Stability induced by *PT*-symmetry breaking in stochastic oscillators

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Non-Hermitian Photonics in Complex Media: PT-Symmetry and Beyond

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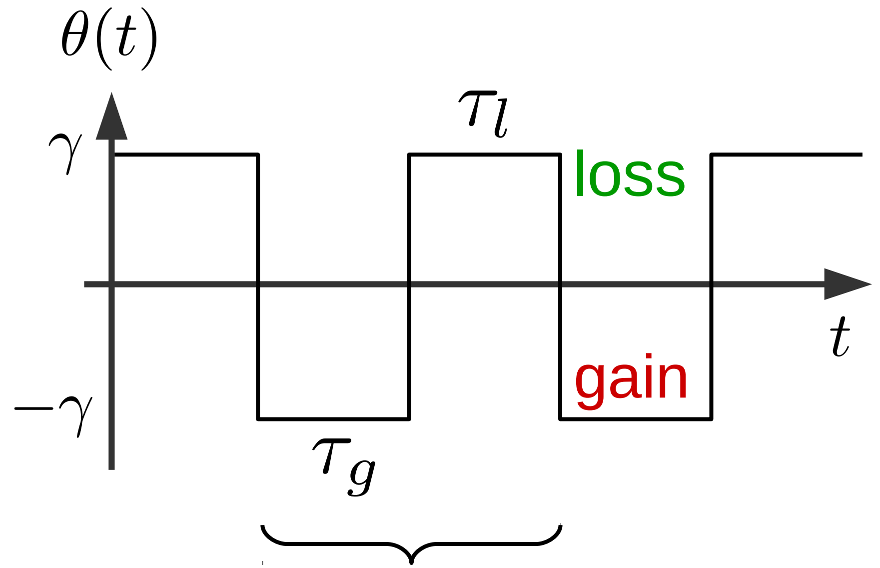
Classical *PT*-Symmetry in Zero Dimensions

$$\ddot{x} + 2\theta(t)\dot{x} + \omega^2 x = 0$$

$$\theta(t) = \pm\gamma$$

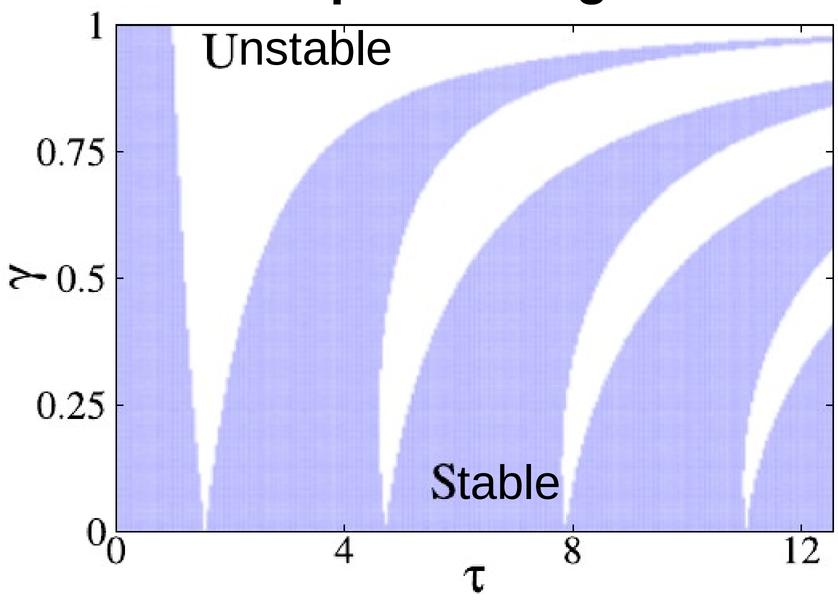
$\theta > 0$ The system is **dissipative**

$\theta < 0$ The system is **active**



$$T = \tau_g + \tau_l = \text{one period}$$

***PT* phase diagram**



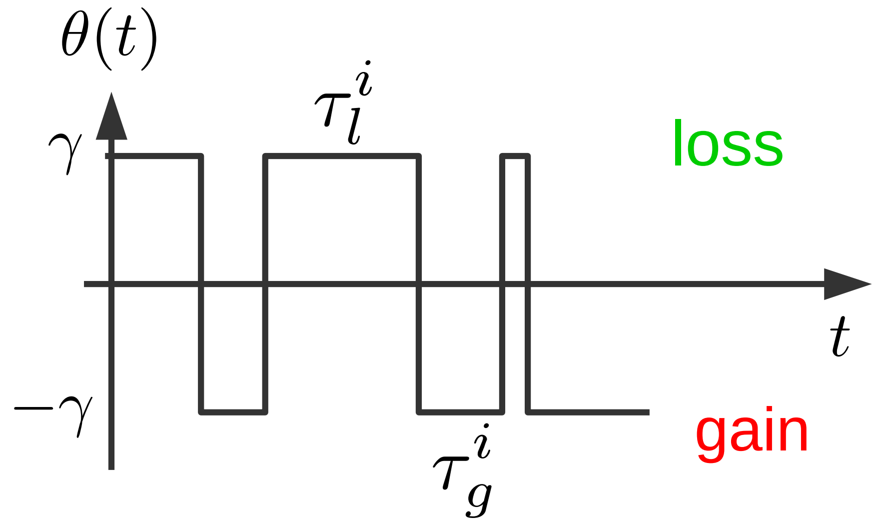
Gain and Loss perfectly **balanced**

$$\tau = \tau_l = \tau_g$$

Tsironis and Lazarides 2013

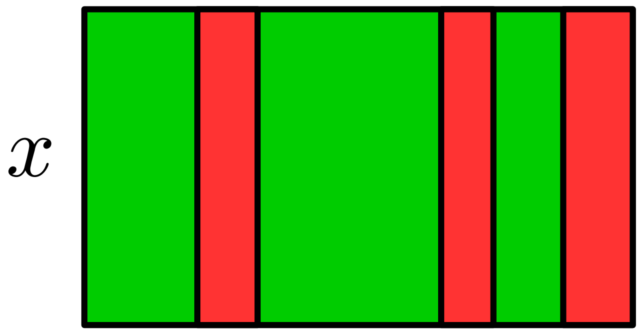
Harmonic Oscillator with **Stochastic** Damping

$$\ddot{x} + 2\theta(t)\dot{x} + \omega^2 x = 0$$



Exponential distribution of gain/loss times:

$$\psi_{g/l}(\tau) = \frac{1}{\tau_{g/l}} \exp\left(-\frac{\tau}{\tau_{g/l}}\right)$$



$\tau_l =$ average loss time

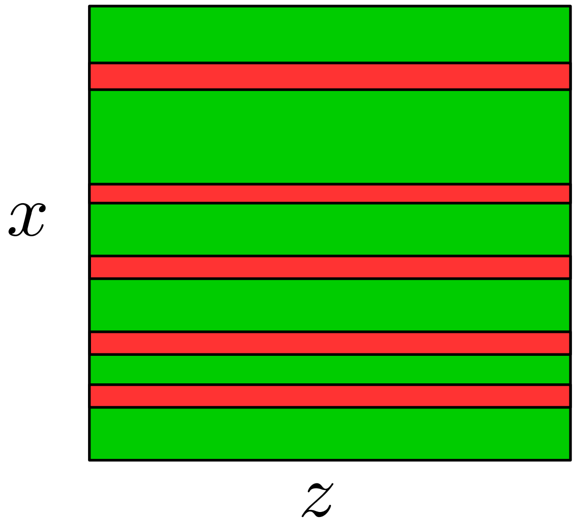
$\tau_g =$ average gain time

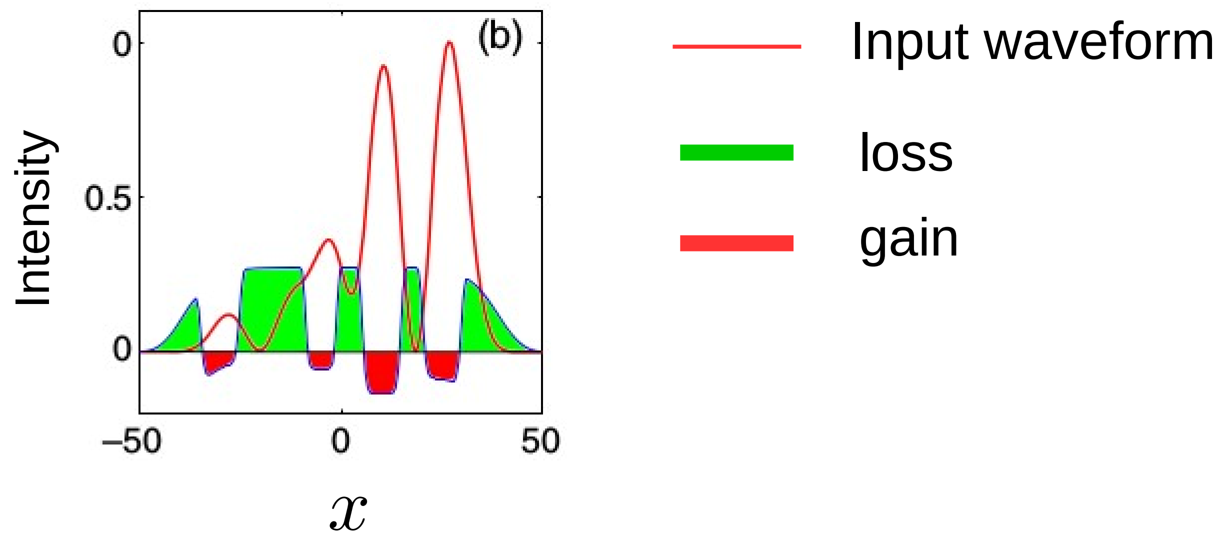
Stochastic system with probability evolution

$$P_g(x, \dot{x}, t) \quad P_l(x, \dot{x}, t)$$

Motivation

Random spatial distribution of gain and loss states – “Non-normal optical potentials”



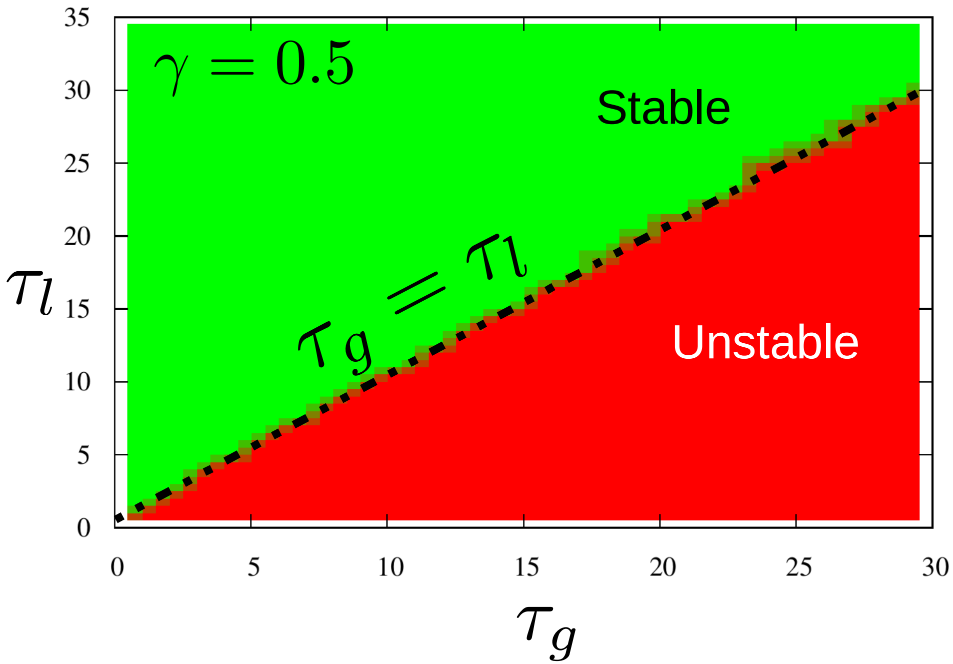


On average the distribution is lossy.

> “Depending on the spatial distribution of the gain and loss, there are optimal initial conditions for which the injected power can be amplified by several orders of magnitude, even though all the eigenmodes of the system are decaying.”

Makris et al., Phys. Rev. X (2014)

Simulation Results: Symmetry… but!



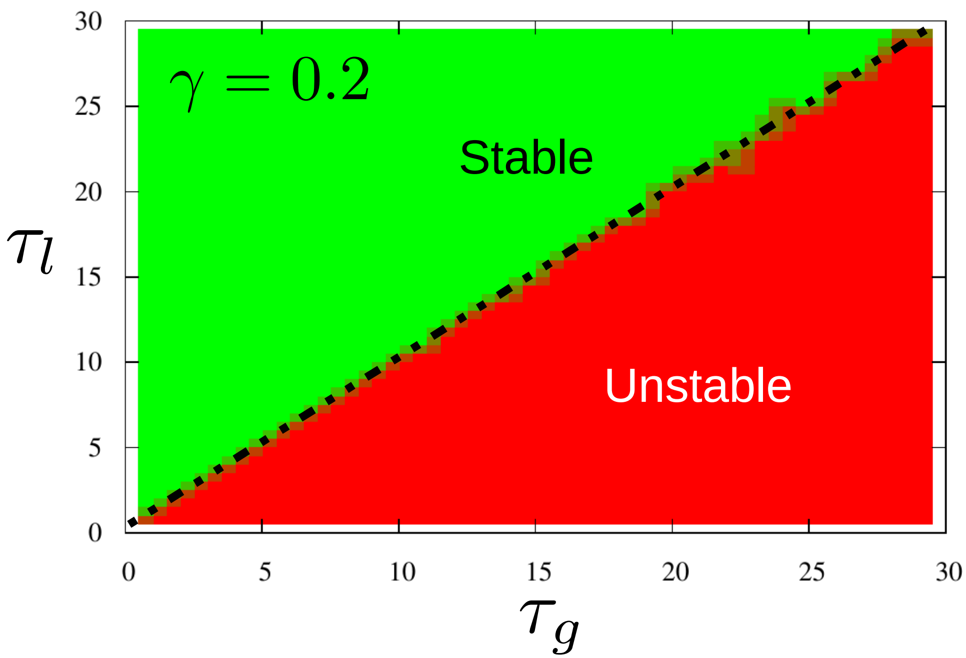
<u>4 parameters</u>

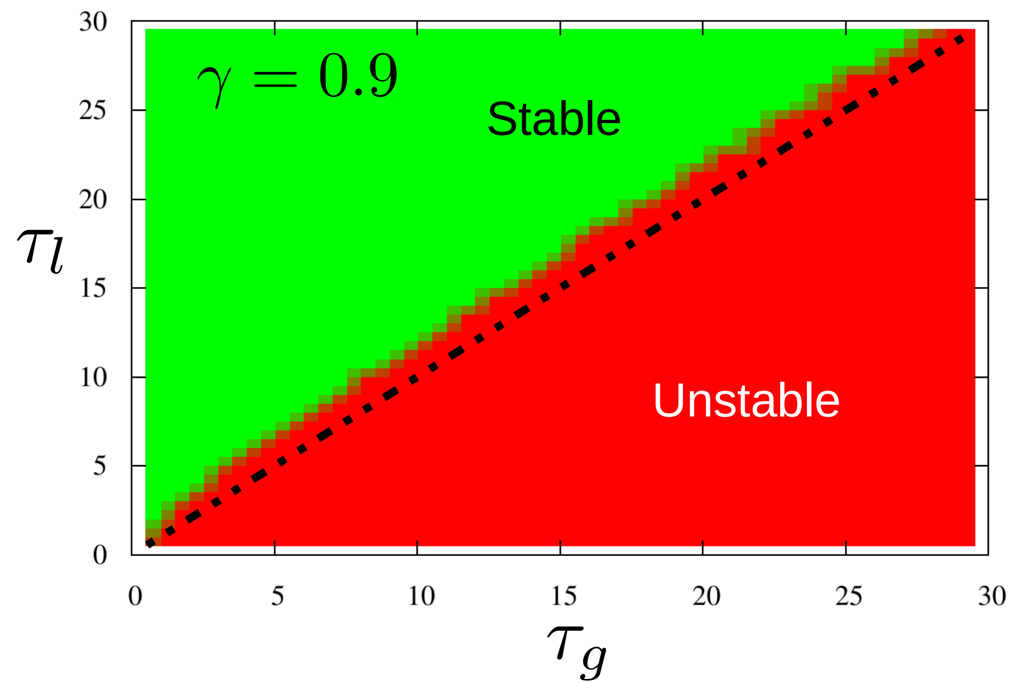
$\omega = 1$

$$\gamma < \omega$$

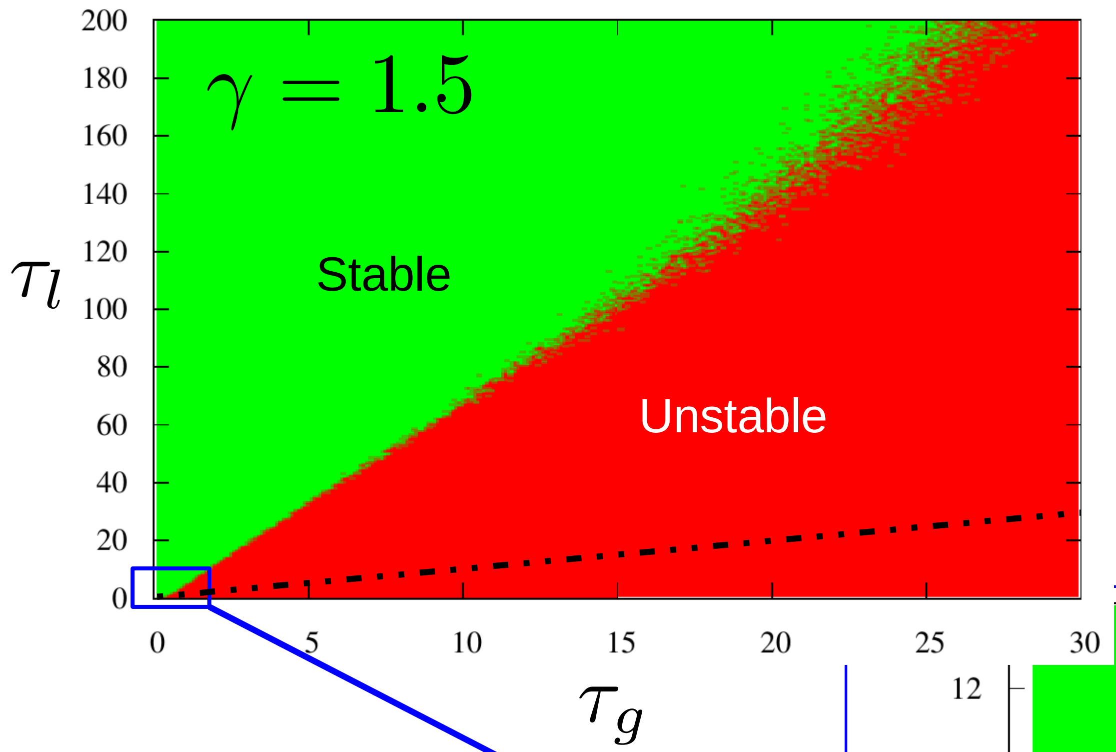
τ_l = average loss time

τ_g = average gain time



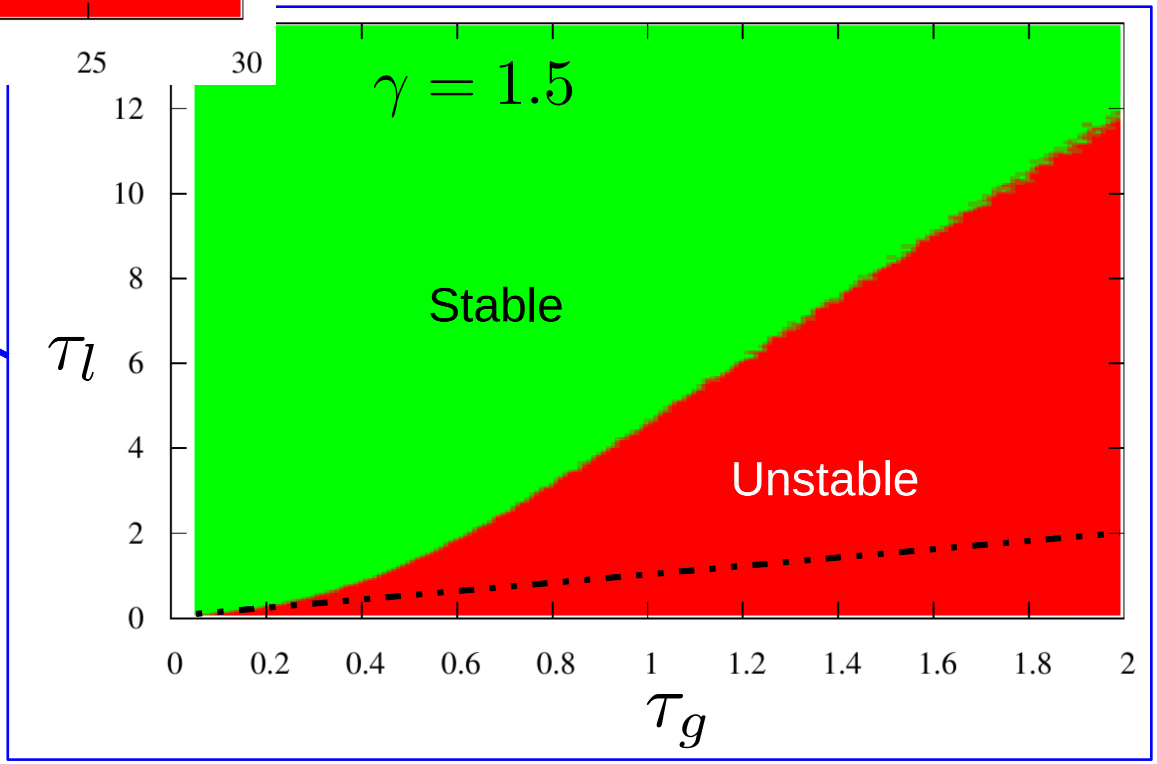


Simulation Results: Breaking of Symmetry



$\omega = 1$

$\gamma > \omega$



What is going on?

For **stochastic** systems:

Can we talk about symmetry?

Master Equation – stochasticity breaks symmetry

$$\ddot{x} + 2\theta(t)\dot{x} + \omega^2 x = 0$$

$$\dot{x} = v$$
$$\dot{v} = -2\theta(t)v - \omega^2 x$$

$$\overbrace{[\partial_t + v\partial_x - \omega^2 x\partial_v] P_{g/l}(x,v,t)}^{\text{Liouville term}} \pm \overbrace{2\gamma\partial_v(vP_{g/l}(x,v,t))}^{\text{loss/gain term}}$$

Evolution equation for

$P_g(x,v,t), P_l(x,v,t)$

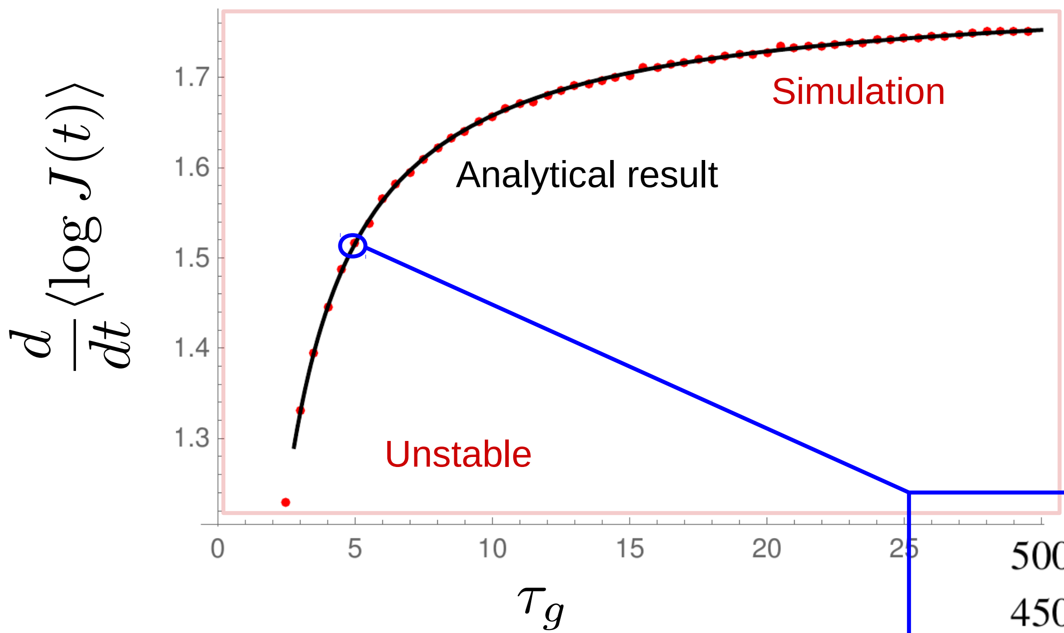
$$= \mp \underbrace{\left(\frac{P_g(x,v,t)}{\tau_g} - \frac{P_l(x,v,t)}{\tau_l}\right)}_{\text{Stochastic switching term (coupling)}}$$

Stochastic switching term

(coupling)

Slope – Theory vs. Simulations $\gamma < \omega$

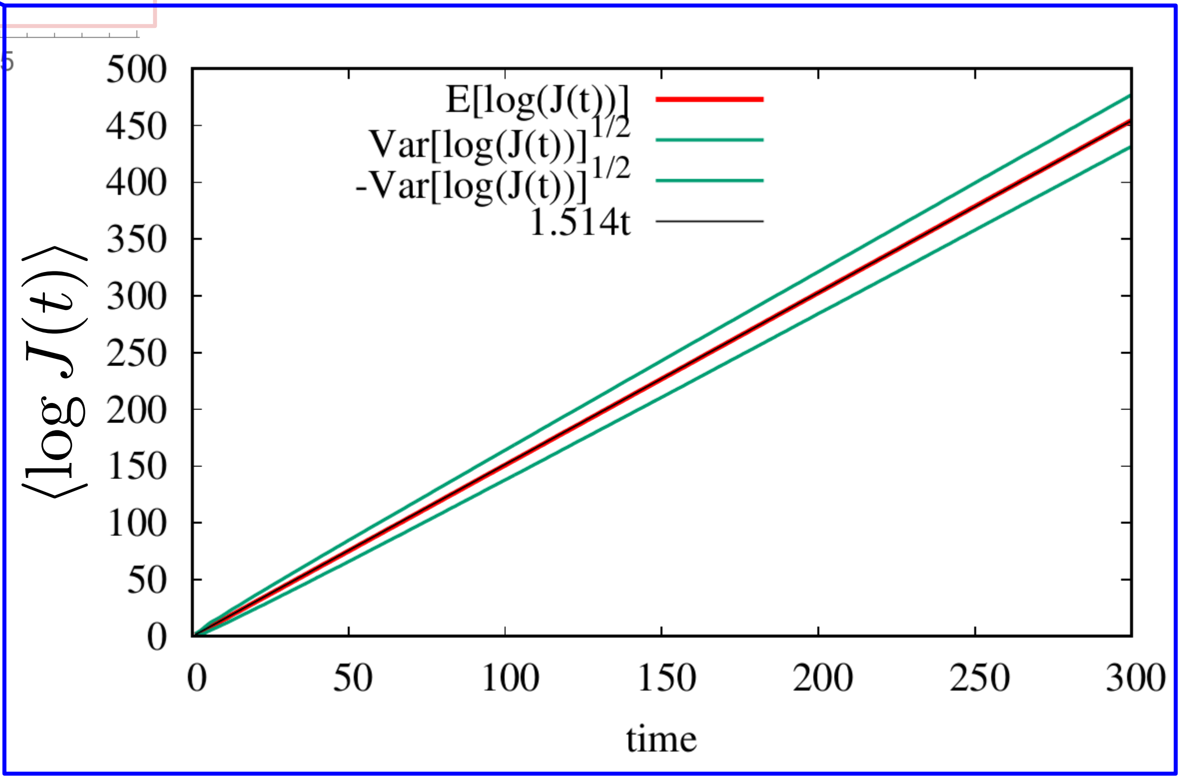
$\gamma = 0.9 \quad \omega = 1 \quad \tau_l = 1$



Relative variance shrinks

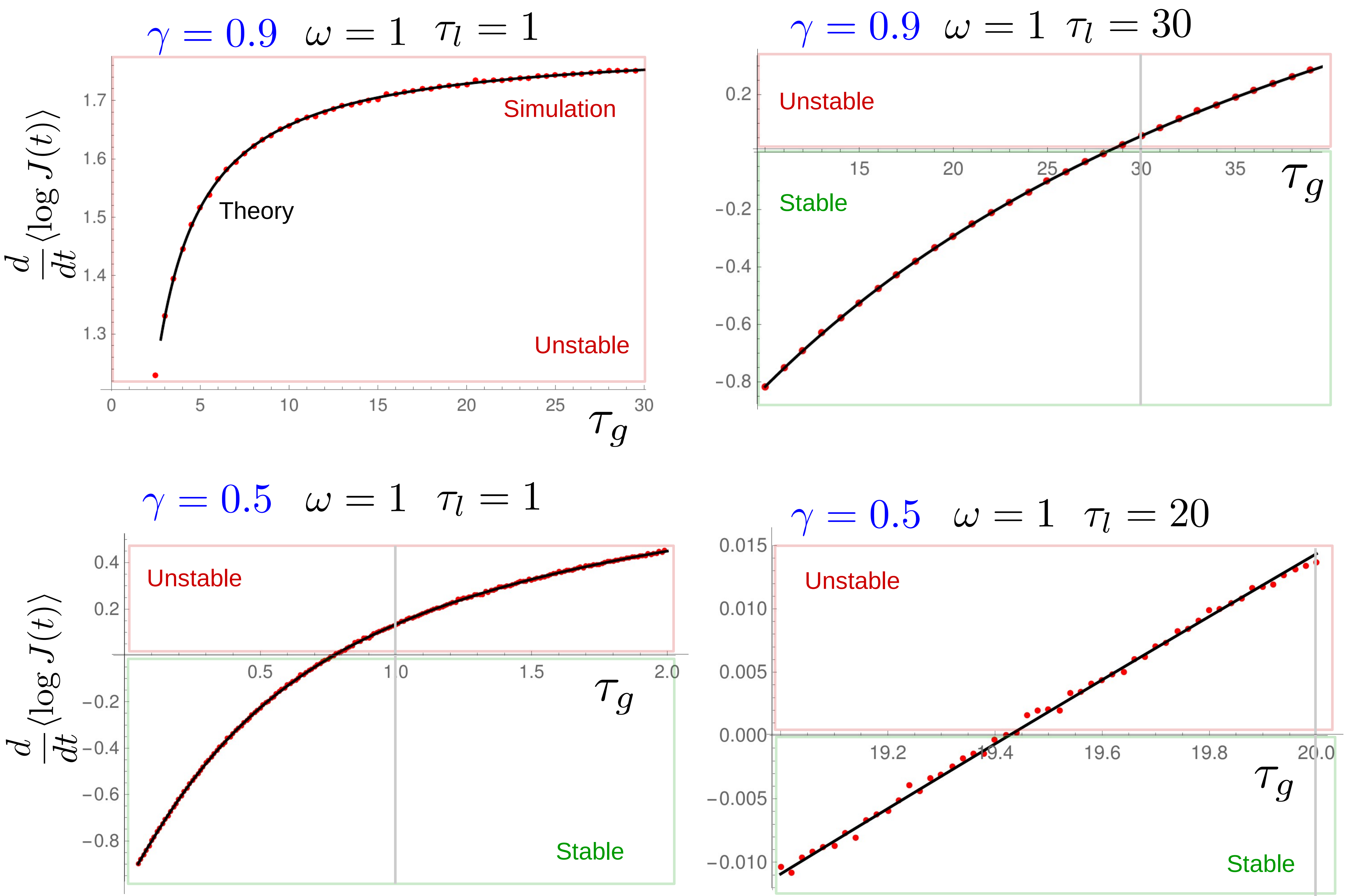
$$\frac{\sqrt{Var[\log J(t)]}}{\langle \log J \rangle} \sim \frac{1}{\sqrt{t}} \to 0$$

$\gamma = 0.9 \quad \omega = 1 \quad \tau_l = 1 \; \tau_g = 5$

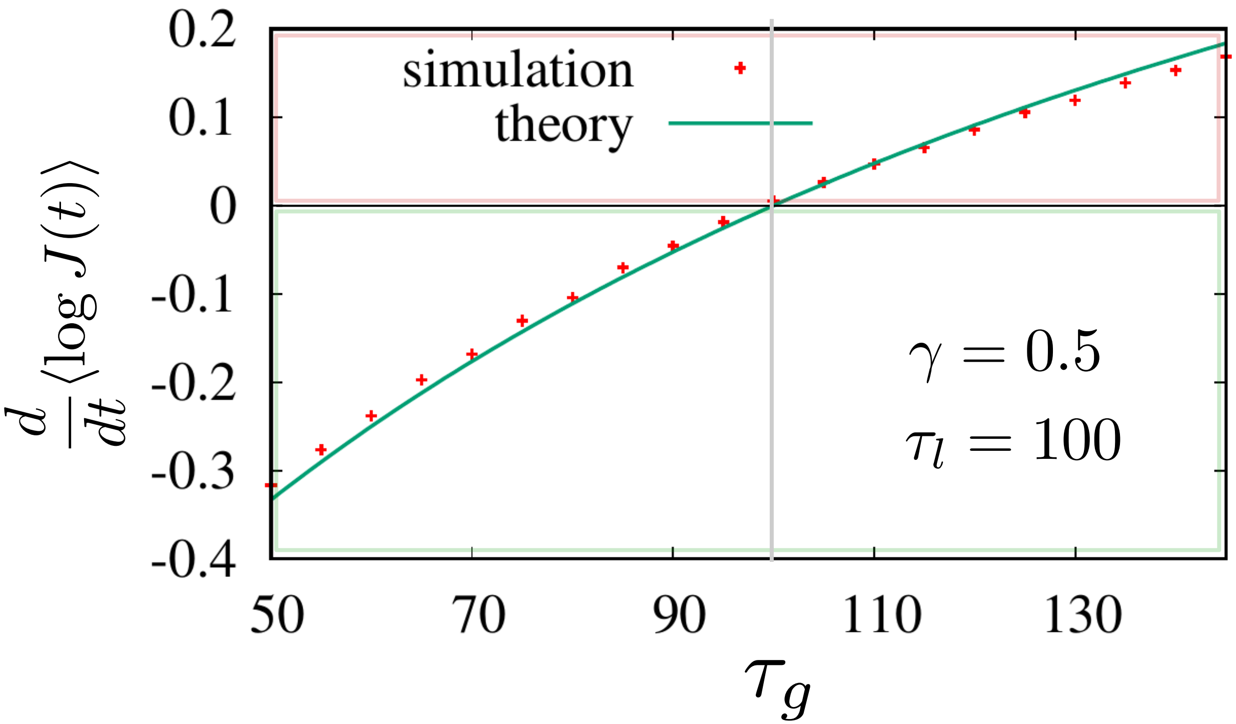


$\log J$ is a 'deterministic' quantity in the asymptotic limit

Slope – Stability Criterion $\gamma < \omega$



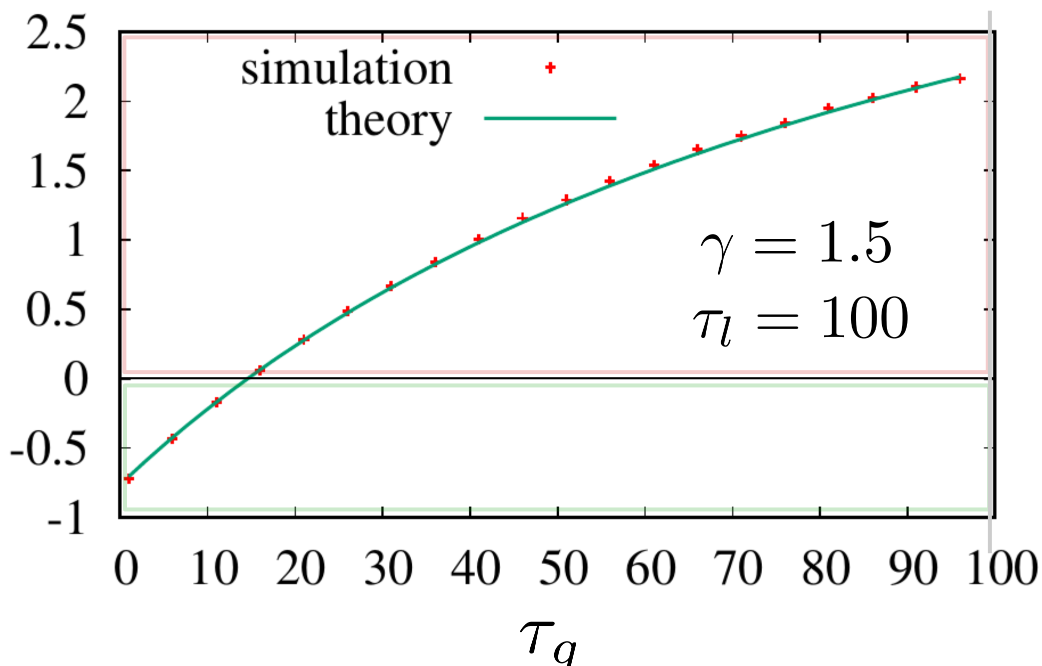
Slope for Large τ



Symmetry for $\gamma < \omega$

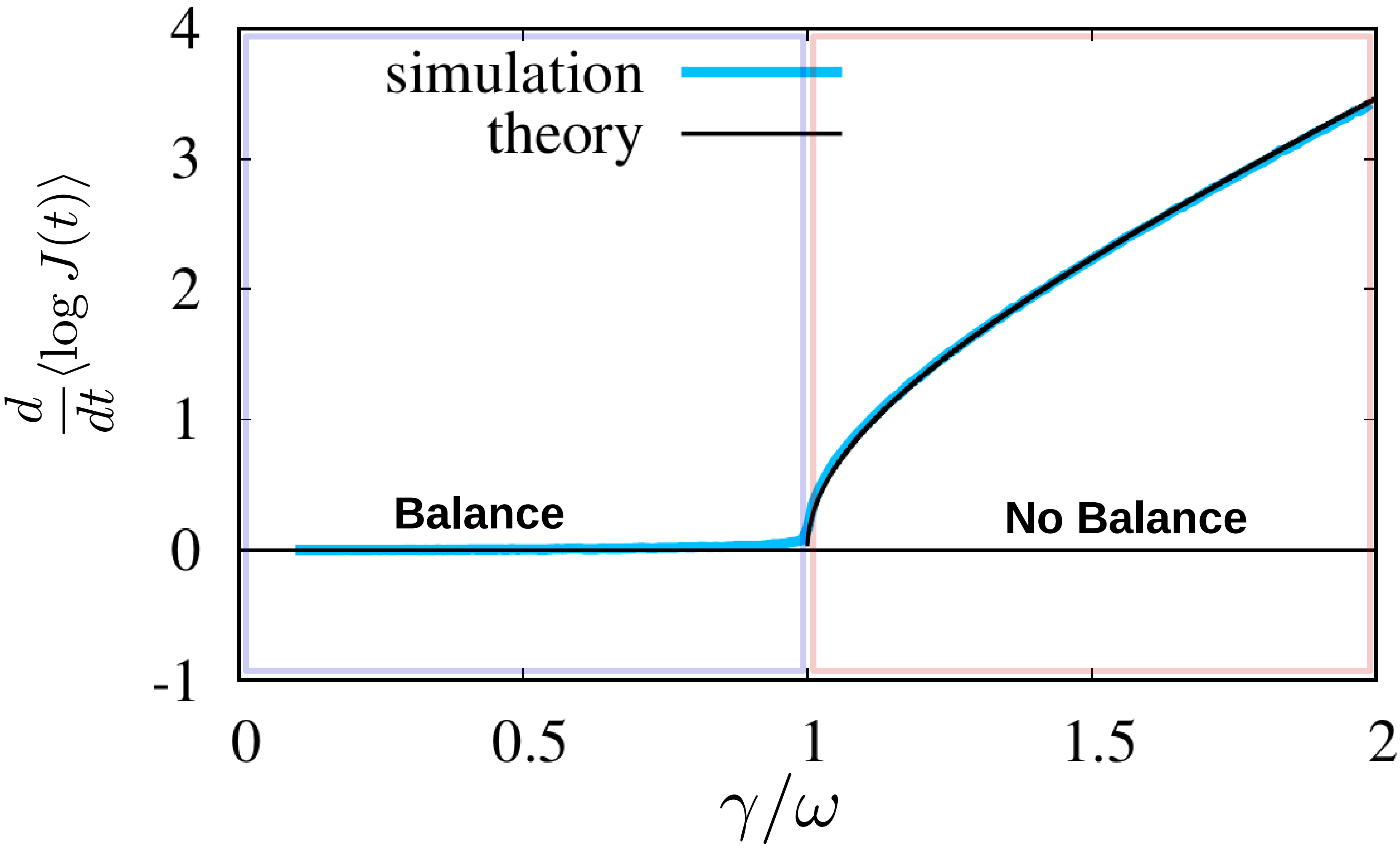
$$\tau_g, \tau_l \to \infty$$

Symmetry breaking for $\gamma > \omega$



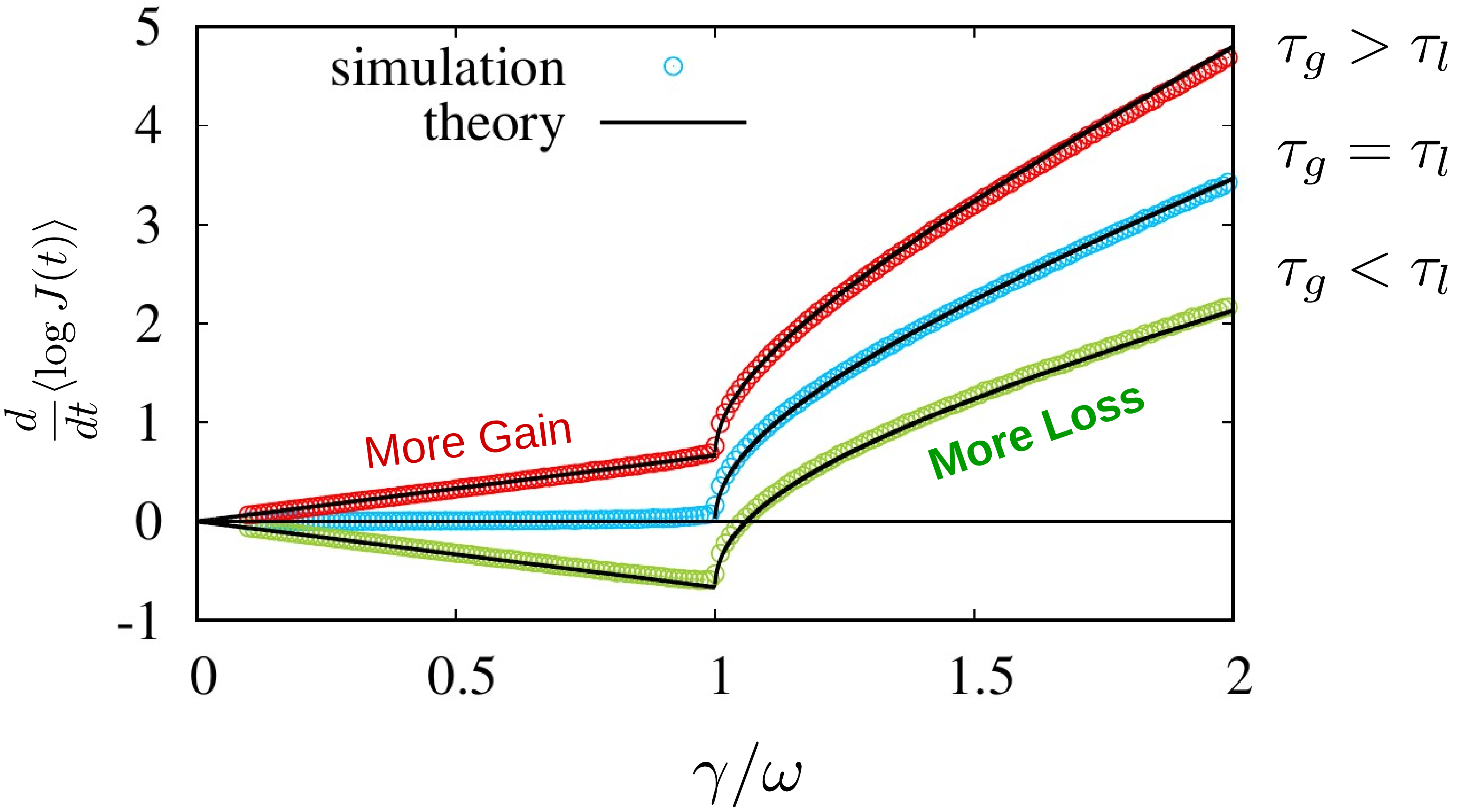
Symmetry Violation

Necessary conditions for symmetry: $\tau_g = \tau_L$ and $\tau_g, \tau_l \to \infty$



Result

We can achieve gain in a system that is dominated by dissipation
– Green curve



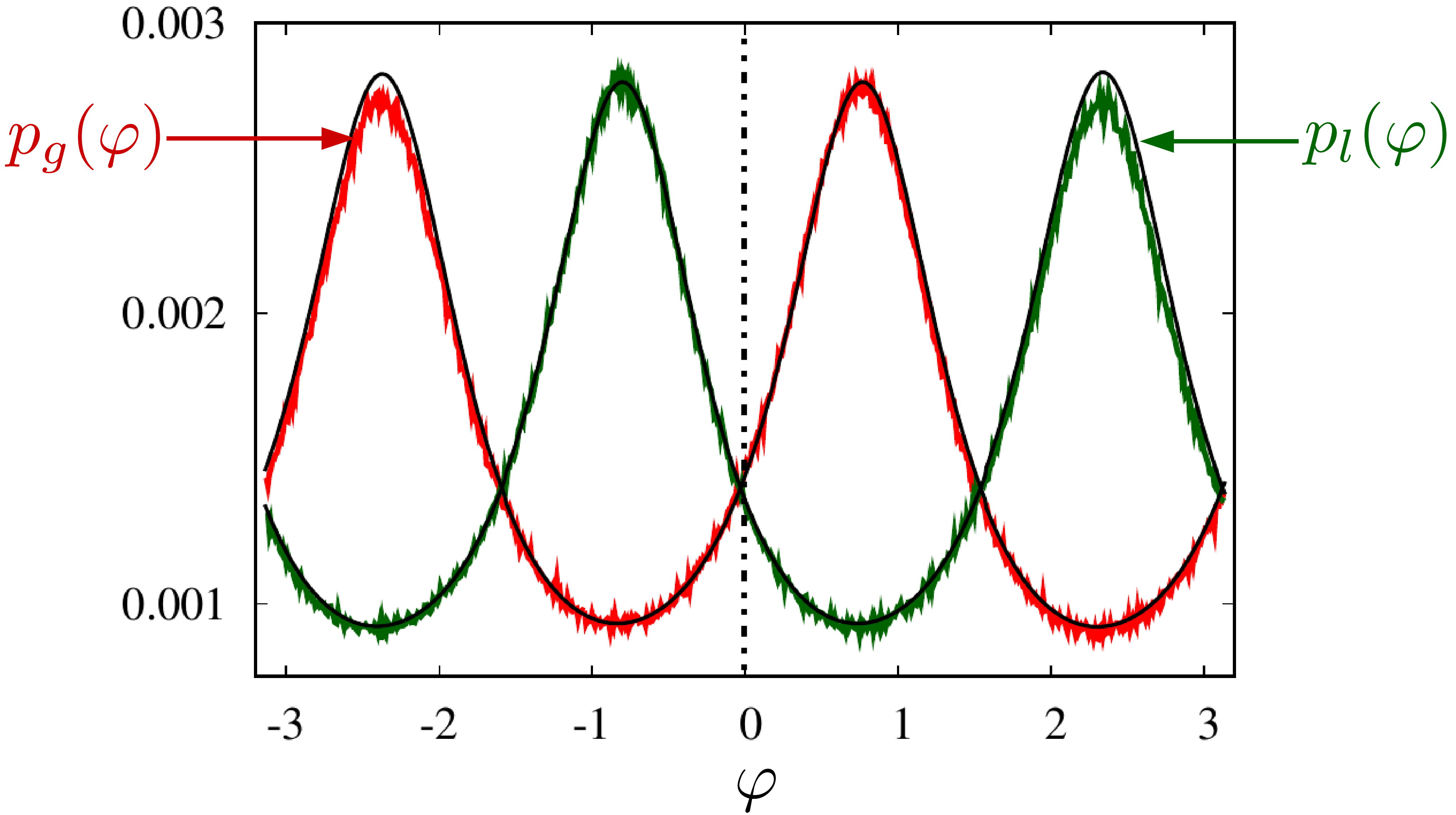
$P(\varphi)$: $\gamma < \omega$ $\quad \tau_g = \tau_l$ Balanced State

Broad distribution – '**disordered**' state

Undetermined φ

$$\frac{d}{dt}\langle \log J(t) \rangle = 0$$

Mirror symmetry



Statistical Symmetry

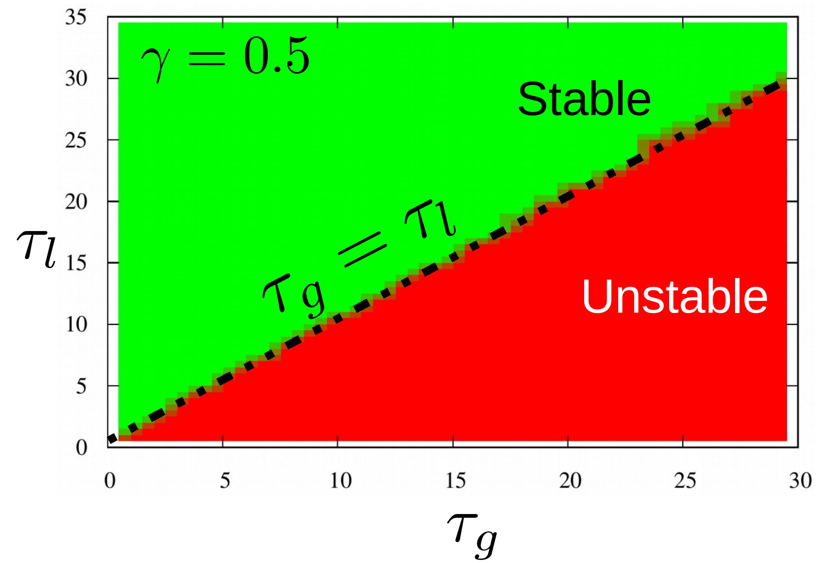
Instead… in the asymptotic limit we get:

<u>Effective **Mirror** symmetry</u>

- Gain-loss State Conjugation (C): $P_g \leftrightarrow P_l$
- PT-symmetry: $\varphi \to -\varphi \; (x \to -x, t \to -t \Rightarrow v \to v)$

$$\varphi = \tan^{-1}\left(\frac{\omega x}{v}\right)$$

Valid only for $\gamma < \omega$ and $\tau_g = \tau_l$!



Symmetry a gain-loss balance at the boundary between regions of stability and instability

$P(\varphi) : \gamma > \omega$ Symmetry violation

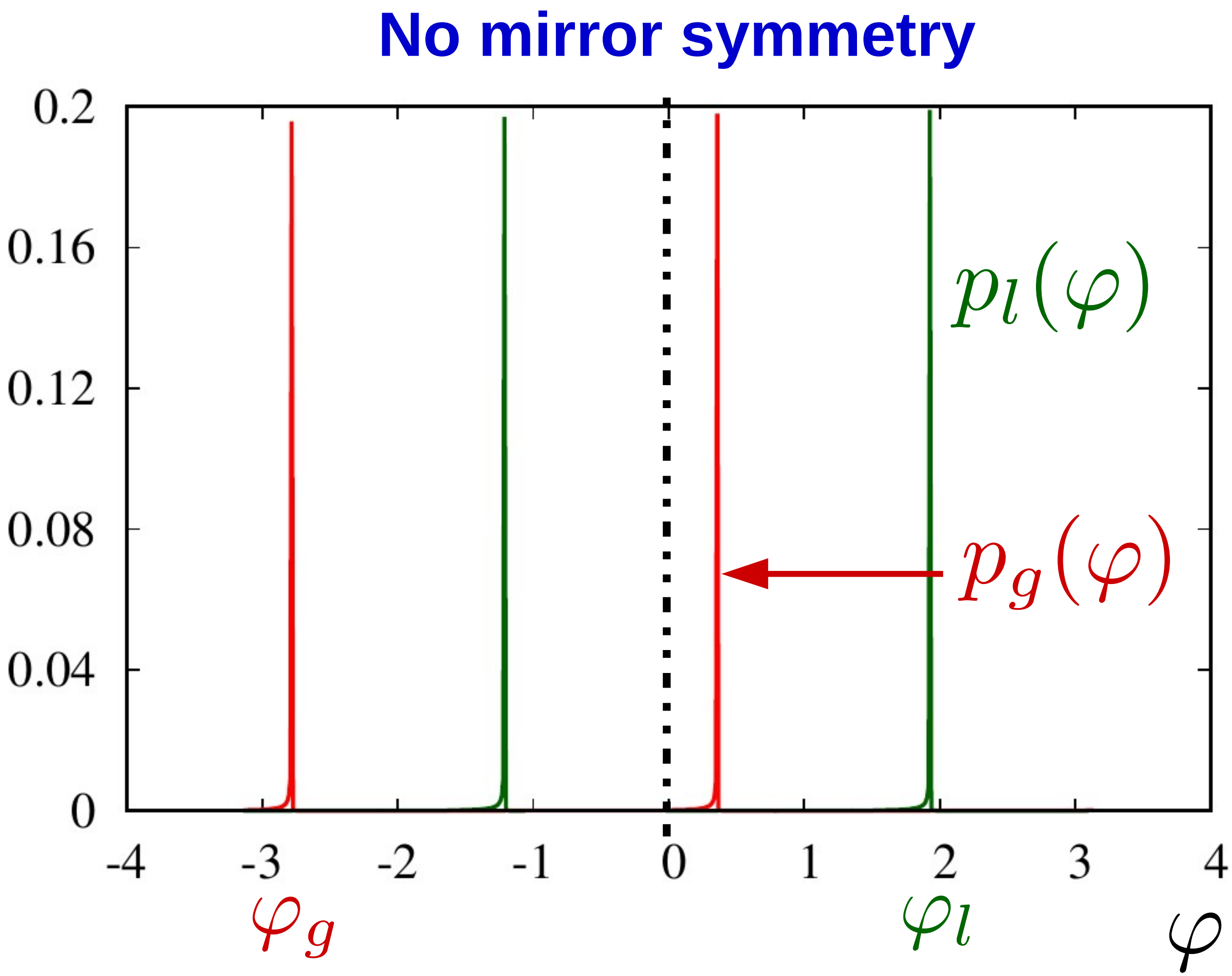
Transition to an **ordered** state: Two values: $\varphi_{g/l}$ and $\varphi_{g/l} + \pi$

$$(x_{g/l}, v_{g/l}) \to (-x_{g/l}, -v_{g/l})$$

Ratio Locking:

Locking of the ratio between the velocity and position

$$\tan \varphi_{g/l} = \omega \frac{x_{g/l}}{v_{g/l}}$$



Conclusion

J : Transition between stability and instability controlled by $\log J$.

φ :

- Disordered state for $\gamma < \omega$ so that we have gain/loss balance, no ratio locking. **Effective PT-symmetry!**
- Once $\gamma > \omega$ we have symmetry breaking, a transition to an ordered state in the large τ limit and ratio locking.

- **Stochastic** system with gain and loss states
- A simple classical model for gain/loss **management**
- The system is **tunable**
- The system displays **amplifications** even in this predominantly **dissipative** case