Introduction: What is a chimera state?

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\[ \frac{\partial \phi}{\partial t} = \omega - \int_{-\pi}^{\pi} G(x - x') \sin [\phi(x, t) - \phi(x', t) + \alpha] \, dx' \]

Spatial coexistence of coherent/synchronized and incoherent/desynchronized domains in a dynamical network

- discovered by Kuramoto and Battogtokh in 2002
- named chimera states by Abrams and Strogatz in 2004
- identical elements, symmetric topology
Recent experimental verifications of chimera states

- **Optical experiment:** Spatial light modulator

- **Chemical experiment:** Light-sensitive BZ reaction

- **Mechanical experiment:** Coupled pendula

- **Electronic experiment:** Frequency-modulated delay oscillator
  Larger, Penkovsky, Maistrenko, *PRL* 111, 054103 (2013)

- **Electrochemical experiment:** Nickel electrodissolution

- **Electrochemical experiment:** Electro-oxidation of Si
Chimeras in nature?

**Unihemispheric sleep:** one half of the brain is highly synchronized (sleeping) while the other half remains desynchronized (awake).

- **dolphins and seals:** escaping from predators and surfacing for air while sleeping
- **humans**: first-night effect – our brain stays alert to protect against unknown danger.

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Chimera states for local coupling?

Superconducting Quantum Interference Devices (SQUIDs)

SQUIDs bring new features to the field of metamaterials

extraordinary electromagnetic properties not found in nature

highly nonlinear oscillator

complex collective behavior


Array of SQUIDs with **local coupling**

![Diagram of SQUIDs with local coupling](image)

- **Josephson junction**
- **Superconducting film**

**magnetic flux threading n-th loop:**

\[
\Phi_n = \Phi_{ext} + L I_n + M (I_{n-1} + I_{n+1})
\]

- **external flux**
- **self-inductance**
- **mutual inductance**

\[
I_n = -C \frac{d^2 \Phi_n}{dt^2} - \frac{1}{R} \frac{d\Phi_n}{dt} - I_c \sin \left(2\pi \frac{\Phi_n}{\Phi_0}\right)
\]

**Josephson relations**

\[
I(t) = I_c \sin \phi_J(t)
\]

\[
U(t) = \frac{\hbar}{2e} \dot{\phi}_J(t)
\]

Array of SQUIDs with \textbf{local coupling}

\[ \ddot{\phi}_n + \gamma \dot{\phi}_n + \phi_n + \beta \sin (2\pi \phi_n) = \lambda (\phi_{n-1} + \phi_{n+1}) + (1 - 2\lambda) \phi_{ac} \cos(\Omega \tau) \]

\[ \lambda = \frac{M}{L} \quad \text{dimensionless coupling coefficient} \]

\[ \phi_n, \phi_{ac} \quad \text{normalized to } \Phi_0 \]

\[ \Omega \quad \text{driving frequency normalized to } \omega_0 = \frac{1}{\sqrt{LC}} \]

\[ \beta = \frac{I_c L}{\Phi_0} = \frac{\beta_L}{2\pi} \quad \text{SQUID parameter} \]

\[ \gamma = \frac{1}{R} \sqrt{\frac{L}{C}} \quad \text{loss coefficient} \]
Array of SQUIDs with **local coupling**

\[ \ddot{\phi}_n + \gamma \dot{\phi}_n + \phi_n + \beta \sin (2\pi \phi_n) = \lambda (\phi_{n-1} + \phi_{n+1}) + (1 - 2\lambda) \phi_{ac} \cos(\Omega \tau) \]

parameters experimentally relevant

- \( \beta = 0.1369 \)
- \( \varphi_{ac} = 0.06 \)
- \( \gamma = 0.024 \)

single SQUID \( (n=1, \lambda=0) \)
Single SQUID snake-like resonance curve \((r=0.024)\)

- co-existence of **stable** and **unstable** periodic solutions
- multiple saddle-node bifurcations of limit cycles
Single SQUID snake-like resonance curve ($\gamma=0.024$)
Single SQUID snake-like resonance curve ($\gamma=0.024$)

\[ \ddot{\phi} + \gamma \dot{\phi} + \beta \sin(2\pi \phi) + \phi = \phi_{ac} \cos(\Omega \tau) \]

- series expansion (5 first terms)
- trial solution: $\phi = \phi_m(\tau) \cos(\Omega \tau + \theta(\tau))$
- Rotating Wave Approximation
- neglecting terms: $\phi_m, \dot{\theta}, \dot{\theta}^2, \phi_m \dot{\theta}$
Single SQUID snake-like resonance curve ($\gamma=0.024$)

$$
\Omega^2 = \Omega_{SQ}^2 - \beta_L \phi_m^2 \left\{ a_1 - \phi_m^2 \left[ a_2 - \phi_m^2 (a_3 - a_4 \phi_m^2) \right] \right\} \pm \frac{\phi_{ac}}{\phi_m}
$$

where:

$$
a_1 = \frac{\pi^2}{2}, \ a_2 = \frac{\pi^4}{12}, \ a_3 = \frac{\pi^6}{144}, \ a_4 = \frac{\pi^8}{2880}
$$
Single SQUID snake-like resonance curve ($\gamma=0.01$)

- As $\gamma$ decreases, the snake curve becomes more winding.
- The number of coexisting periodic orbits increases.
- (Un)stable branches more and smaller.
Two coupled SQUIDs ($\gamma=0.024, \lambda=-0.025$)

system complexity and multistability increases
Two coupled SQUIDs ($\gamma=0.024$, $\lambda=-0.025$) system complexity and multistability increases coexisting chaotic attractors around resonance frequency.
$N=256$ coupled SQUIDs ($\gamma=0.024$, $\lambda=-0.025$)

Stroboscopic maps of various SQUID oscillators

Huge multiplicity of attractors: attractor crowding

K. Wiesenfeld and P. Hadley
Multi-clustered chimera states ($\gamma=0.024$, $\lambda=-0.025$)
Multi-clustered chimera states ($\gamma=0.024$, $\lambda=-0.025$)

two “levels” of coherent clusters corresponding to two long stable branches

oscillators escaping incoherent cluster solitary states


Multi-clustered chimera states ($\gamma=0.0024$, $\lambda=-0.025$)

**coherent** clusters are at one "level" - long stable branch of low amplitude

**incoherent** clusters are more chaotic - multiple smaller branches of high amplitude
Chimera states videos

https://www.researchgate.net/publication/301688296_Chimeras_in_locally_coupled_SQUIDs_Lions_goats_and_snakes
Chimeras in locally coupled SQUIDs: Lions, goats and snakes

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CCQCN, University of Crete
Quantum Metamaterials &
Quantum Technology Conference
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Measuring synchrony: Kuramoto local order parameter

\[ Z_n = \left| \frac{1}{2\delta} \sum_{|j-n| \leq \delta} e^{i\phi_j} \right| \]

Dynamic regimes in the $(\lambda,\gamma)$ parameter space
Conclusions & Outlook

- Robust multi-chimera states have been found in an array of SQUIDs with nearest-neighbor interactions
- Chimeras emerge due to extreme multistability
- Attractor crowding around the geometric resonance frequency some of which are chaotic
- Chimeras characterized through local synchronization
- Dynamical regimes in relevant parameter space reveals coexistence of multi-clustered chimera states
- Experimental verification of SQUID chimeras?
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