Chimeras in locally coupled SQUIDs: Lions, goats and snakes

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Quantum Metamaterials &

Quantum Technology Conference

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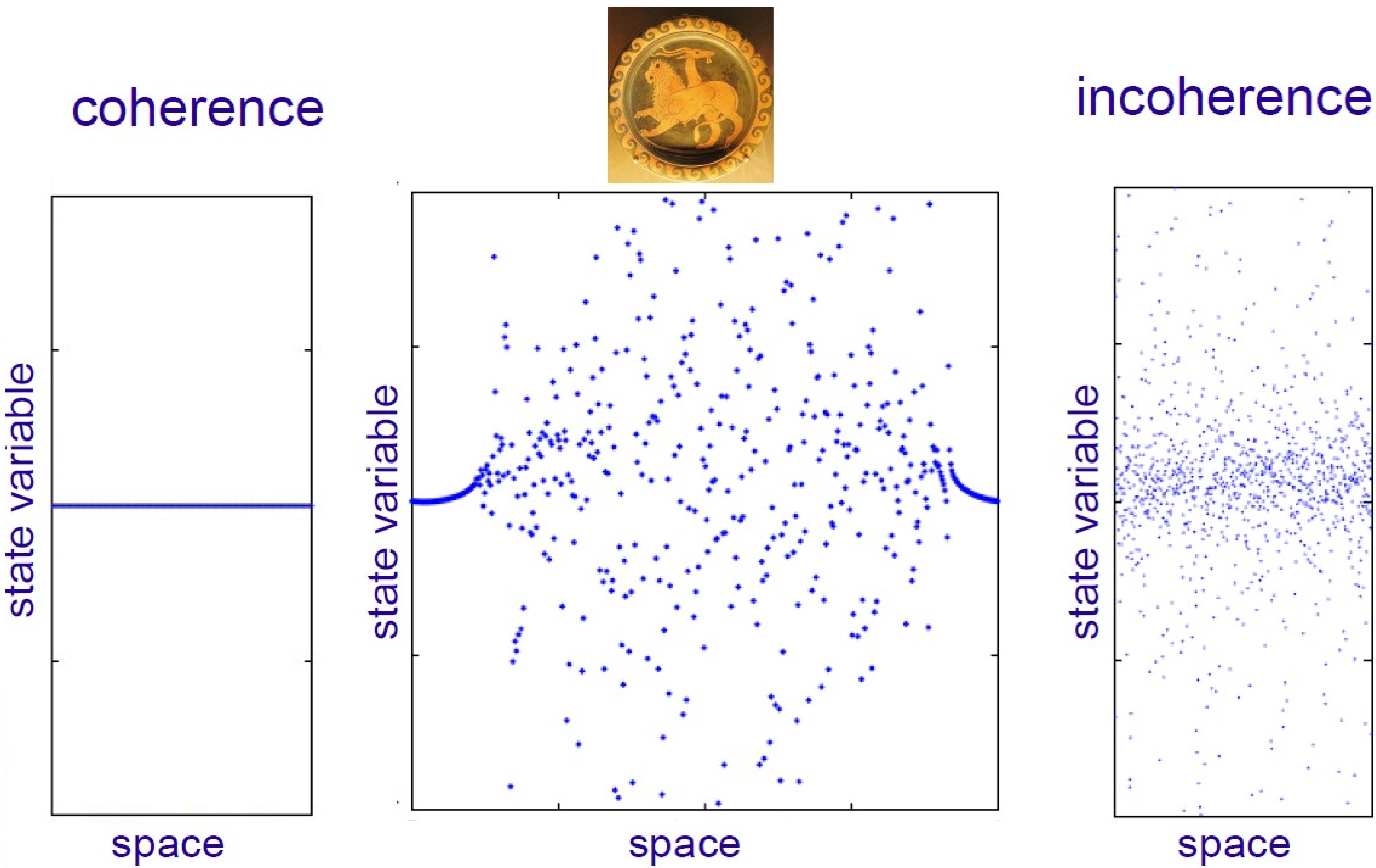
Introduction: What is a chimera state?



Recent review:

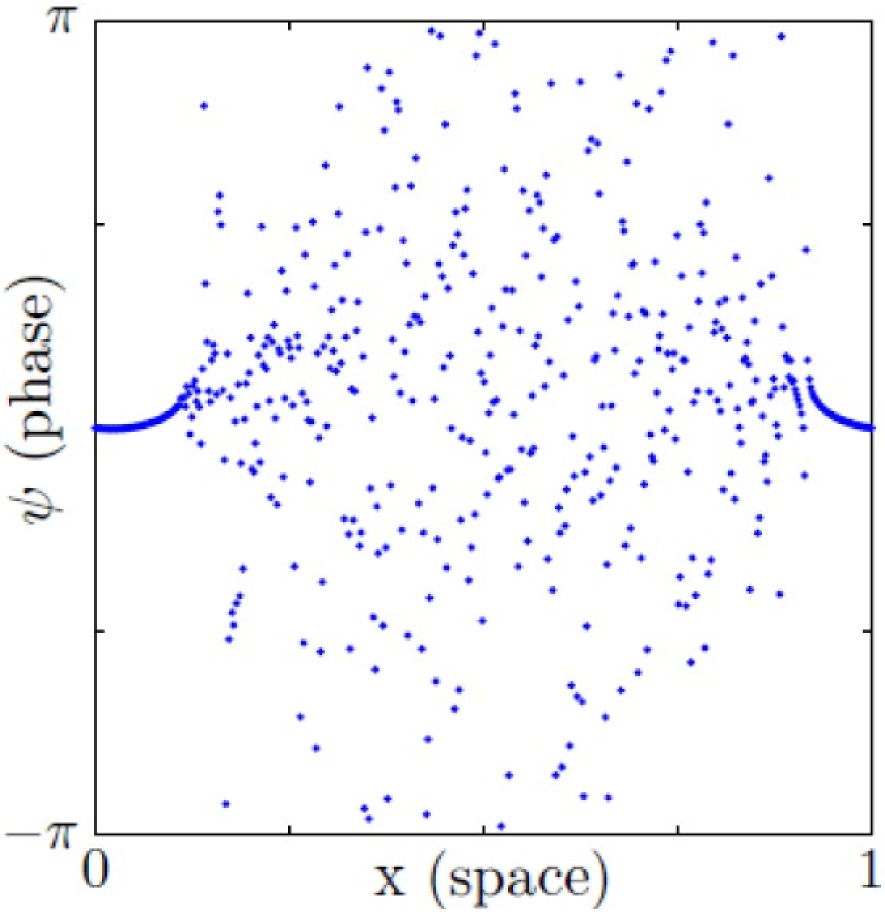
M. J. Panaggio and D. M. Abrams, Nonlinearity 28, R67 (2015)

Introduction: What is a chimera state?

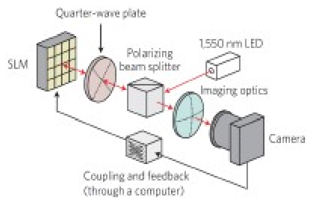
$$\frac{\partial \phi}{\partial t} = \omega - \int_{-\pi}^{\pi} G(x-x')\sin\left[\phi(x,t)-\phi(x',t)+\alpha\right]dx'$$

Spatial coexistence of coherent/synchronized and incoherent/desynchronized domains in a dynamical network

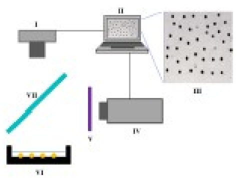
- discovered by Kuramoto and Battogtokh in 2002
- named ***chimera states*** by Abrams and Strogatz in 2004
- identical elements, symmetric topology



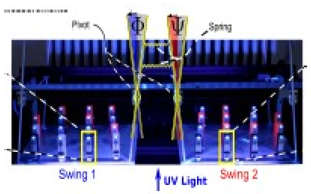
Recent experimental verifications of chimera states



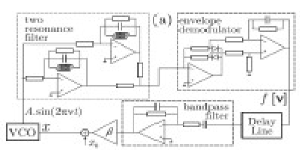
● Optical experiment: Spatial light modulator
Hagerstrom, Murphy, Roy, Hövel, Omelchenko, Schöll, ***Nature Phys.*** **8**, 658 (2012)



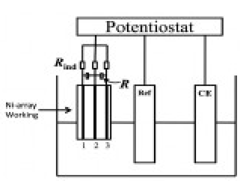
● Chemical experiment: Light-sensitive BZ reaction
Tinsley, Nkomo, Showalter, ***Nature Phys.*** **8**, 662 (2012)



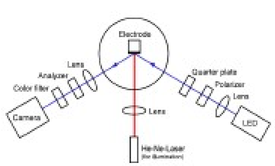
● Mechanical experiment: Coupled pendula
Martens, Thutupalli, Fourriere, Hallatschek, ***PNAS*** **110**, 10563 (2013)



● Electronic experiment: Frequency-modulated delay oscillator
Larger, Penkovsky, Maistrenko, ***PRL*** **111**, 054103 (2013)



● Electrochemical experiment: Nickel electrodissolution
Wickramasinghe, Kiss, ***PLoS ONE 8***, e80586 (2013)



● Electrochemical experiment: Electro-oxidation of Si
Schmidt, Schönleber, Krischer, Garcia-Morales, ***Chaos*** **24**, 013102 (2014)

Chimeras in nature?

Unihemispheric sleep: one half of the brain is highly synchronized (sleeping) while the other half remains desynchronized (awake).

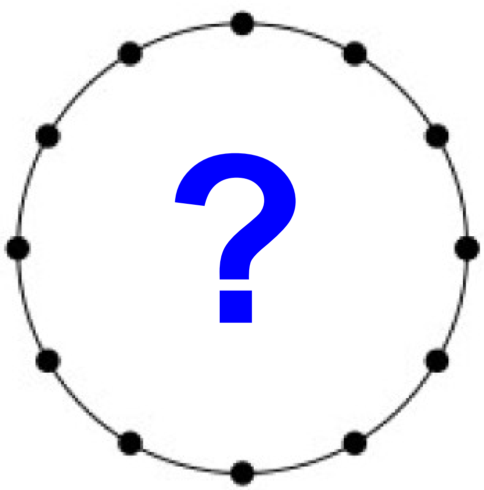
- dolphins and seals: escaping from predators and surfacing for air while sleeping

- humans[1]: first-night effect – our brain stays alert to protect against unknown danger.

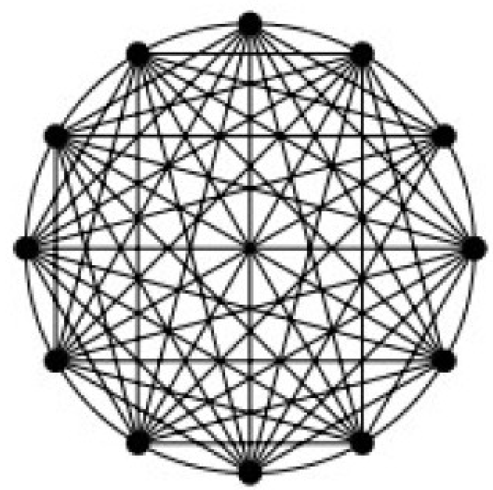
[1]M. Tamaki, J. W. Bang, T. Watanabe, Y. Sasaki, Night Watch in One Brain Hemisphere during Sleep Associated with the First-Night Effect in Humans, Curr Biol. 26, 5 (2016)

Chimera states for local coupling?

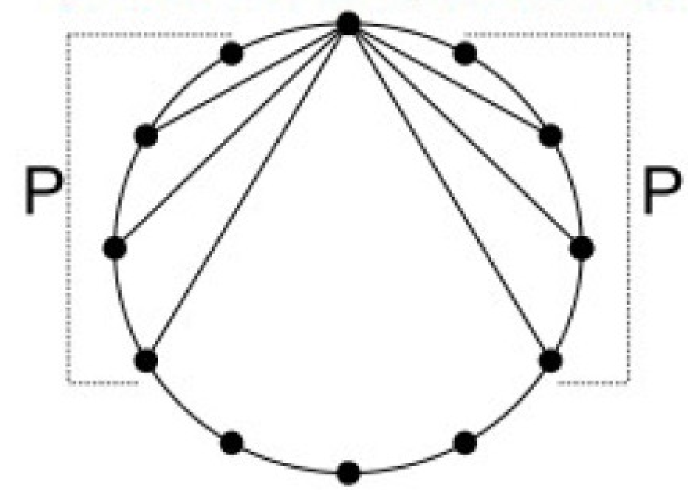
Local coupling



Global coupling



Nonlocal (intermediate) coupling



C. Laing, Phys. Rev. E **92**, 050904(R) (2015).
B. K. Bera, D. Ghosh, and M. Lakshmanan Phys. Rev. E **93**, 012205 (2016).

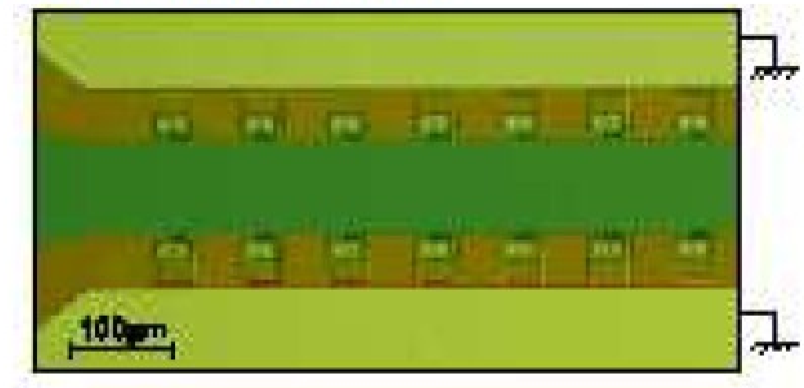
Superconducting Quantum Interference Devices (SQUIDs)

SQUIDs bring new features to the field of metamaterials

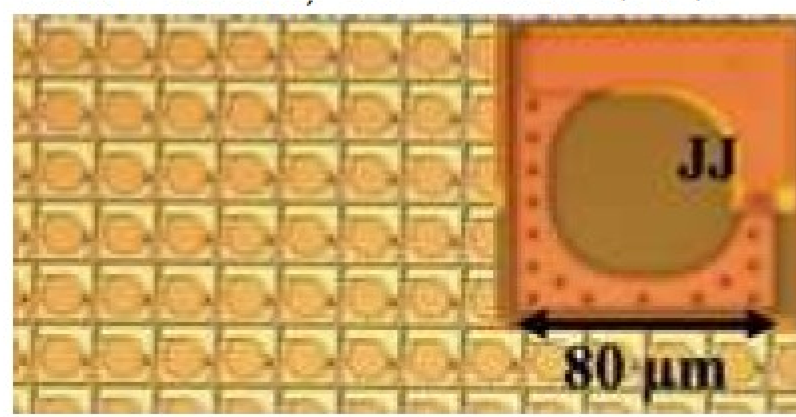
extraordinary electromagnetic properties not found in nature

highly **nonlinear oscillator**

complex **collective behavior**

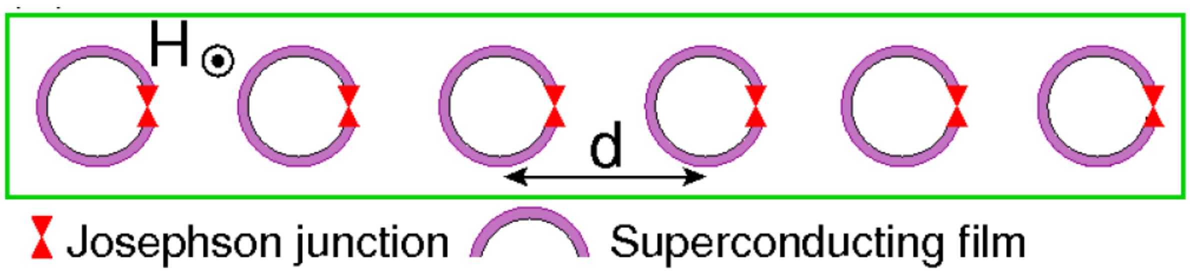


S. Butz, P. Jung, L. V. Filippenko, V. P. Koshelets, and A. V. Ustinov, Opt. Express **21**, 22540 (2013).



M. Trepanier, D. Zhang, O. Mukhanov, and S. M. Anlage, Phys. Rev. X **3**, 041029 (2013).

Array of SQUIDs with **local coupling**



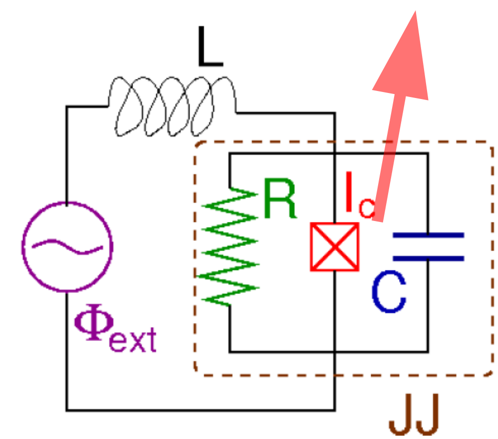
magnetic flux threading n-th loop:

$$\Phi_n = \underbrace{\Phi_{ext}}_{\text{external flux}} + \underbrace{L\, I_n}_{\text{self-inductance}} + \underbrace{M\,(I_{n-1} + I_{n+1})}_{\text{mutual inductance}}$$

$$I_n = -C\frac{d^2\Phi_n}{dt^2} - \frac{1}{R}\frac{d\Phi_n}{dt} - I_c \sin\left(2\pi\frac{\Phi_n}{\Phi_0}\right)$$

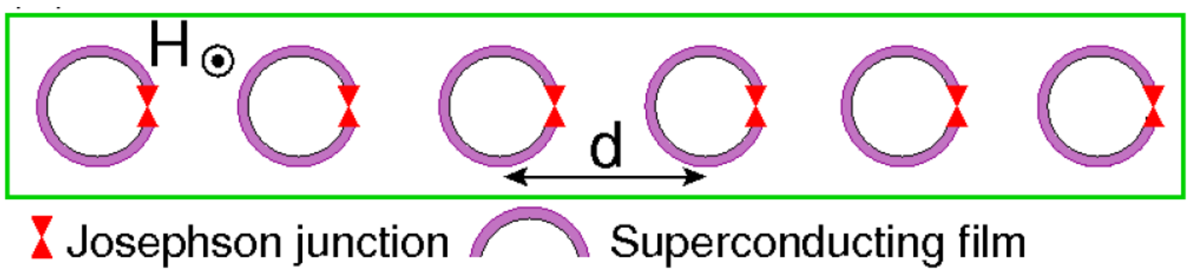
Josephson relations

$$\boxed{\begin{aligned} I(t) &= I_c \sin\phi_J(t) \\ U(t) &= \frac{\hbar}{2e}\dot{\phi}_J(t) \end{aligned}}$$



K. K. Likharev *Dynamics of Josephson Junctions and Circuits*, Gordon and Breach, Philadelphia (1986).

Array of SQUIDs with **local coupling**



$$\ddot{\phi}_n + \gamma\dot{\phi}_n + \phi_n + \beta \sin\left(2\pi\phi_n\right) = \lambda(\phi_{n-1} + \phi_{n+1}) + (1-2\lambda)\phi_{ac}\cos(\Omega\tau)$$

$\lambda = M/L$ dimensionless coupling coefficient

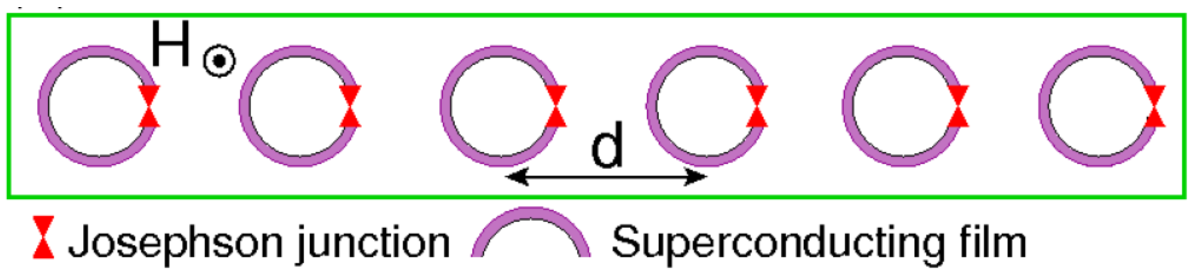
ϕ_n, ϕ_{ac} normalized to Φ_0

Ω driving frequency normalized to $\omega_0 = 1/\sqrt{LC}$

$\beta = \frac{I_c L}{\Phi_0} = \frac{\beta_L}{2\pi}$ SQUID parameter

$\gamma = \frac{1}{R}\sqrt{\frac{L}{C}}$ loss coefficient

Array of SQUIDs with **local coupling**



$$\ddot{\phi}_n + \gamma\dot{\phi}_n + \phi_n + \beta \sin\left(2\pi\phi_n\right) = \lambda(\phi_{n-1} + \phi_{n+1}) + (1-2\lambda)\phi_{ac}\cos(\Omega\tau)$$

parameters experimentally relevant

$\beta = 0.1369$

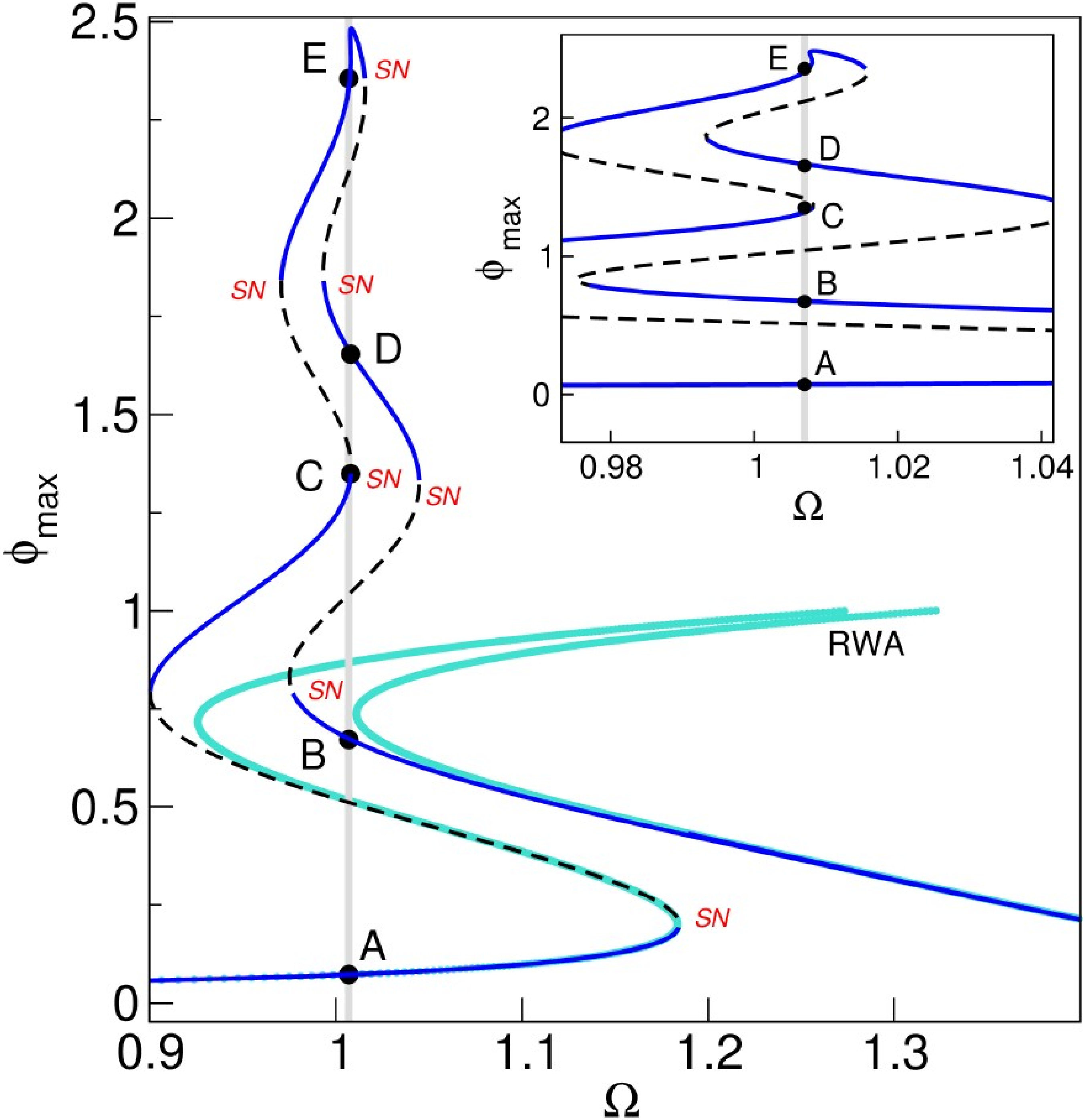
$\varphi_{ac} = 0.06$

$\gamma = 0.024$

M. Trepanier, D. Zhang, O. Mukhanov, and S. M. Anlage, Phys. Rev. X **3**, 041029 (2013).

single SQUID ($n=1, \lambda=0$)

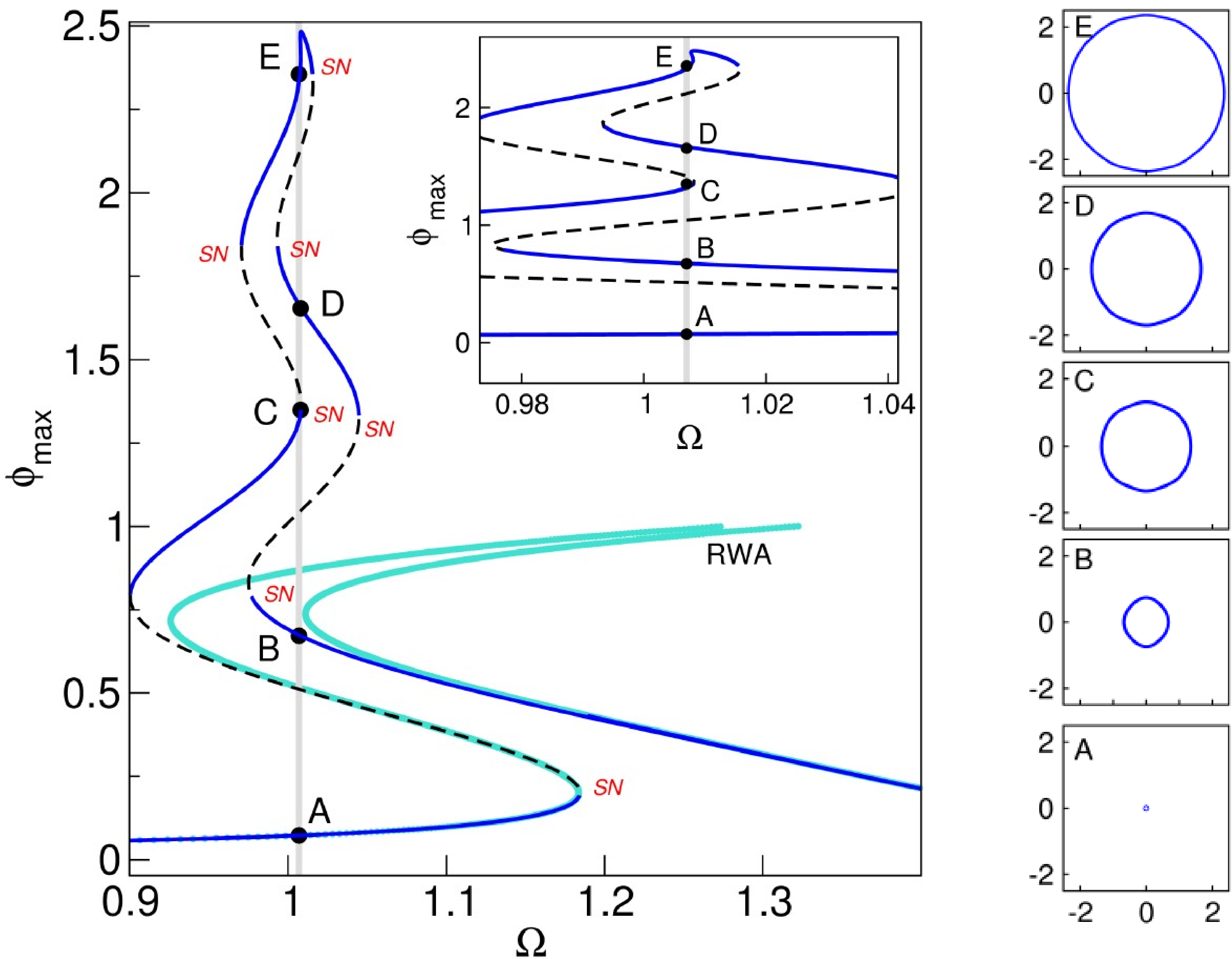
Single SQUID snake-like resonance curve *(γ=0.024)*



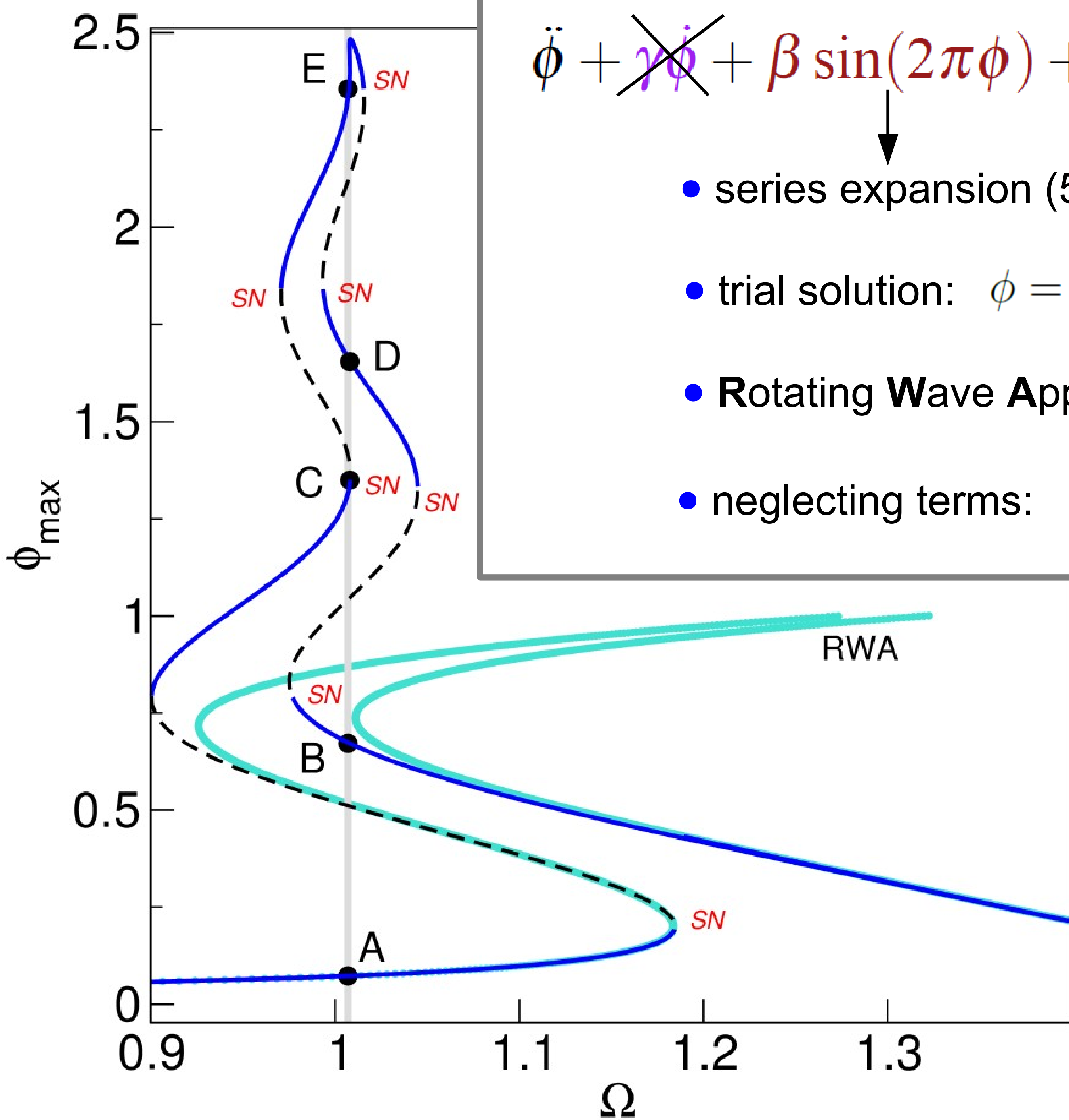
co-existence of **stable** and **unstable** periodic solutions

multiple saddle-node bifurcations of limit cycles

Single SQUID snake-like resonance curve $(\gamma=0.024)$



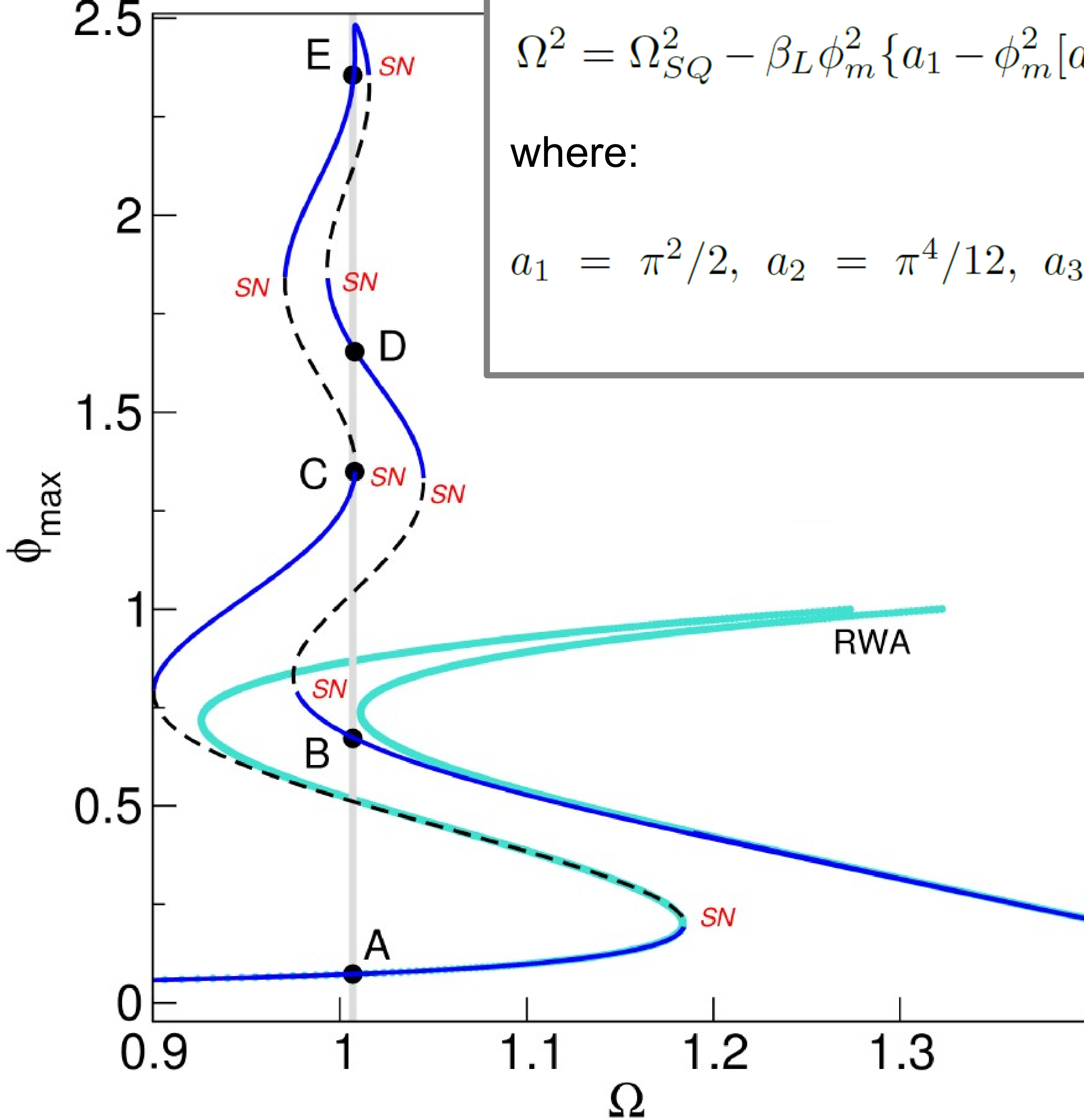
Single SQUID snake-like resonance curve *(γ=0.024)*



$$\ddot{\phi} + \cancel{\gamma\dot{\phi}} + \beta\sin(2\pi\phi) + \phi = \phi_{ac}\cos(\Omega\tau)$$

- series expansion (5 first terms)
- trial solution: $\phi = \phi_m(\tau)\,\cos[\Omega\tau + \theta(\tau)]$
- **R**otating **W**ave **A**pproximation
- neglecting terms: $\ddot{\phi}_m,\ \ddot{\theta},\ \dot{\theta}^2,\ \dot{\phi}_m\dot{\theta}$

Single SQUID snake-like resonance curve *(γ=0.024)*

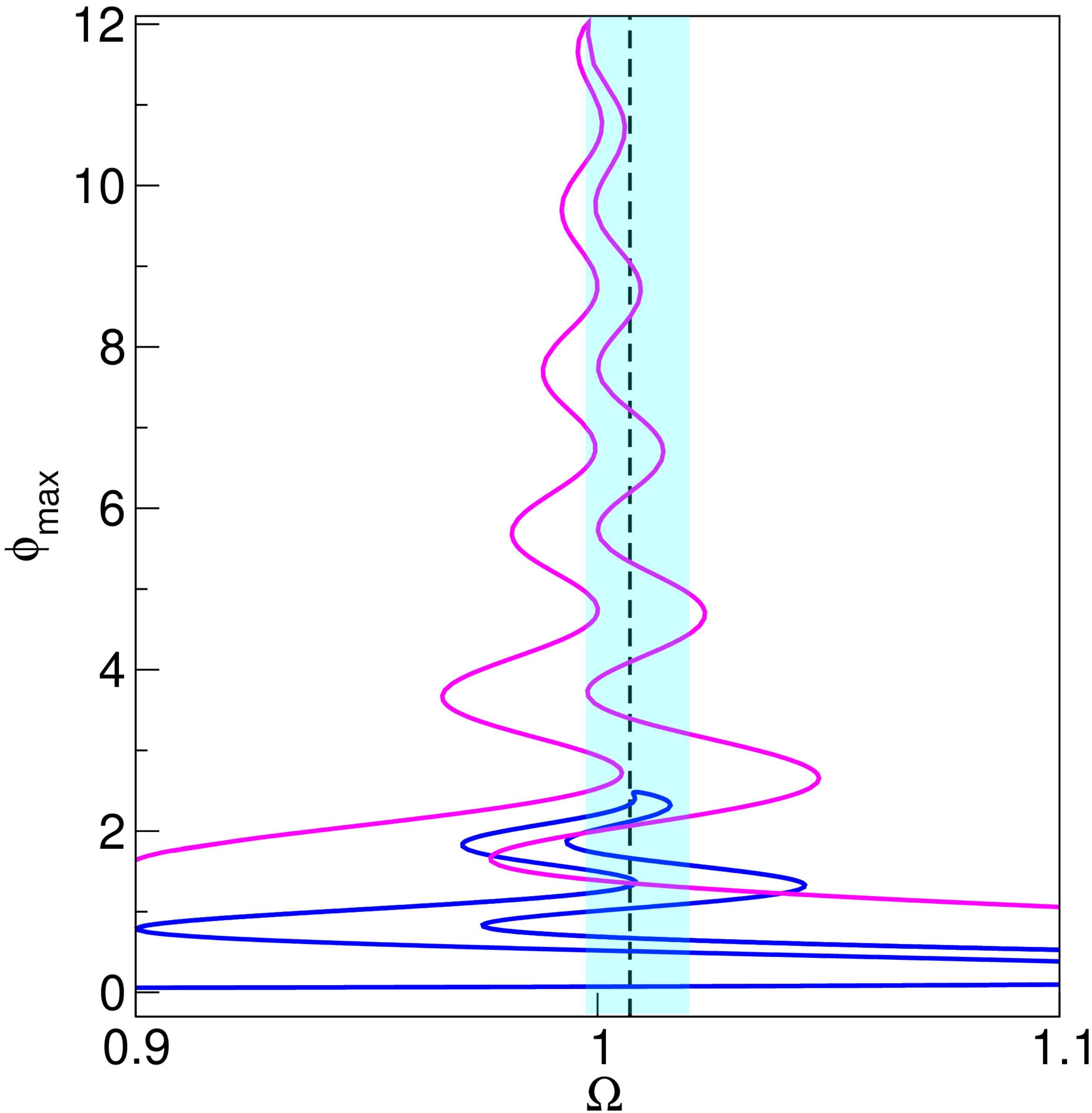


$$\Omega^2 = \Omega_{SQ}^2 - \beta_L \phi_m^2 \{a_1 - \phi_m^2 [a_2 - \phi_m^2 (a_3 - a_4 \phi_m^2)]\} \pm \frac{\phi_{ac}}{\phi_m}$$

where:

$$a_1 = \pi^2/2, \; a_2 = \pi^4/12, \; a_3 = \pi^6/144, \; a_4 = \pi^8/2880$$

Single SQUID snake-like resonance curve *(γ=0.01)*

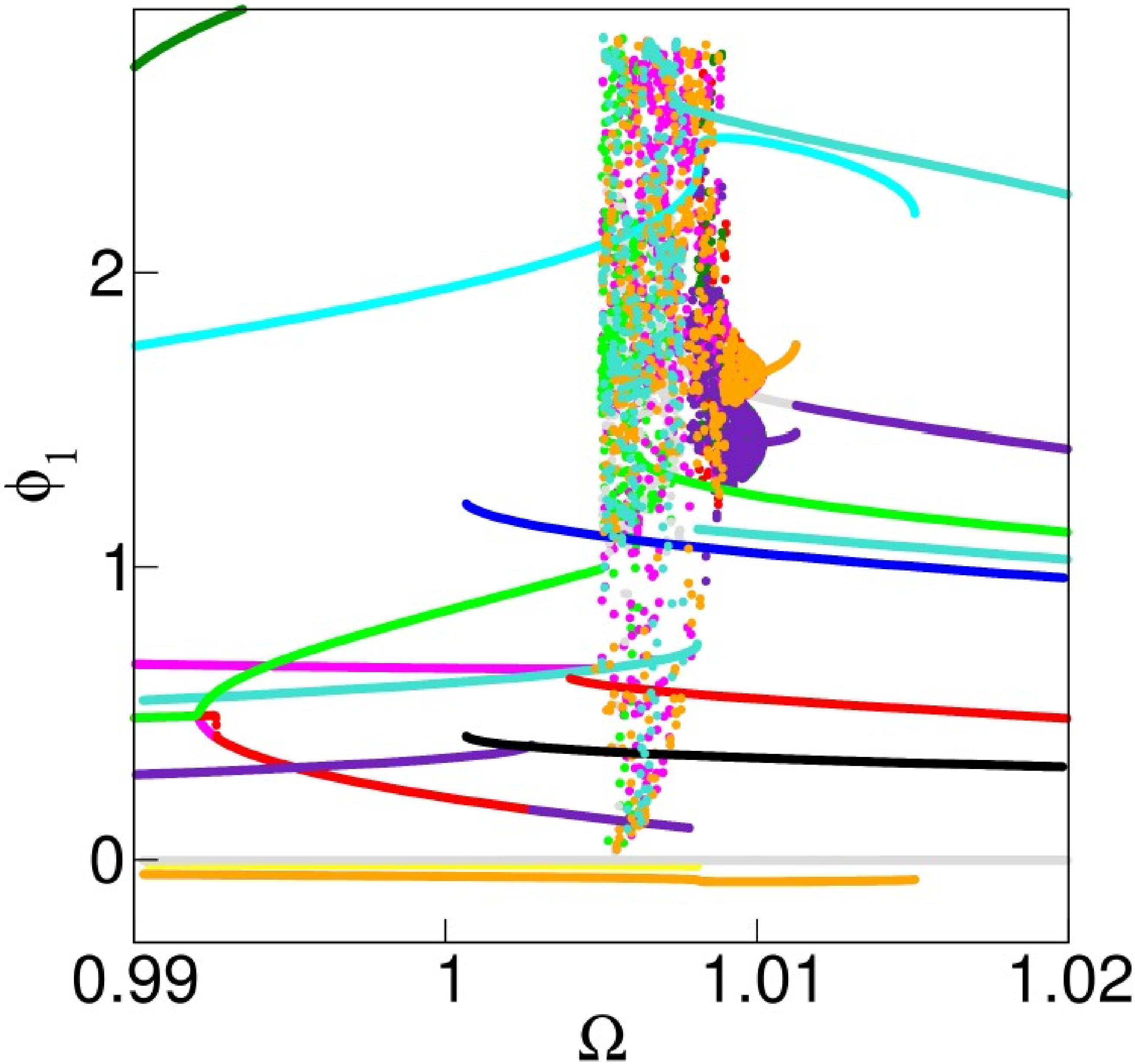


as γ decreases the snake curve becomes more **winding**

number of coexisting periodic orbits **increases**

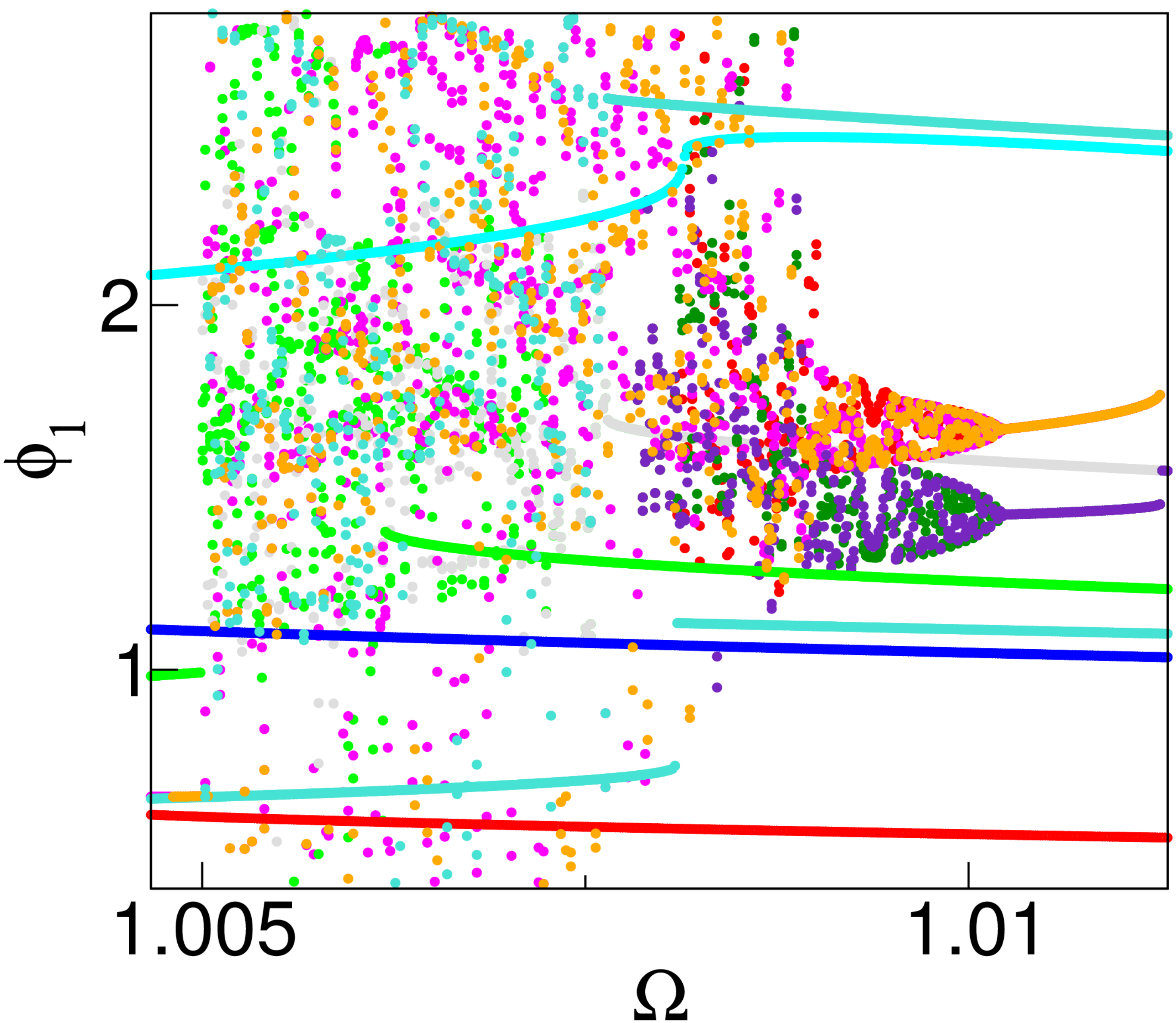
(un)stable branches more and **smaller**

Two coupled SQUIDs *(γ=0.024, λ=-0.025)*



system complexity and multistability **increases**

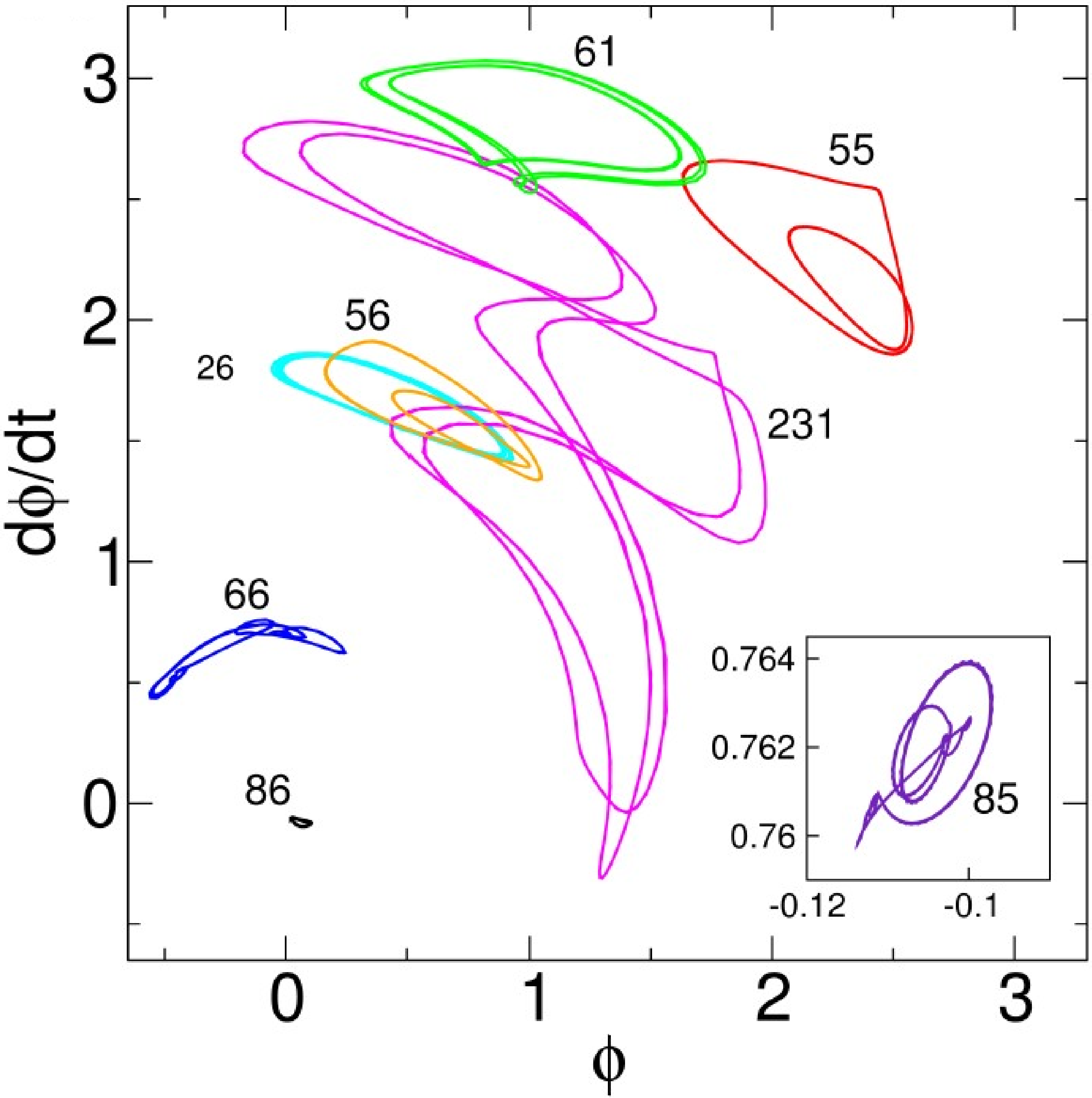
Two coupled SQUIDs *(γ=0.024, λ=-0.025)*



system complexity and multistability **increases**

coexisting **chaotic** attractors around resonance frequency

N=256 coupled SQUIDs *(γ=0.024, λ=-0.025)*

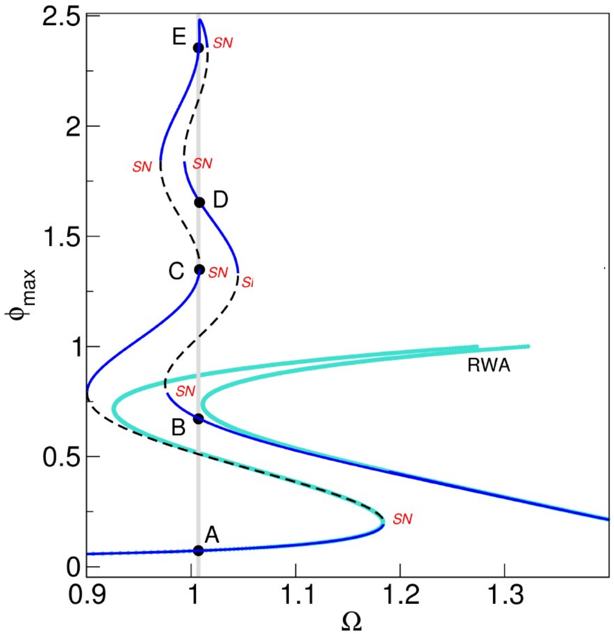


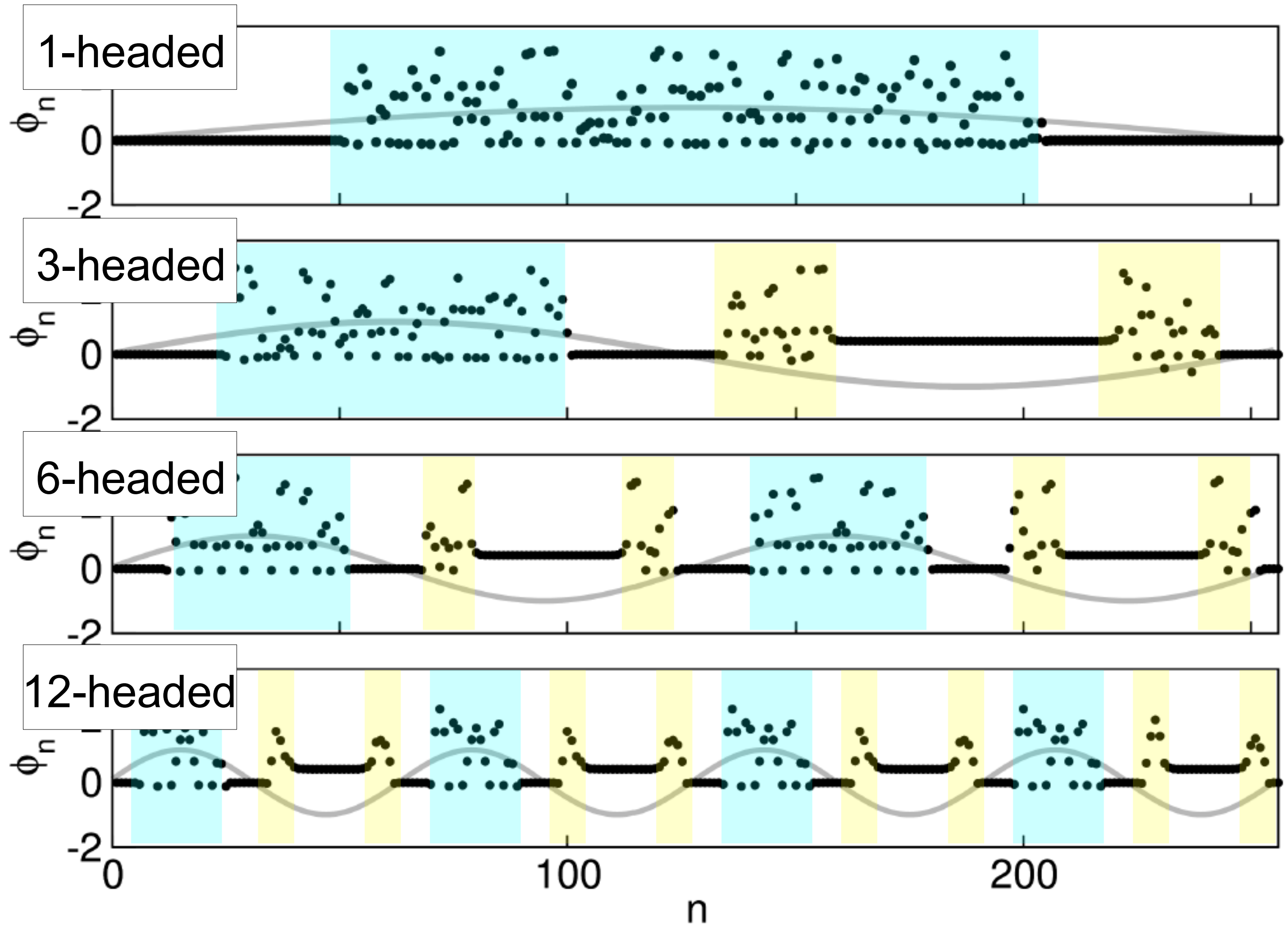
stroboscopic maps of various SQUID oscillators

huge multiplicity of attractors: **attractor crowding**

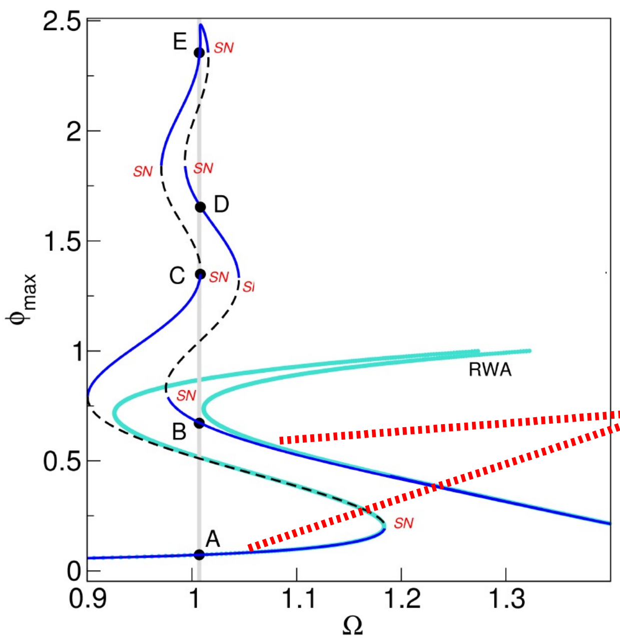
K. Wiesenfeld and P. Hadley Phys. Rev. Lett. **62**, 1335 (1989)

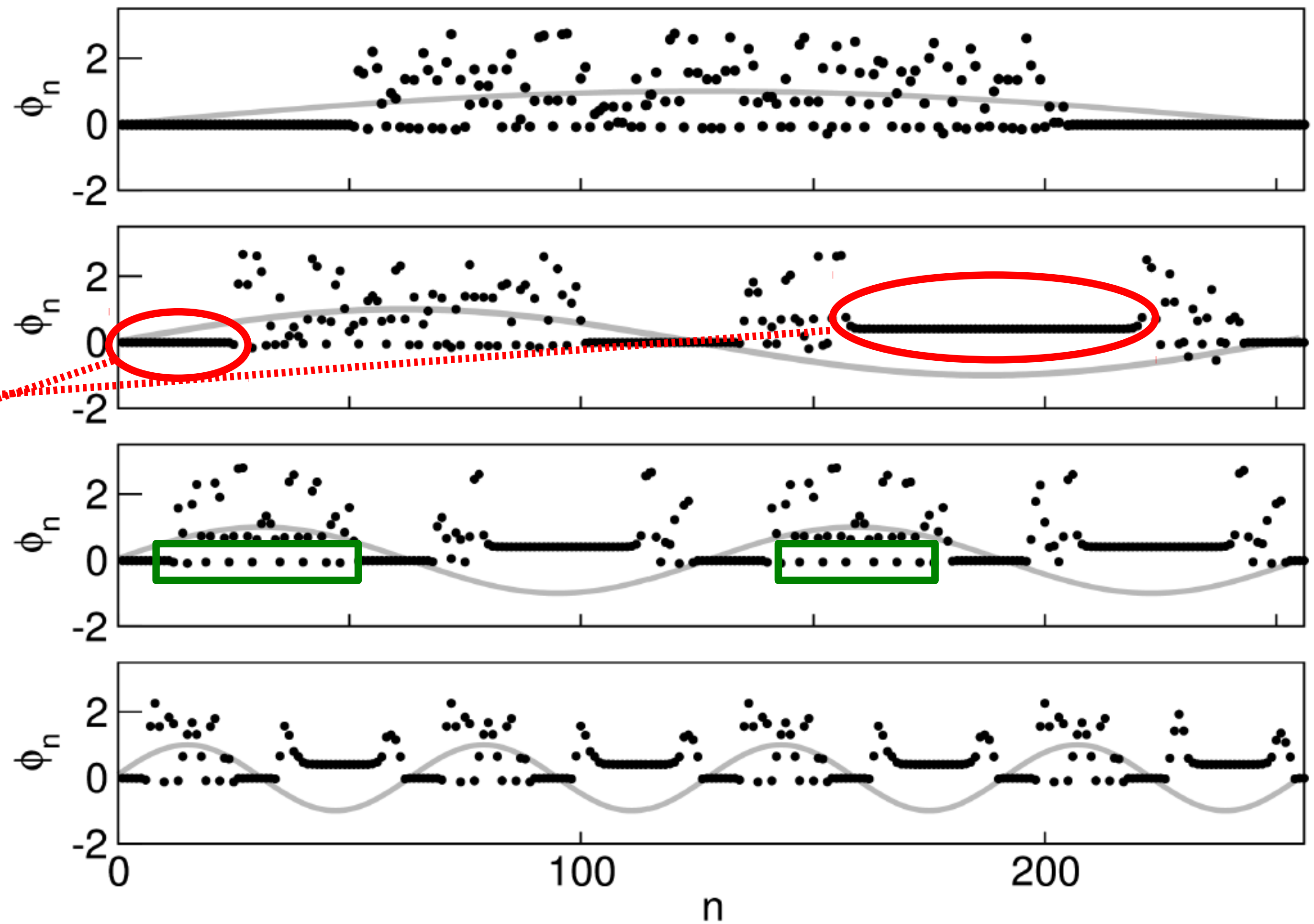
Multi-clustered chimera states (γ=0.024, λ=-0.025)





Multi-clustered chimera states *(γ=0.024, λ=-0.025)*





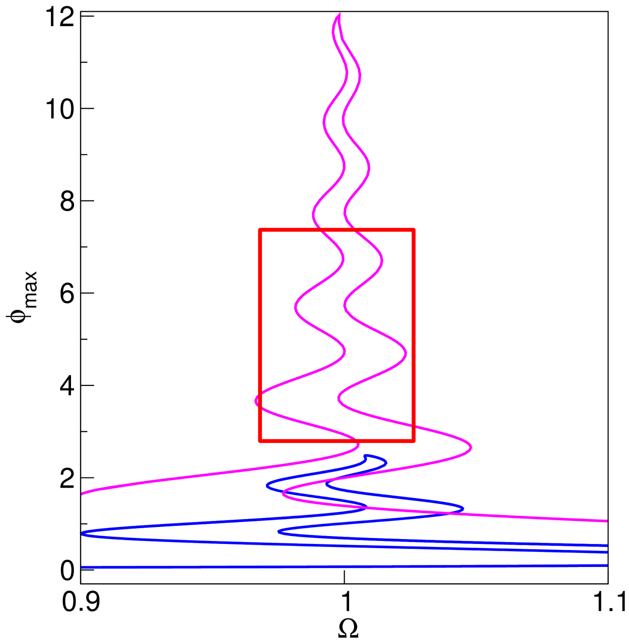
two “levels” of coherent clusters corresponding to two long stable branches

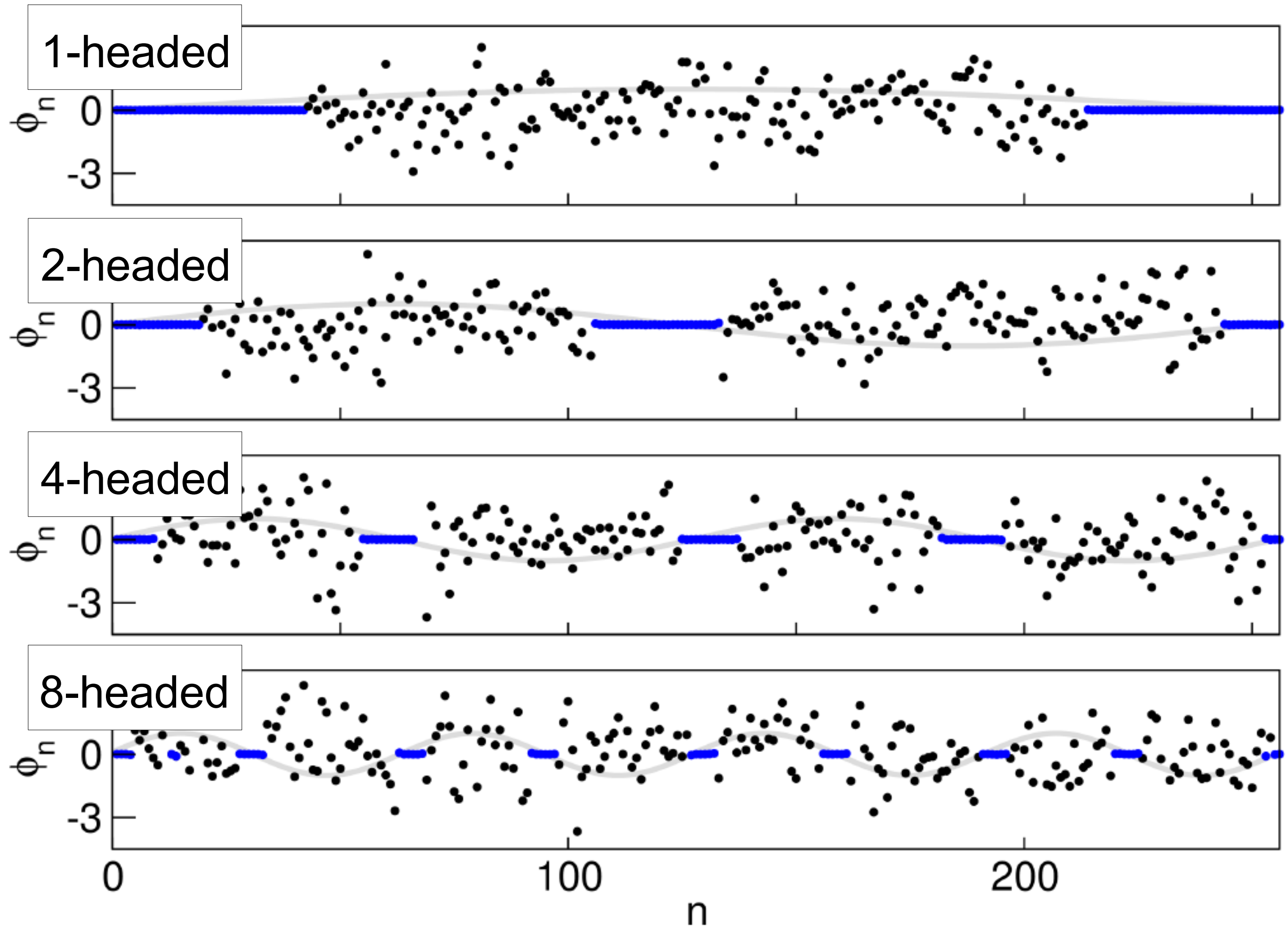
oscillators escaping incoherent cluster
solitary states

P. Jaros, Y. Maistrenko, and T. Kapitaniak, Phys. Rev. E **91**, 022907 (2015).

J. Hizanidis, N. Lazarides, G. Neofotistos, and G. P. Tsironis, Eur. Phys. J. Special Topics (to appear) (2016).

Multi-clustered chimera states *(γ=0.0024, λ=-0.025)*





coherent clusters are at one "level" - **long** stable branch of **low** amplitude

incoherent clusters are more chaotic - **multiple smaller** branches of **high** amplitude

Chimera states videos

https://www.researchgate.net/publication/301688296_Chimeras_in_locally_coupled_SQUIDs_Lions_goats_and_snakes

Chimeras in locally coupled SQUIDs: Lions, goats and snakes

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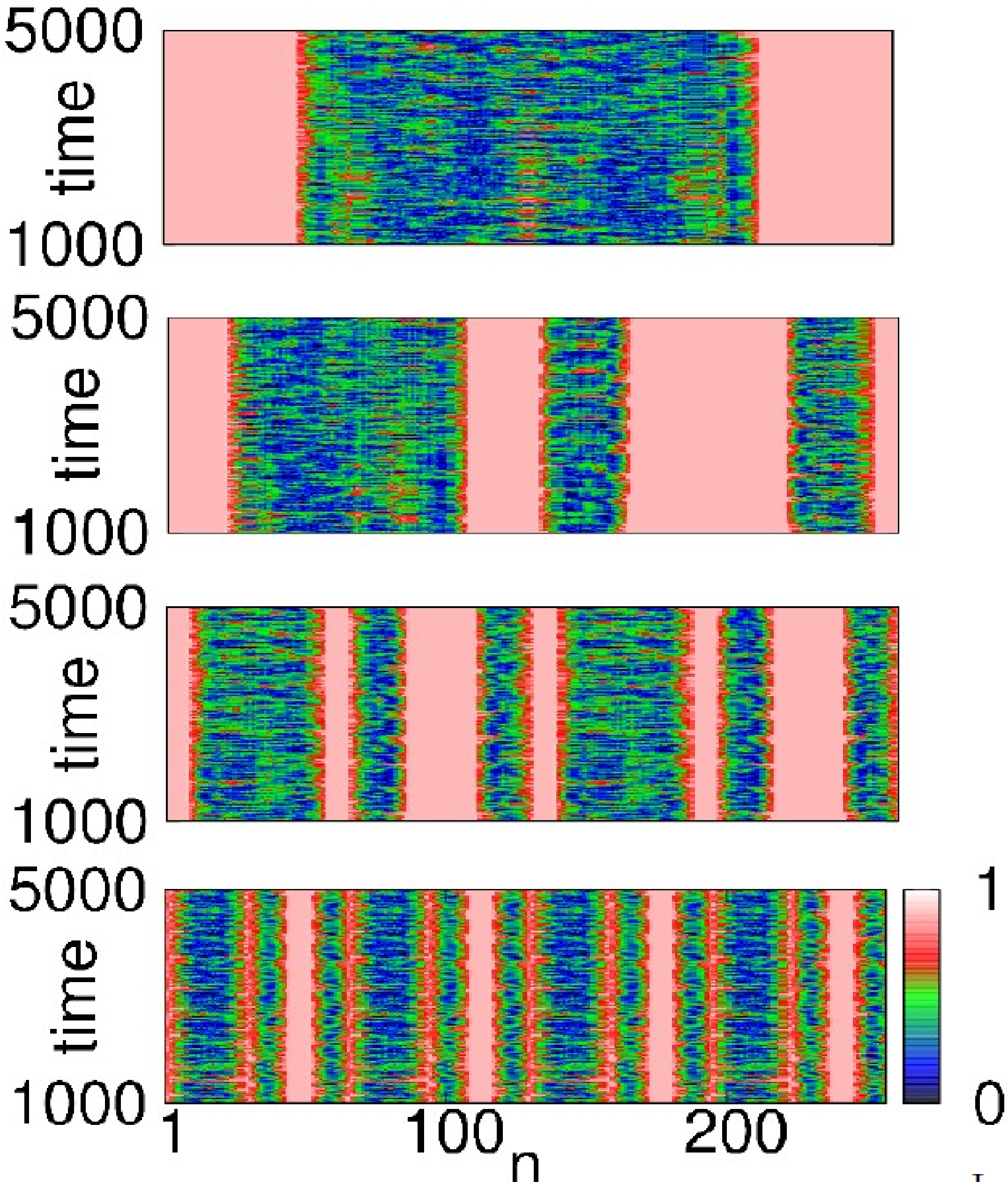
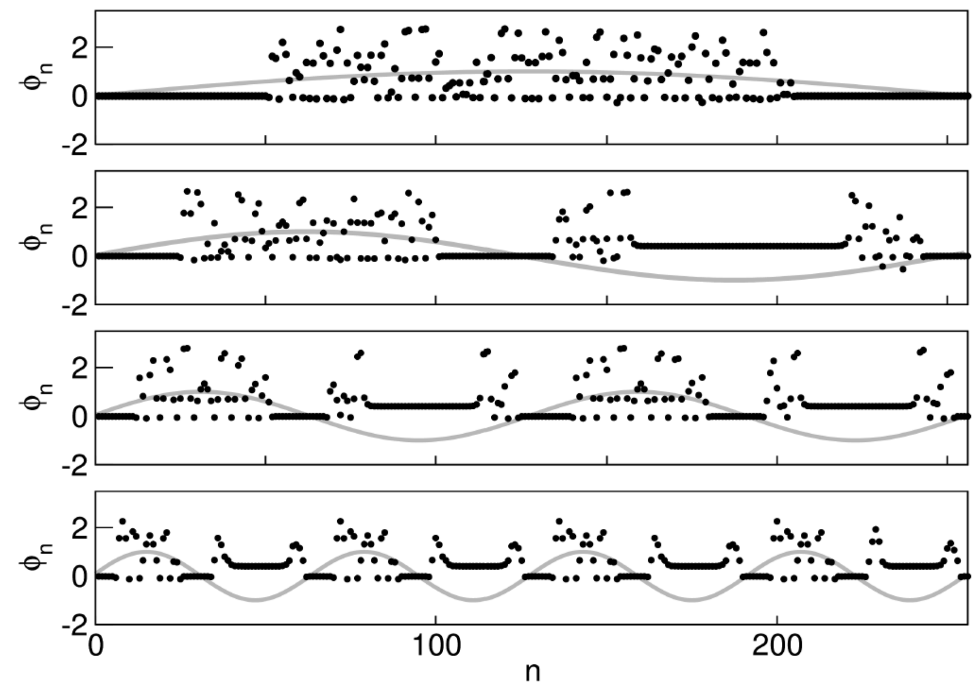
Measuring synchrony: Kuramoto local order parameter

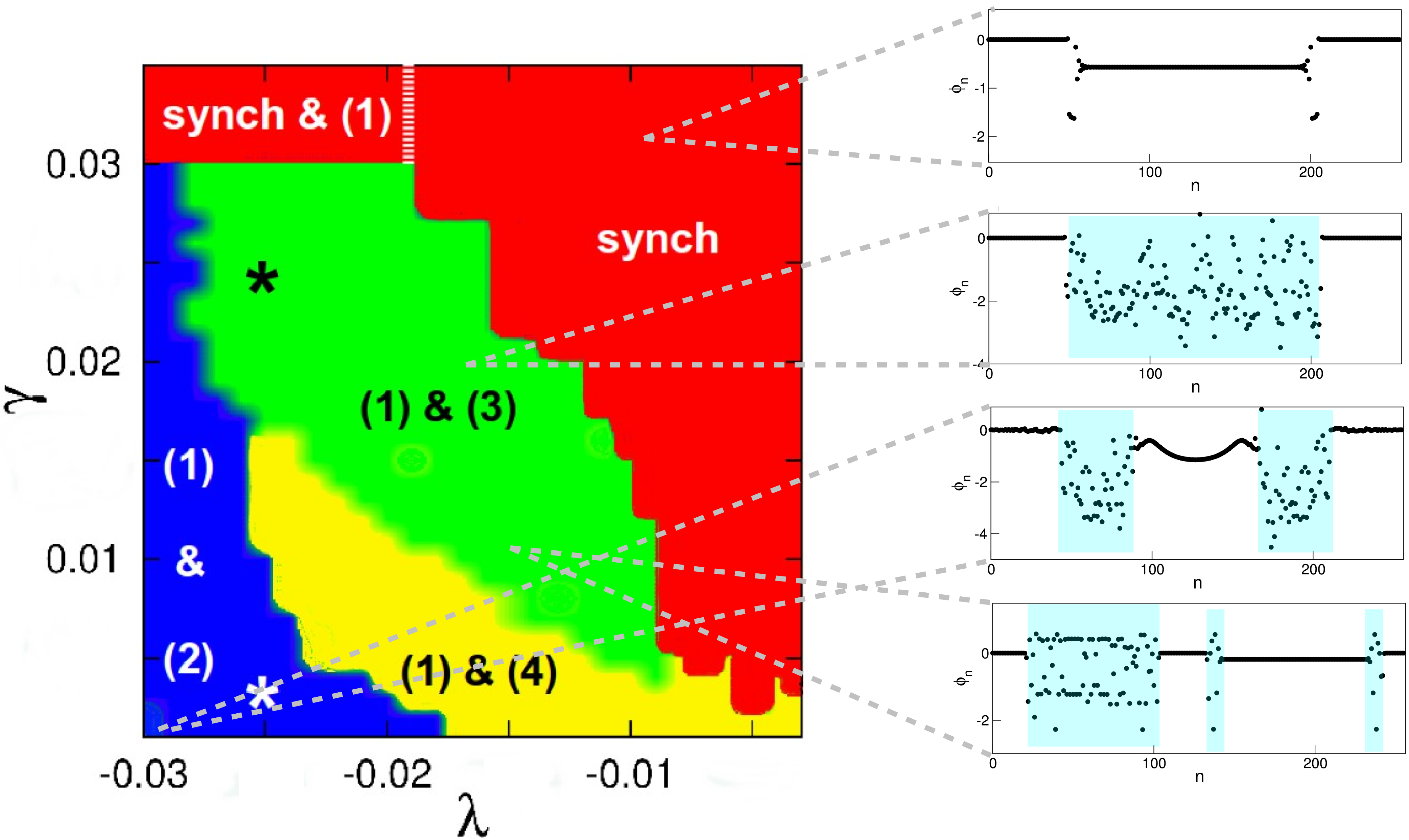




$$Z_n = \left| \frac{1}{2\delta} \sum_{|j-n| \leq \delta} e^{i\phi_j} \right|$$

I. Omelchenko, O. E. Omel'chenko, P. Hövel, and E. Schöll, Phys. Rev. Lett. **110**, 224101 (2013).

Dynamic regimes in the (λ,γ) parameter space



Conclusions & Outlook

- **Robust** multi-chimera states have been found in an array of SQUIDs with **nearest-neighbor interactions**
- Chimeras emerge due to **extreme multistability**
- **Attractor crowding** around the geometric resonance frequency some of which are chaotic
- Chimeras characterized through local synchronization
- Dynamical regimes in relevant parameter space reveals coexistence of multi-clustered chimera states
- **Experimental verification of SQUID chimeras?**

Acknowledgements

Crete Center for Quantum Complexity and Nanotechnology

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