Emergent and Nonlinear Properties of Macroscopic Quantum Metamaterials

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Outline

What are Metamaterials?

What are SQUIDs?

Nonlinear Metamaterials

Experiments

  Transparency
  Coherence
  Intermodulation

Models of SQUID Dynamics

Conclusions
What is a Metamaterial?

An artificial structure made up of repeating units of engineered objects having desired properties.

Electromagnetic metamaterials:  
The engineered objects (called ‘meta-atoms’) are electrically small (dimension $<<$ wavelength) and have ‘designer’ electromagnetic properties.

Large numbers of meta-atoms collected together can create an effective medium with novel coarse-grained properties:

- $\varepsilon_{\text{eff}}$, $\mu_{\text{eff}}$, index of refraction,
- enhanced $\chi_{\text{NL}}$,
- transparency,
- novel boundary conditions, etc.
Metamaterial vs. Photonic Crystal

Metamaterial

- Create an “effective medium,” using engineered “atoms,” with macroscopic $\varepsilon_{\text{eff}}$, $\mu_{\text{eff}}$, $n$ properties.

- Use constructive and destructive interference to engineer properties of light $\rightarrow \omega(\tilde{k})$
  - band structure
  - band gaps
  - defect states
  - negative group velocity …

Photonic Crystal

- Elementary units or “meta-atoms”
- $a \sim \lambda$
Why Superconducting Metamaterials?

Many exciting applications of metamaterials:
- Metasurface Light Manipulation
- Super-resolution Imaging
- Cloaking Devices
etc. …

… have strict REQUIREMENTS on the metamaterials:
- Ultra-Low Losses
- Ability to scale down in size (e.g. $\lambda/10^2$) and texture the “atoms”
- Nonlinearity with wide and fast tunability of the index of refraction $n$

… and superconductors bring these new capabilities to the metamaterials field:
- Flux quantization and Josephson effects
- Quantized energy states and quantum interactions with light
- Strong diamagnetism

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Macroscopic Quantum Effects In Superconductors

Superconductor is described by a single-valued Macroscopic Quantum Wavefunction

\[ \Psi = \left| \Psi \right| e^{i\theta} \]

Consequence:

Magnetic flux is quantized in units of \( \Phi_0 = h/2e \) (= 2.07 x 10^{-15} Tm²)

\[ \Phi = n \Phi_0, \quad n = \text{integer} \]

Example of Flux Quantization

One flux quantum in this loop requires a field of \( B = \Phi_0/\text{Area} = 1 \mu T \)

Earth’s magnetic field \( B_{\text{earth}} \sim 50 \mu T \)
Macroscopic Quantum Effects

Josephson Effects (Tunneling of Cooper Pairs)

\[ I = I_c \sin(\delta) \quad \text{DC} \]
\[ \frac{d\delta}{dt} = \frac{2e}{\hbar} V \quad \text{AC} \]

Gauge-invariant phase difference

\[ \delta = \theta_1 - \theta_2 - \frac{2e}{\hbar} \int_1^2 \vec{A} \cdot d\vec{l} \]

\( \delta(t) \) is the solution of a nonlinear diff. Eq.
Why Josephson / SQUID Metamaterials?

Josephson Inductance is **large, tunable and nonlinear**

\[ L_{JJ} \approx \frac{\Phi_0}{2\pi I_c \cos(\delta)} \]

Resistively and Capacitively Shunted Junction (RCSJ) Model

When the JJ is incorporated into a loop and flux \( \Phi \) is applied

Superconducting Loop

Combines the Josephson effects with flux quantization

External magnetic field essentially acts as a surrogate for the gauge-invariant phase
rf SQUID Meta-Atoms

A ‘Macroscopic Quantum’ Split-Ring Resonator

SQUID = Superconducting QUantum Interference Device
A self-resonant meta-atom with very nonlinear properties
Resonant Frequency of rf SQUID

\[ f_0 (\text{GHz}) \]
rf SQUID Superconducting Metamaterial

- Low loss
- Small Size
  - $\lambda \sim 3 \text{ cm (}\sim 10 \text{ GHz})$
  - $2r = 20 \sim 800 \ \mu\text{m}$

The SQUIDs interact by means of dipole – dipole coupling
$\rightarrow$ Collective Behavior

$\lambda \gg r, d$
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Modeling RF SQUIDs: Nonlinearity

\[ \Phi_{\text{applied}} + \Phi_{\text{induced}} = n\Phi_0 \]

\[ \Phi_{DC} + \Phi_{rf} \sin \omega t = \frac{\Phi_0 \delta}{2\pi} + L \left( I_C \sin \delta + \frac{L}{R} \frac{\Phi_0}{2\pi} \frac{d\delta}{dt} + C \frac{\Phi_0}{2\pi} \frac{d^2 \delta}{dt^2} \right) \]

Flux quantization linearizes the rf SQUID at high excitation levels!

Solve for \( \delta(t) \), calculate \( L_{JJ}, I(t), \mu_r(f), \) S-parameters, etc.

Why **Nonlinear** Metamaterials?

**Tunability** – Change the properties of the metamaterial after it has been fabricated – improved design flexibility

**Nonlinear response** – the metamaterial ‘looks’ different when probed at different intensity/power, frequency, direction, etc.

- Engineer enhanced coupling to light
- Self-induced nonlinear response

Some examples:
- Tunable band-pass filter
- Power limiters
- Nonlinear gain media
Why **Nonlinear** Metamaterials?

Quantum-Limited Amplifiers

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**A near-quantum-limited Josephson traveling-wave parametric amplifier**


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**A wideband, low-noise superconducting amplifier with high dynamic range**

Byeong Ho Eom, Peter K. Day, Henry G. LeDuc and Jonas Zmuidzinas
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Measurement of rf SQUID Metamaterial

The propagating EM wave provides the rf flux bias $\Phi_{rf}$.
DC magnetic flux tuned resonance

Coherent!

11x11 array, 4.4 K, -70 dBm

\[ |S_{21}| \text{ (dB)} \]

Frequency (GHz)

K-band cutoff

X: 18.23
Y: -5.412

Freq (GHz)

X: 18.23
Y: -5.412

X: 18.23
Y: -5.412
Overview

**Transparency in rf SQUID Metamaterials**
Under certain circumstances the metamaterial will ‘disappear’
Due to bi-stability in rf SQUID
Analytical model for the onset of transparency

**Coherence of rf SQUID Metamaterial Collective Response**
Disorder prevents all of the SQUIDs from responding coherently
Study ways to overcome disorder to enhance coherence

**Intermodulation in rf SQUID Metamaterials**
Characterizing the nonlinear properties
Transparency in a SQUID Meta-atom

Data and model agree that the single-SQUID “disappears” over a range of incident power

\[ \log_{10}(\Phi_{rf}/\Phi_0) \]

Transparency!

\[ |S_{21}| \text{(dB)} \]

Transparency in a SQUID Meta-atom

\[ \text{Transparency}(\Phi_{rf}/\Phi_0) = 1 - \frac{S_{21}(\Phi_{rf}/\Phi_0)}{S_{21}(\Phi_{rf}/\Phi_0 = 10^{-3})} \]

Broad-band effect
Bi-Stability of Transparency of an 11x11 SQUID array Metamaterial

Transparency shows up when bistability begins. \( \rightarrow \) Evident from Duffing Oscillator

\( \sin \delta \cong \delta - \delta^3 / 3! \)

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Intermodulation in rf SQUID Metamaterials
Characterizing the nonlinear properties
Flux-Tuning of 2x2 SQUID Array

Incoherent!

Insufficient Magnetic Shielding

\[ \Delta |S_{21}| (\text{dB}) \]

How do we make the rf SQUID metamaterial have a strong coherent response with \( \mu_{\text{eff}} < 0 \)?

rf power = -70 dBm
Temperature = 6.8 K
Solving for $\delta(t)$: SQUID Arrays

One SQUID

$$\frac{2\pi}{\Phi_0} (\Phi_{dc} + \Phi_{rf} \sin \Omega \tau) = \delta + \beta_{rf} \sin \delta + \gamma \delta' + \delta''$$

Once $\delta(t)$ is known all observable quantities can be calculated

SQUID Array

$$\frac{2\pi}{\Phi_0} (\hat{\Phi}_{dc} + \hat{\Phi}_{rf} \sin \Omega \tau) = \hat{\delta} + \kappa (\beta_{rf} \sin \hat{\delta} + \gamma \hat{\delta}' + \hat{\delta}'')$$

$\kappa = \frac{M}{L}$ mutual inductance

self inductance

$\delta(t)$ solutions are nearly sinusoidal:

$$\delta_j(t) \sim A_j e^{i(\Omega \tau + \phi_j)}$$
Quantifying Coherence

- Kuramoto order parameter*

\[ r e^{i \psi} = \frac{1}{N} \sum_{j} e^{i \phi_j} \]

- \( r = 1 \): complete coherence
- \( 0 < r < 1 \): partial coherence
- \( r = 0 \): complete incoherence

For SQUID metamaterials:

\[ \delta_j(t) \sim A_j e^{i(\Omega \tau + \phi_j)} \]

\[ r_A = \left| \frac{\sum_{j} A_j e^{i \phi_j}}{\sum_{j} A_j} \right| \]

Coupling Introduces Magneto-Inductive Modes

Simulation results for strong coupling (21 x 21 array), low power (linear)

\[ r_A = \left| \frac{\sum_j^N A_j e^{i\theta_j}}{\sum_j^N A_j} \right| \]

\[ \delta_j(t) \]

\[ \Omega_k \]

Lazarides, arXiv:1310.5445

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Magnetoinductive breathers in metamaterials

M. Eleftheriou, N. Lazarides, and G. P. Tsironis
Experimental Evidence for MI Modes

21 x 21 rf SQUID metamaterial ($\kappa = 0.02$ [strong coupling], linear response limit)

Flux gradient added to simulation
Simulated $|I_{rf}|$ in each SQUID

Simulated phase in each SQUID

Cryogenic environment

Simulation Results
High Power (-20 dBm)
Coherent Mode

A. P. Zhuravel, P. Jung, S. Anlage, A. Ustinov, KIT, Germany
How to Re-Establish Coherence in the Presence of Disorder?

dc flux gradients are the biggest culprit in experiments

1. Increased Coupling Between Meta-Atoms

2. Increasing rf Flux / Power

3. Increasing Temperature

By using these strategies to maximize coherence and taking steps to minimize uneven dc flux bias, rf SQUID metamaterials can be tuned coherently over a broad frequency range
Overview

Transparency in rf SQUID Metamaterials
Under certain circumstances the metamaterial will ‘disappear’
Due to bi-stability in rf SQUID
Analytical model for the onset of transparency

Coherence of rf SQUID Metamaterial Collective Response
Disorder prevents all of the SQUIDs from responding coherently
Study ways to overcome disorder to enhance coherence

Intermodulation in rf SQUID Metamaterials
Characterizing the nonlinear properties
Why **Nonlinear** Metamaterials?

Quantum-Limited Amplifiers

Two input tones: weak (to be amplified) strong (pump signal)
Linear systems vs. Nonlinear systems

- **Linear**
  - $f_1$
  - $f_1$

- **Nonlinear**
  - $f_1$

Harmonics
- $f_1$, $2f_1$, $3f_1$, ...

Intermodulation Distortion (IMD)
- $f_1 - 2f_2$, $f_2 - 2f_1$
- $2f_1 - 3f_2$, $f_1$, $f_2$, $2f_2 - 3f_1$

Harmonics
- $f_1$, $2f_1$, $3f_1$, ...
- $f_2$, $2f_2$, $3f_2$, ...
Measurement of Intermodulation in rf SQUID Metamaterials

Why measure IMD?
“zero” background (large S/N)
large dynamic range
Related to engineering specs for wireless communications
(Hypres directly-digitized radio)
Intermodulation measurement, 27x27 SQUID metamaterial
Resonance: 21.5 GHz
Tone power: -45 dBm

![Graph showing intermodulation distortion](image)

- Main tones
- 3rd IMD
- 5th IMD
- Output (dBm) 6.5K (above $T_c$ of Nb)
- Output (dBm) 15K

Nonlinear system

- $f_1$
- $f_2$
- $2f_1 - f_2$
- $f_1$
- $f_2$
- $2f_2 - f_1$
IMD Spectrum of a $27 \times 27$ SQUID Metamaterial
- very rich generation of IMD
  Out to 51st-order

Fixed input power, fixed center frequency for the two tones
P_{IMD} Generation: Experiment and Simulation

(a) Experiment, 4.6 K
(b) Simulation, 4.6 K

Log_{10}(\Phi_{rf}/\Phi_0)

Resonant frequency gap

\delta(t)

17.35 GHz
19.00 GHz

10 -20
12 -10
14 -8
16 -6
18 -4
20 -2

-100 -80 -60 -40 -20

-2.5 -2 -1.5 -1
-2.5 -2 -1.5 -1

-80 -70 -60 -50 -40 -30 -20 -10

-110 -100 -90 -80 -70 -60 -50 -40

-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1

-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1

-80 -70 -60 -50

-110 -100 -90 -80 -70 -60 -50

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-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1

-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1

-110 -100 -90 -80 -70 -60 -50 -40

-80 -70 -60 -50

-110 -100 -90 -80 -70 -60 -50

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1

-110 -100 -90 -80 -70 -60 -50 -40

-80 -70 -60 -50

-110 -100 -90 -80 -70 -60 -50

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-110 -100 -90 -80 -70 -60 -50

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1

-110 -100 -90 -80 -70 -60 -50 -40

-80 -70 -60 -50

-110 -100 -90 -80 -70 -60 -50

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1

-110 -100 -90 -80 -70 -60 -50 -40

-80 -70 -60 -50
Some Observations About The Data

Sharp onset of $P_{\text{IMD}}$ as a function of increasing frequency at fixed $\Phi_{\text{rf}}$

The onset of $P_{\text{IMD}}$ as a function of increasing frequency coincides with the onset of jumps between two bi-stable states

There is a prominent gap in the 3rd-order IMD as a function of frequency and rf flux. A very robust feature seen in single SQUIDs and metamaterials.

This gap occurs very close to the resonant frequency
Analytical Treatment of IMD
single SQUID

Slowly varying envelope approximation
\[
\frac{d^2 \delta}{d\tau^2} + \frac{1}{Q} \frac{d\delta}{d\tau} + \delta + \beta_{rf} \sin \delta = \phi_{dc} + \bar{\phi}_{rf} \sin(\Omega \tau)
\]

Two tone: \( \bar{\phi}_{rf} \) is slowly varying envelope, the period depends on the tone distance frequency.

Slowly varying in time
Steady-state response: \( \delta = \bar{\delta} + \tilde{\delta} \sin(\Omega \tau + \theta) \)

Expand: \( \sin \delta = \sin \tilde{\delta} J_0(\tilde{\delta}) + 2 J_1(\tilde{\delta}) \sin(\Omega \tau + \theta) + H.O.T. \)

\[
(1 - \Omega^2) \tilde{\delta} + 2 \beta_{rf} J_1(\tilde{\delta}) = \bar{\phi}_{rf} \cos \theta
\]

\[
\frac{\Omega}{Q} \tilde{\delta} = -\bar{\phi}_{rf} \sin \theta
\]

(set \( \bar{\delta} = \phi_{dc} = 0 \))

Collaboration with Tom Antonsen and Ed Ott
Bi-Stability and Intermodulation Generation

\[ \delta(t) \]

\[ 17.30 \text{ GHz} \]

\[ \text{time} \]

Single-tone behavior

\[ \delta_1, \delta_2, \delta_3, \delta_4 \]

\[ 16 \text{ GHz}, 17.5 \text{ GHz}, 19 \text{ GHz} \]

\[ \bar{\phi}_{rf} \]

\[ 0, 1 \]

\[ (1 - \Omega^2)\delta + 2\beta_{rf} J_1(\delta) = \bar{\phi}_{rf} \cos \theta \]

\[ \frac{\Omega}{Q} \delta = -\bar{\phi}_{rf} \sin \theta \]
Do You Hear a Ringing?

Numerical solution

Analytical model (steady state)

Develop a dynamical model
Dynamical Model of $\delta(t)$ for a single SQUID with 2-Tone Stimulus

The gauge-invariant phase difference,

$$\delta = Re\{\hat{\delta} e^{i\Omega t}\} \quad \hat{\delta} = \delta_R + i\delta_I$$

evolves on a Hamiltonian landscape given by,

$$H = \frac{1}{2\Omega} [(1 - \Omega^2) \frac{|\hat{\delta}|^2}{2} - 2\beta_{rf} J_0(|\hat{\delta}|) - \delta_R \bar{\phi}_{rf}]$$

according to these equations,

$$\frac{d}{d\tau}\delta_R = -\frac{1}{2Q} \delta_R - \frac{\partial}{\partial \delta_I} H(|\hat{\delta}|)$$

$$\frac{d}{d\tau}\delta_I = -\frac{1}{2Q} \delta_I + \frac{\partial}{\partial \delta_R} H(|\hat{\delta}|)$$

Snapshot of $H$
Dynamical Model of $\delta(t)$ for a single SQUID with 2-Tone Stimulus

Hamiltonian in a beat

$\delta$ trajectory in a beat

$\delta(t)$ envelope amplitude

time
Extracted envelope amplitude for three frequencies at input power of -65 dBm

17.3 GHz (before onset)

The ringing should been seen in spectrum.

Driving tones $f_1$, $f_2$

Ringing Sidebands

0.7 ns, $f = 1.43$ GHz

1 ns, $f = 1$ GHz

Microwave Power (relative dB)

Frequency (GHz)
Conclusions

• RF SQUID meta-atoms and metamaterials show transparency and bi-stable response

• Coherence of rf SQUID metamaterials is enhanced by strong coupling and nonlinearity

• Two-tone intermodulation tuned through rf and dc flux

• Analytical and numerical modeling: onset of IMD arises from access to multiple states

Thanks for your attention!

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