaterials?

## Quantum-Limited Amplifiers

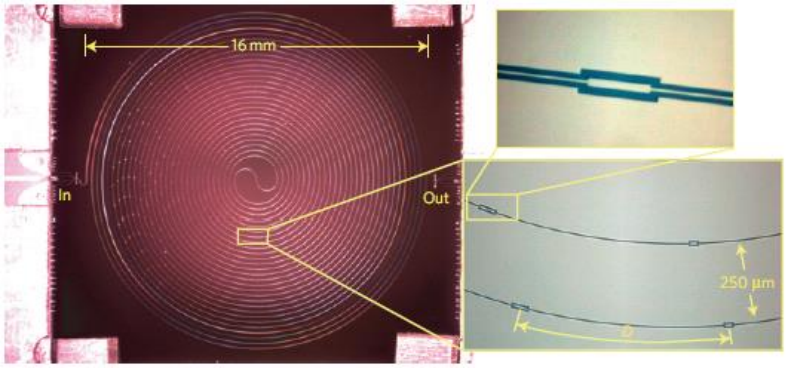



**A near-quantum-limited Josephson traveling-wave parametric amplifier**

C. Macklin,[1,2*] K. O'Brien,[3] D. Hover,[4] M. E. Schwartz,[1] V. Bolkhovsky,[4] X. Zhang,[3,5] W. D. Oliver,[4,7] I. Siddiqi[1]

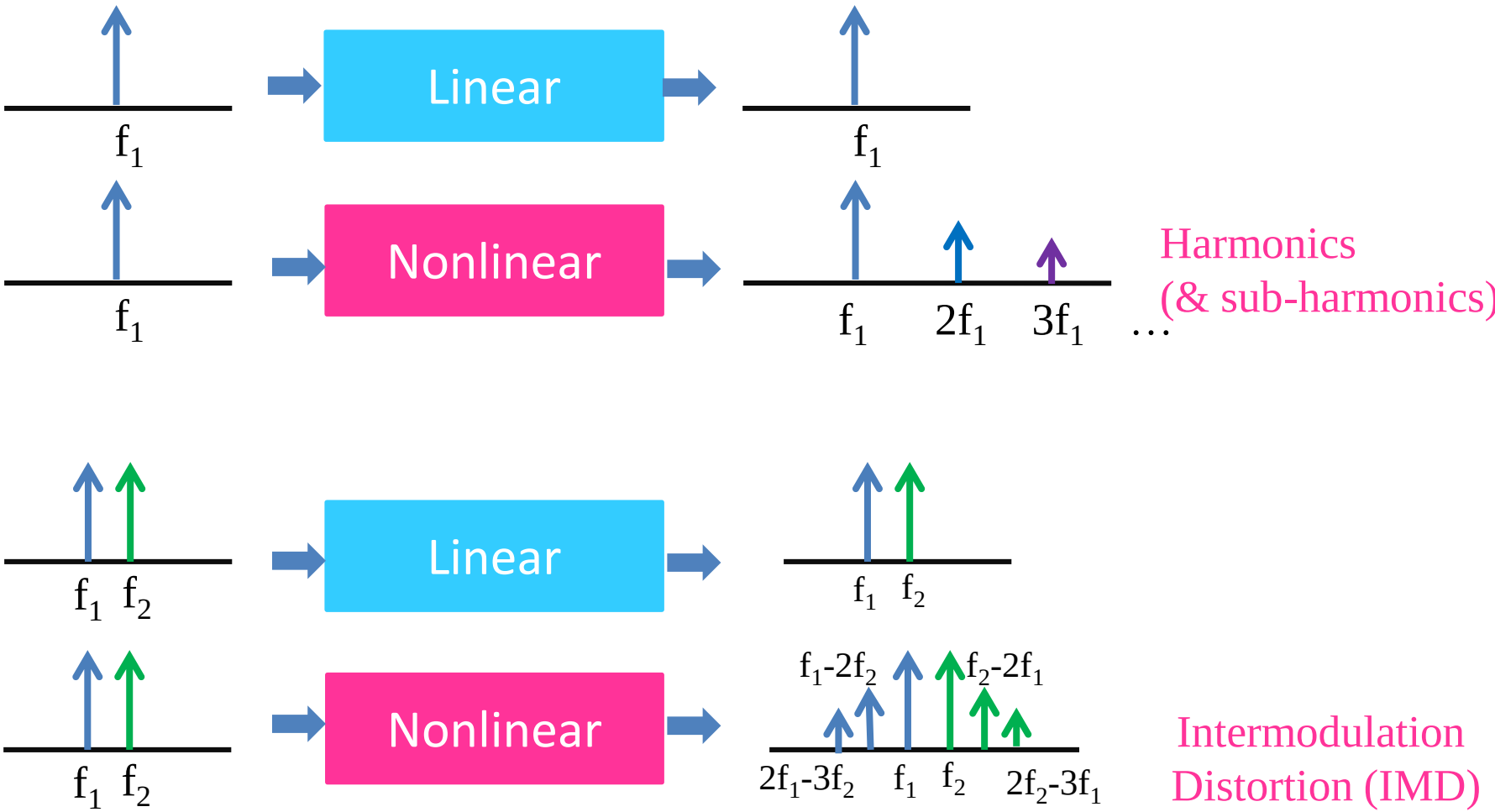
**Two input tones:**
**weak (to be amplified)**
**strong (pump signal)**




**A wideband, low-noise superconducting amplifier with high dynamic range**

Byeong Ho Eom[1], Peter K. Day[2]*, Henry G. LeDuc[2] and Jonas Zmuidzinas[1,2]

# Linear systems vs. Nonlinear systems

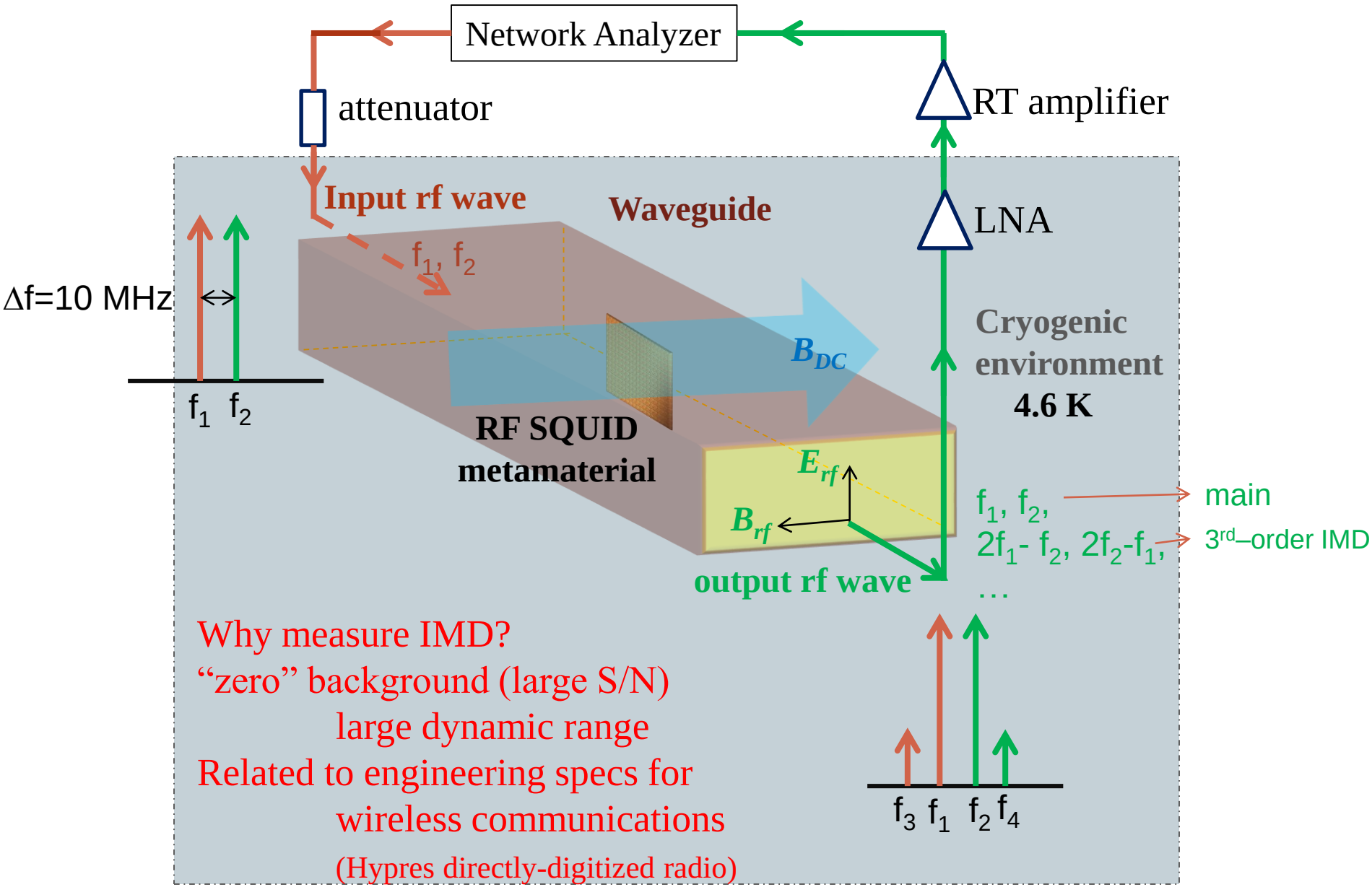



$f_1$, $2f_1$, $3f_1$, …

$f_2$, $2f_2$, $3f_2$, …

Harmonics

# Measurement of Intermodulation in rf SQUID Metamaterials




Why measure IMD?

"zero" background (large S/N)

large dynamic range

Related to engineering specs for

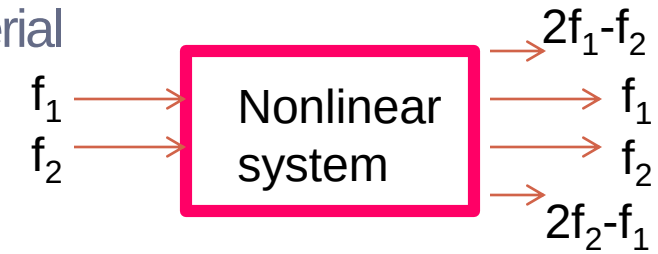
wireless communications

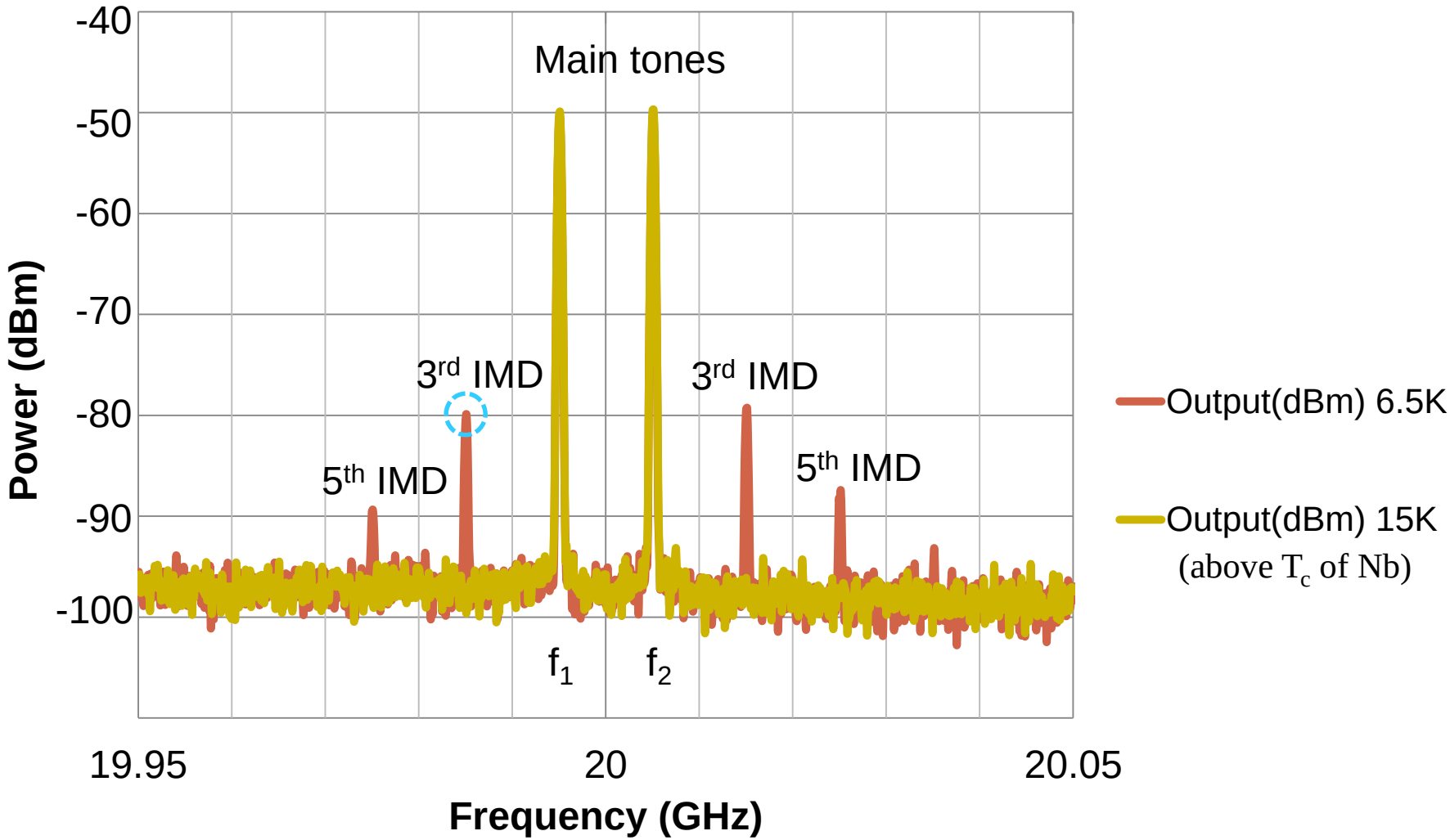
(Hypres directly-digitized radio)

Intermodulation measurement, 27x27 SQUID metamaterial

Resonance: 21.5 GHz

Tone power: -45 dBm

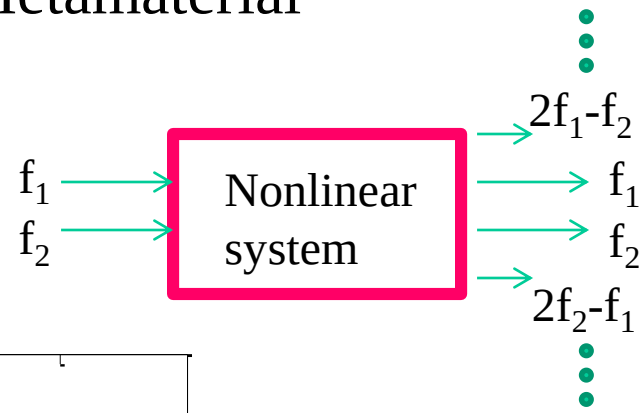


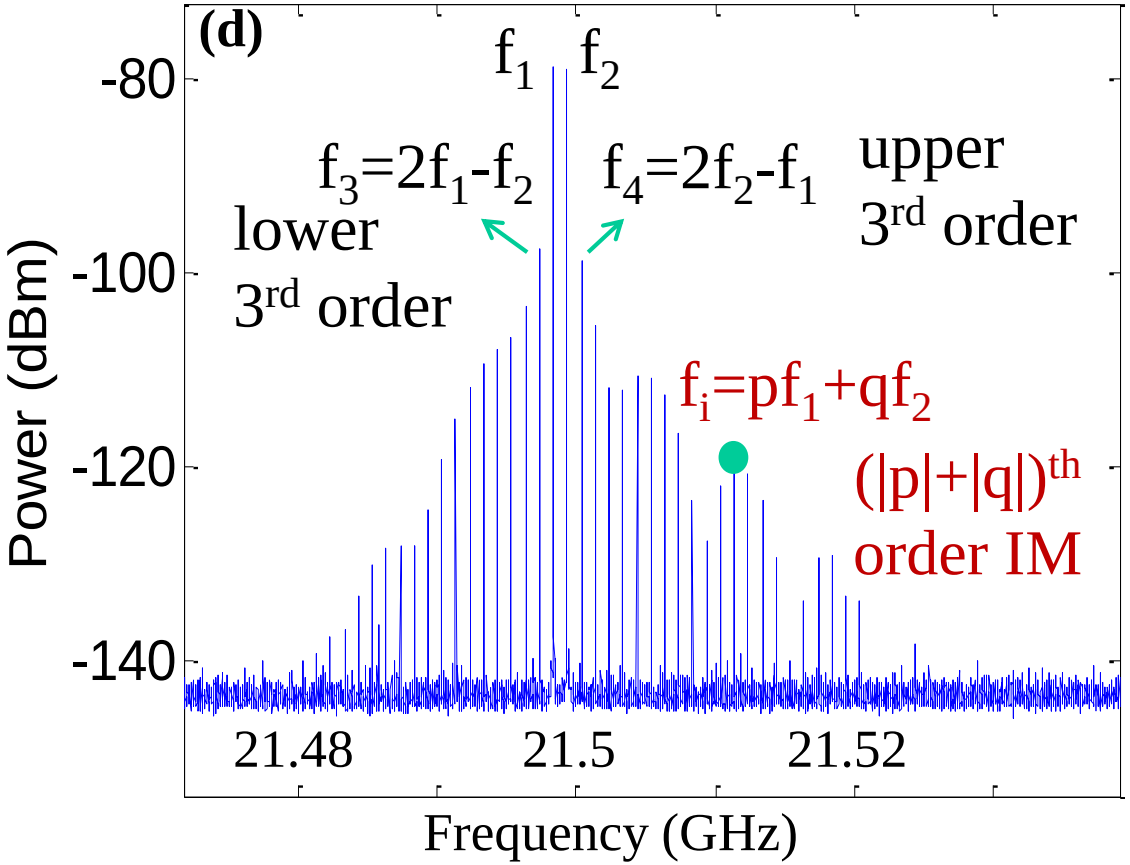



# IMD Spectrum of a 27 x 27 SQUID Metamaterial

- very rich generation of IMD

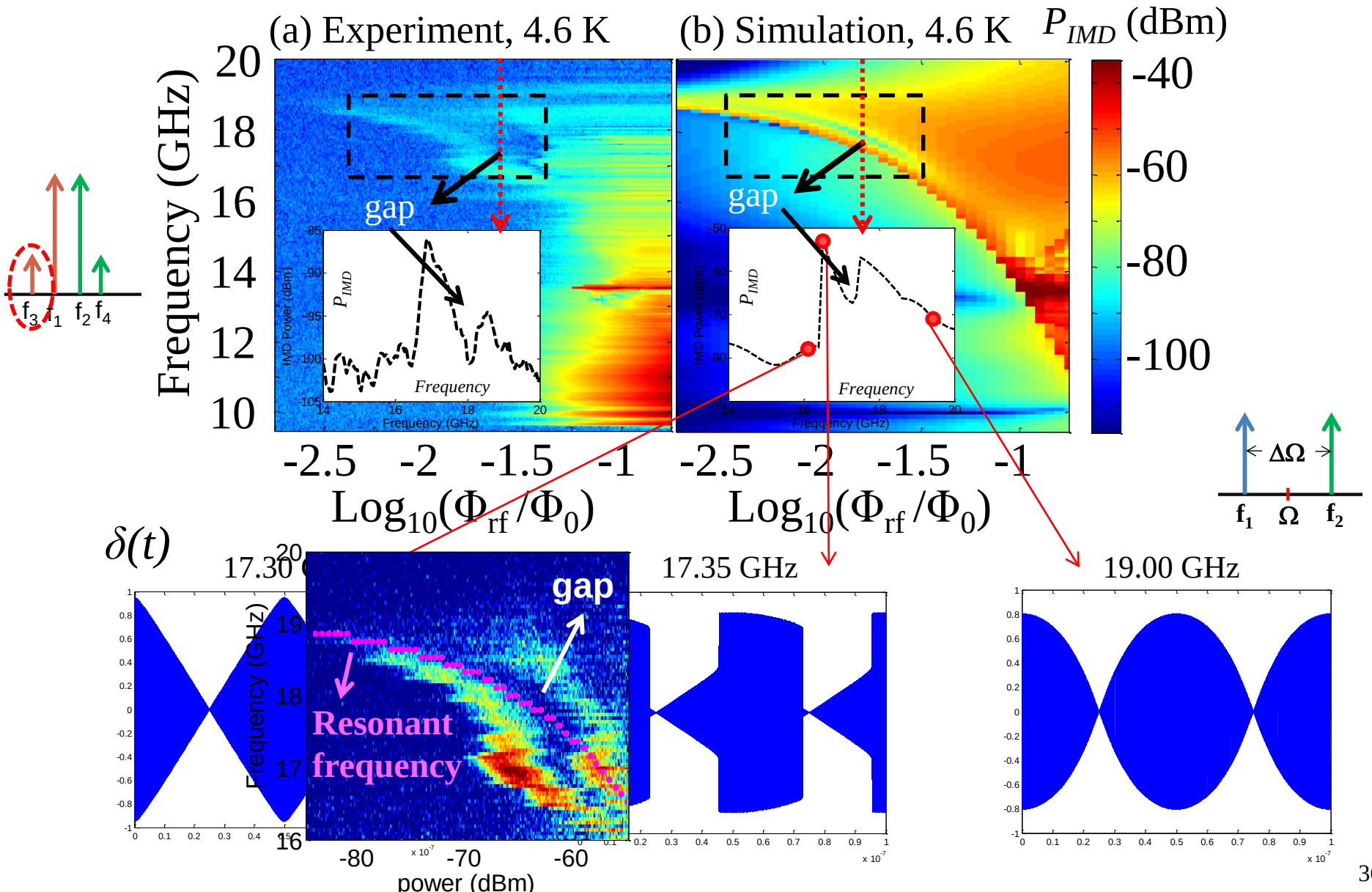
Out to 51st-order




Fixed input power, fixed center frequency for the two tones

# $P_{IMD}$ Generation : Experiment and Simulation



# Some Observations About The Data

Sharp onset of $P_{IMD}$ as a function of increasing frequency at fixed $\Phi_{rf}$

The onset of $P_{IMD}$ as a function of increasing frequency coincides with the onset of jumps between two bi-stable states

There is a prominent gap in the 3rd-order IMD as a function of frequency and rf flux. A very robust feature seen in single SQUIDs and metamaterials.

This gap occurs very close to the resonant frequency

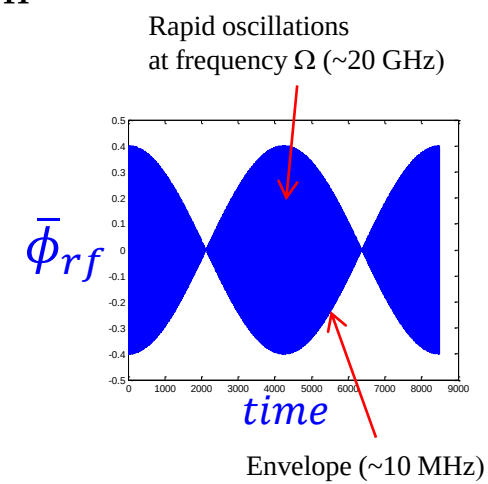
# Analytical Treatment of IMD

single SQUID

Slowly varying envelope approximation

$$\frac{d^2\delta}{d\tau^2} + \frac{1}{Q}\frac{d\delta}{d\tau} + \delta + \beta_{rf}\sin\delta = \phi_{dc} + \bar{\phi}_{rf}\sin(\Omega\tau)$$

Two tone: $\bar{\phi}_{rf}$ is slowly varying envelope, the period depends on the tone distance frequency.



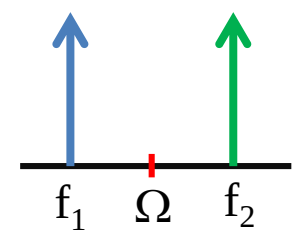

Slowly varying in time

Steady-state response: $\delta = \bar{\delta} + \tilde{\delta}\sin(\Omega\tau + \theta)$

Expand: $\sin\delta = \sin\bar{\delta}\, J_0(\tilde{\delta}) + 2J_1(\tilde{\delta})\sin(\Omega\tau + \theta) + H.O.T.$

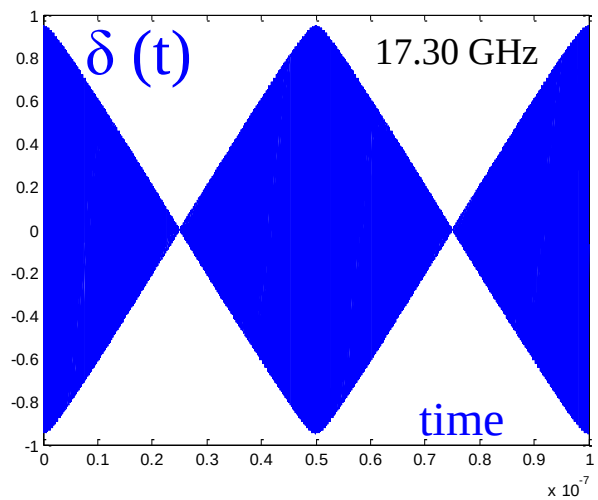
$$\boxed{\begin{aligned} &(1-\Omega^2)\tilde{\delta} + 2\beta_{rf}\, J_1(\tilde{\delta}) = \bar{\phi}_{rf}\cos\theta \\ &\frac{\Omega}{Q}\tilde{\delta} = -\bar{\phi}_{rf}\sin\theta \end{aligned}}$$

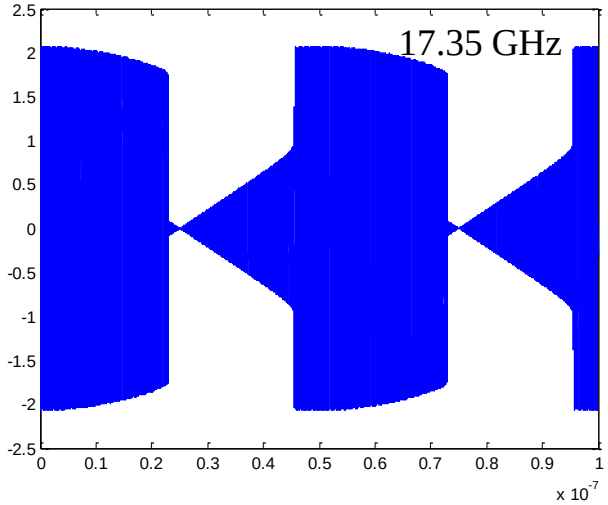
(set $\bar{\delta} = \phi_{dc} = 0$)

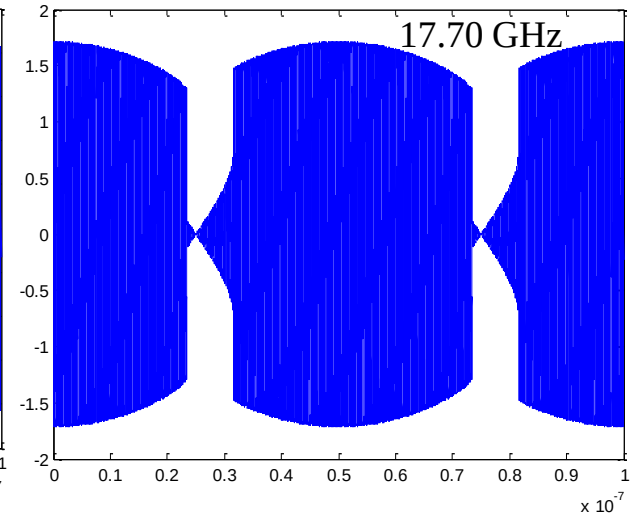



Collaboration with Tom Antonsen and Ed Ott

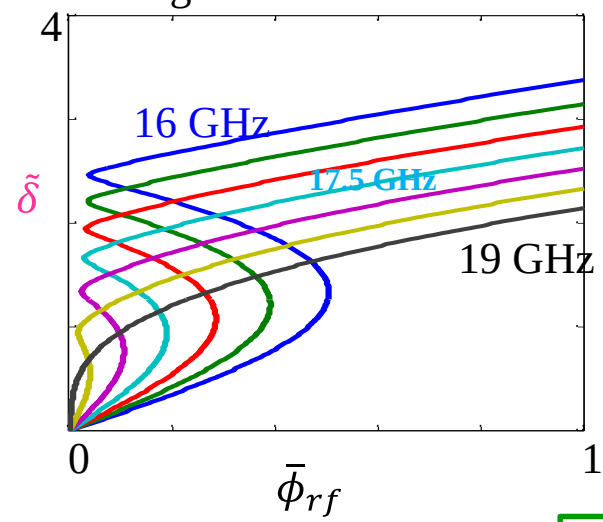
# Bi-Stability and Intermodulation Generation

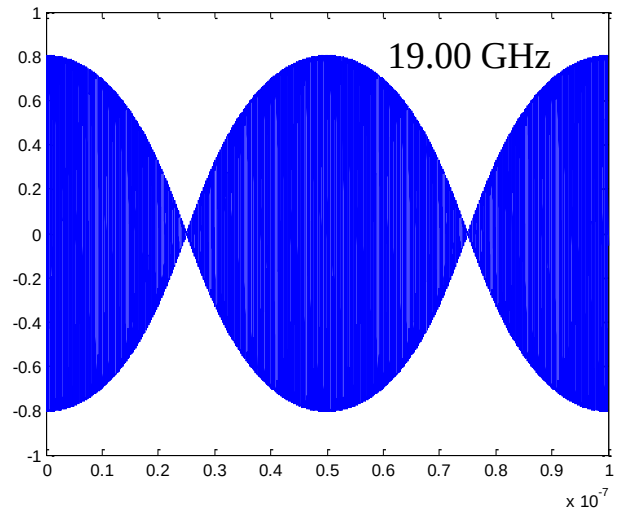


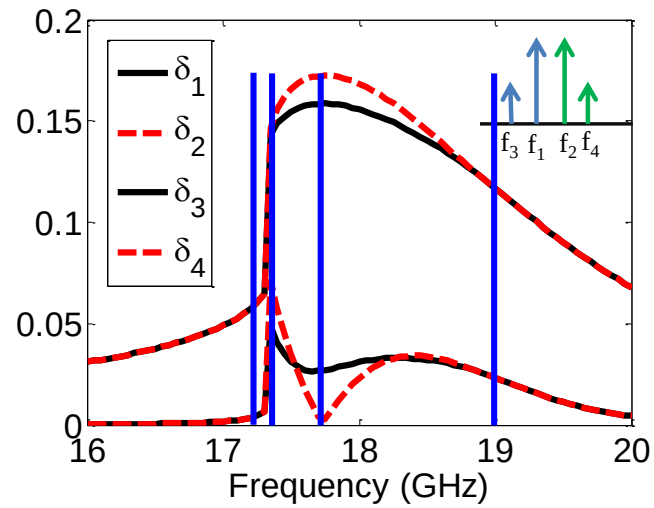




Single-tone behavior



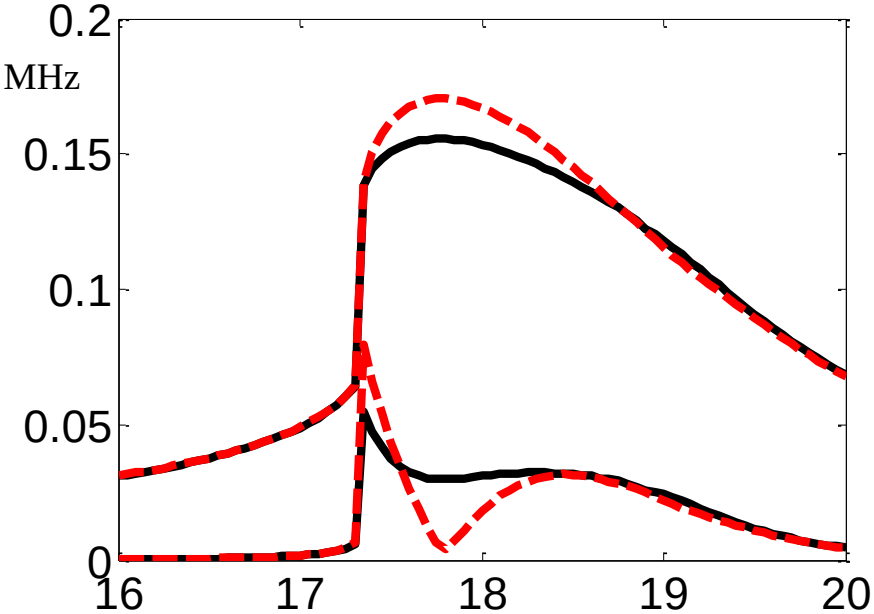

$$\left(1-\Omega^2\right)\tilde{\delta} + 2\beta_{rf} J_1\left(\tilde{\delta}\right) = \bar{\phi}_{rf}\cos\theta$$

$$\frac{\Omega}{Q}\tilde{\delta} = -\bar{\phi}_{rf}\sin\theta$$

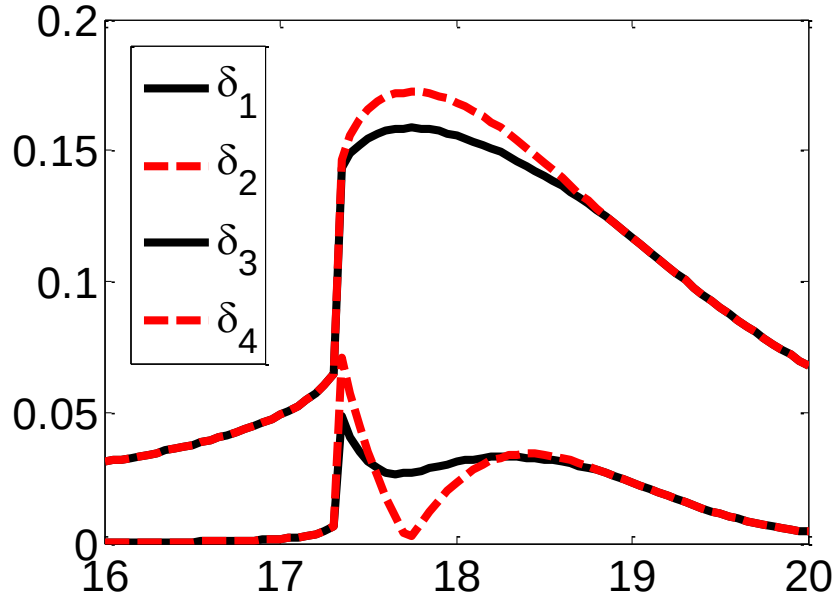
# Do You Hear a Ringing?

$f_2 - f_1 = 10$ MHz

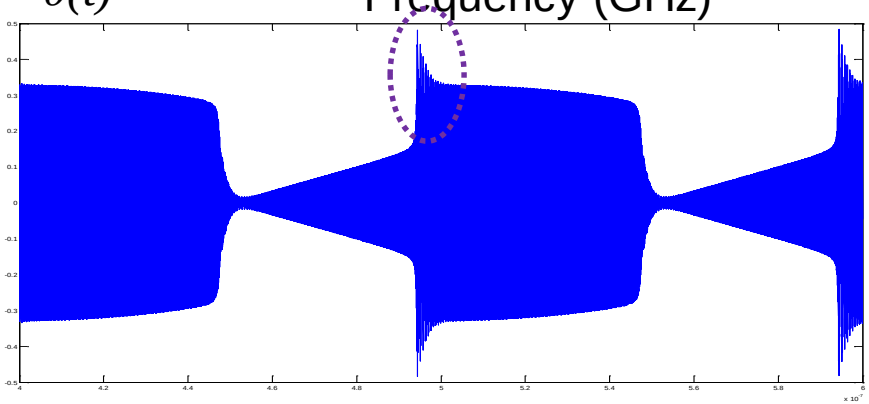
Numerical solution

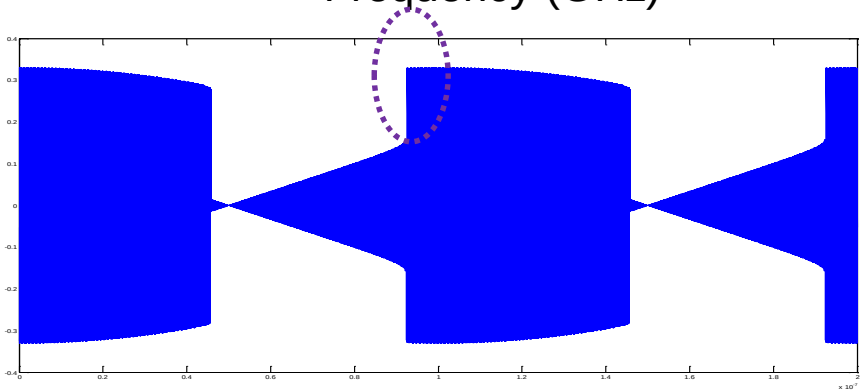



Analytical model (steady state)




$\delta(t)$





**Develop a dynamical model**

# Dynamical Model of $\delta(t)$ for a single SQUID with 2-Tone Stimulus

The gauge-invariant phase difference,

$$\delta = Re\{\hat{\delta} e^{i\Omega t}\} \qquad \hat{\delta} = \delta_R + i\delta_I$$

evolves on a Hamiltonian landscape given by,

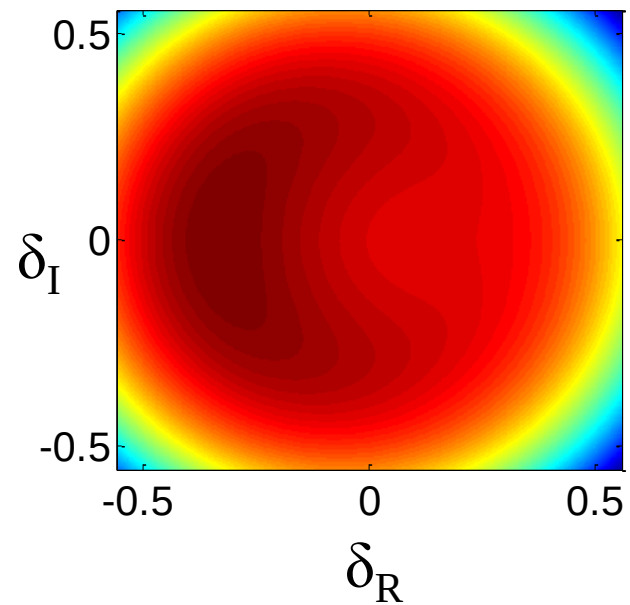
$$H = \frac{1}{2\Omega}\left[\left(1-\Omega^2\right)\frac{\left|\hat{\delta}\right|^2}{2} - 2\beta_{rf} J_0\left(\left|\hat{\delta}\right|\right) - \delta_R \bar{\phi}_{rf}\right]$$

according to these equations,

$$\frac{d}{d\tau}\delta_R = -\frac{1}{2Q}\delta_R - \frac{\partial}{\partial \delta_I} H\left(\left|\hat{\delta}\right|\right)$$

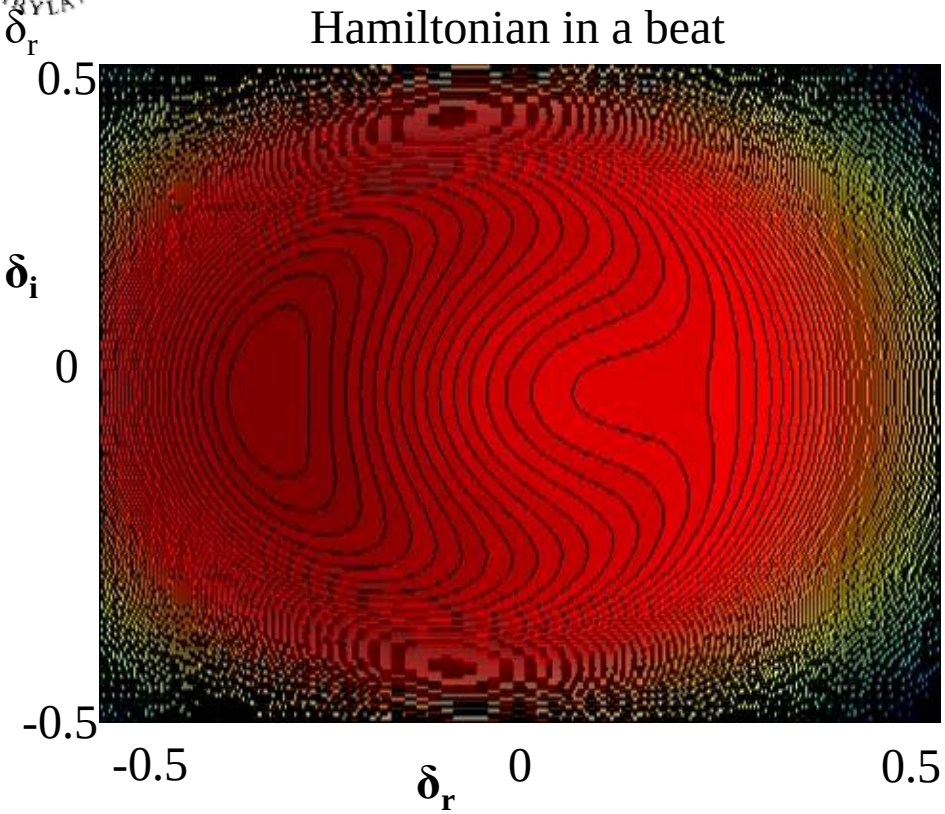
$$\frac{d}{d\tau}\delta_I = -\frac{1}{2Q}\delta_I + \frac{\partial}{\partial \delta_R} H\left(\left|\hat{\delta}\right|\right)$$

Snapshot of $H$

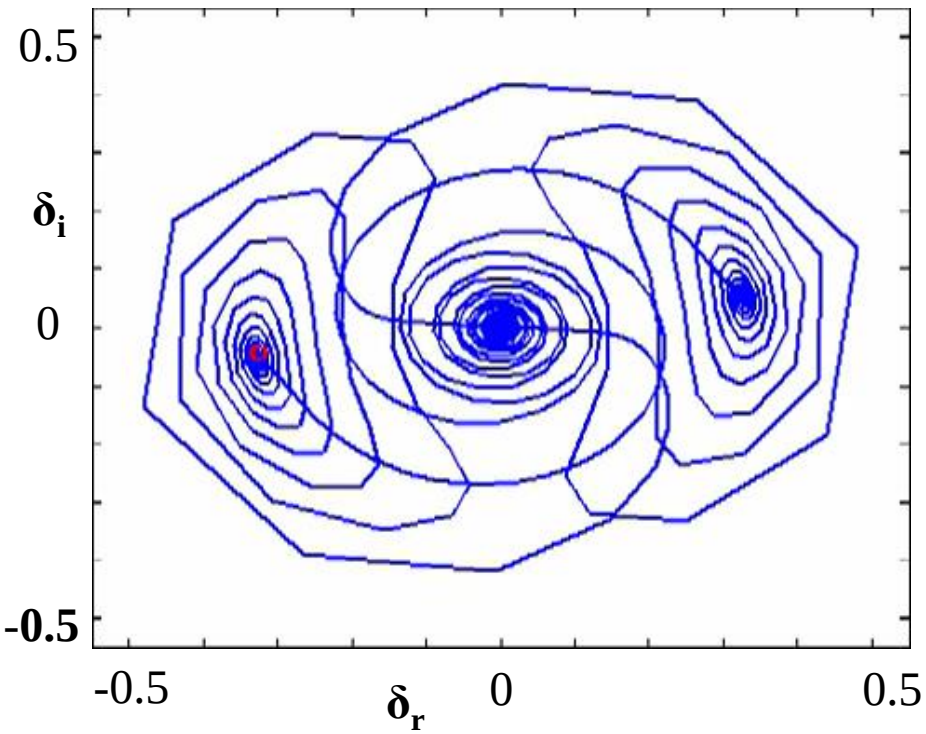


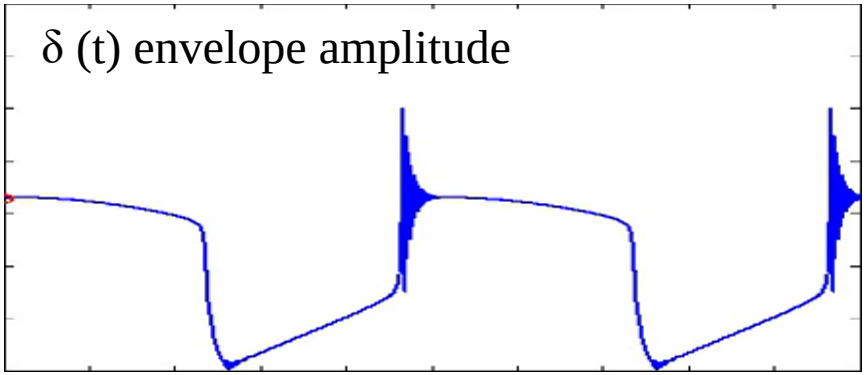
# Dynamical Model of $\delta(t)$ for a single SQUID with 2-Tone Stimulus

Hamiltonian in a beat



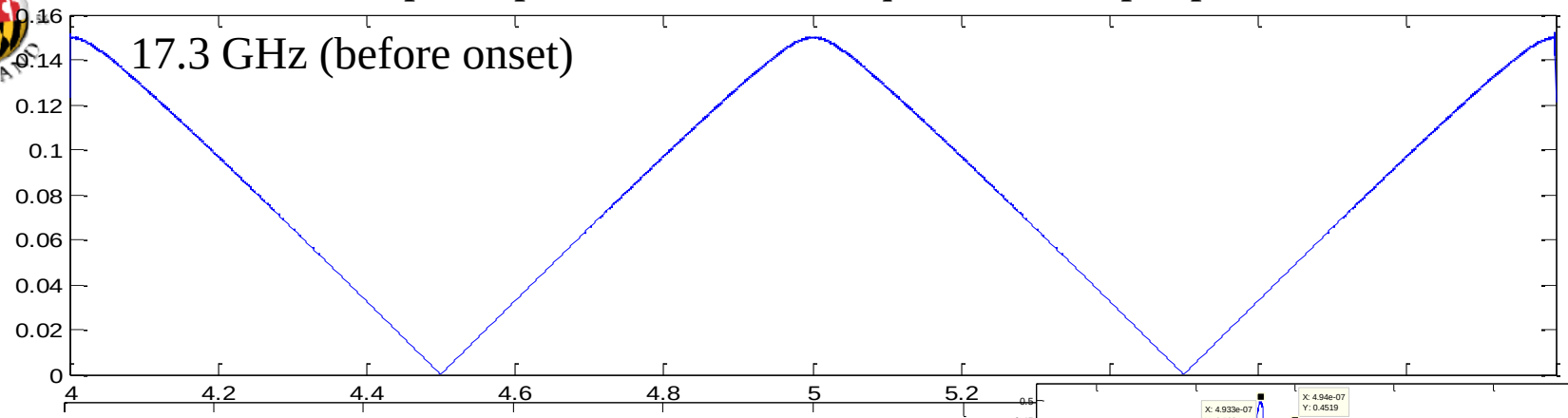

δ trajectory in a beat



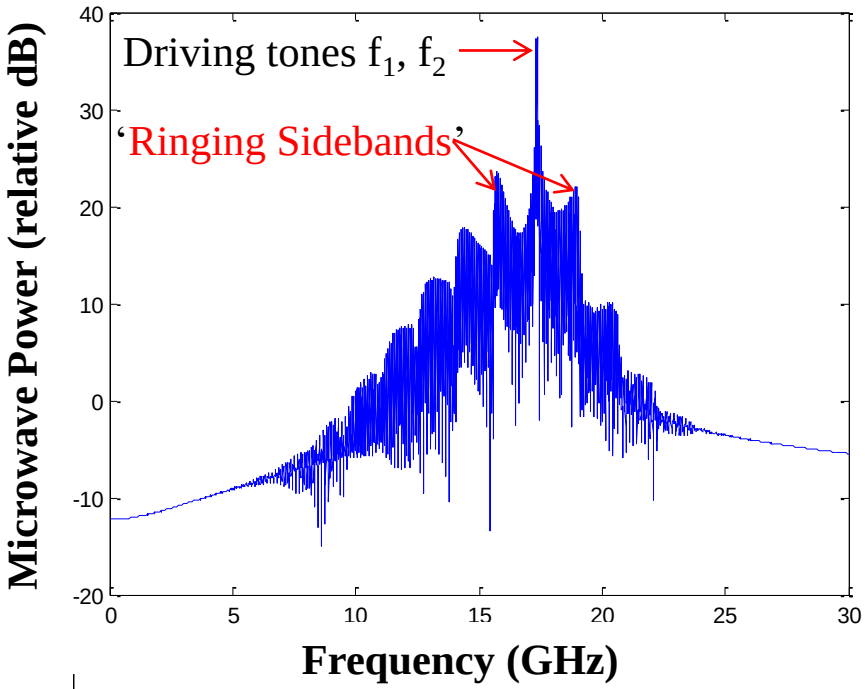



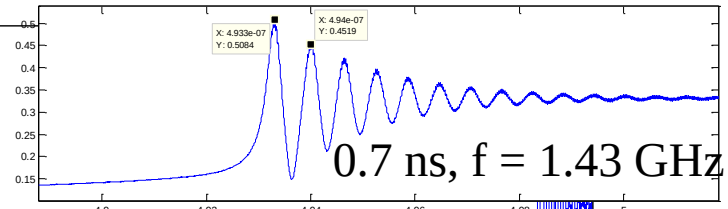

time

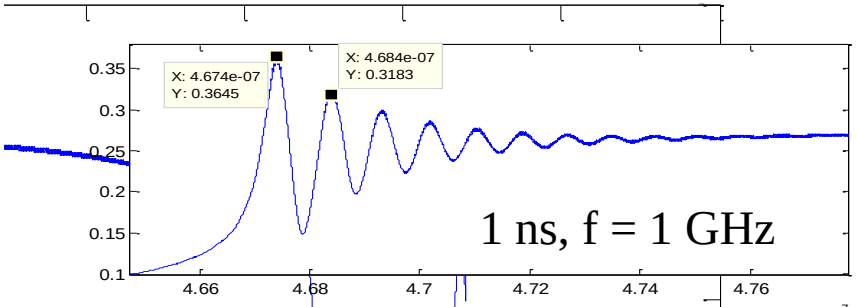
# Extracted envelope amplitude for three frequencies at input power of -65 dBm

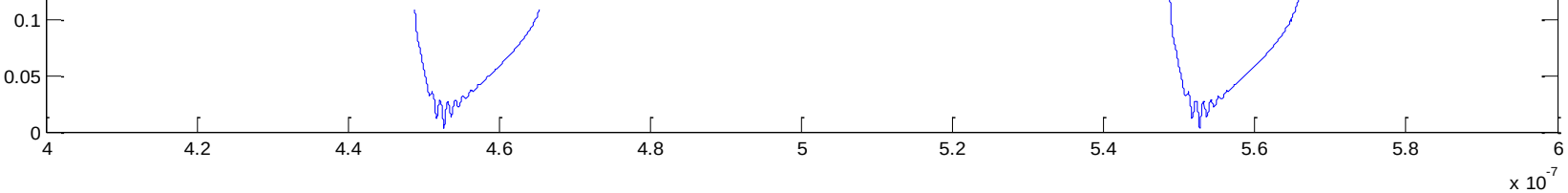



The ringing should been seen in spectrum.



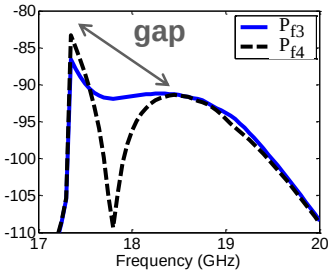



# Conclusions

- RF SQUID meta-atoms and metamaterials show transparency and bi-stable response

- Coherence of rf SQUID metamaterials is enhanced by strong coupling and nonlinearity

- Two-tone intermodulation tuned through rf and dc flux




- Analytical and numerical modeling:
  onset of IMD arises from access to multiple states

**Thanks for your attention!**

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**Melissa Trepanier, *et al*. Phys. Rev. X. 3, 041029 (2013)**

P. Jung, A. Ustinov and S. M. Anlage, "**Progress in Superconducting Metamaterials**," Supercond. Sci. Technol. 27, 073001 (2014)

**Daimeng Zhang, *et al*., Phys. Rev. X 5, 041045 (2015)**