

The role of charge conjugation symmetry in topological photonics

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Heraklion, 16 June 2016

Classification of electronic topological systems

Time reversal symmetry	\mathcal{T}	:	$\mathcal{T} H \mathcal{T} = H$	reciprocity
Chiral symmetry	\mathcal{X}	:	$\mathcal{X} H \mathcal{X} = -H$	sublattices, Dirac systems
Charge conj. symmetry	$C = \mathcal{X} \mathcal{T}$:	$C H C = -H$	superconductors

→ **10 universality classes** with different response to defects, e.g.

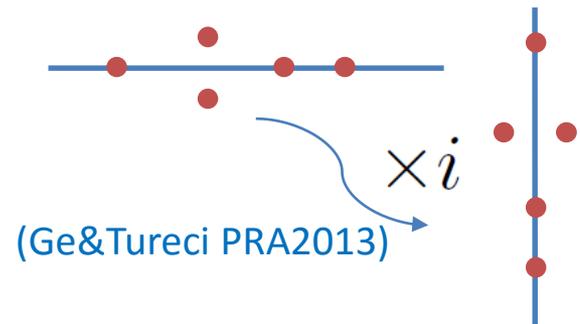
- edge and defect states
- parity/chiral anomaly
- charge fractionalization...

Here: extend to optical systems with loss and gain:
consequence of topological defects and anomalies?

PT symm.: $\mathcal{P} = \sigma_x \otimes 1_N$ (map between equivalent subspaces)

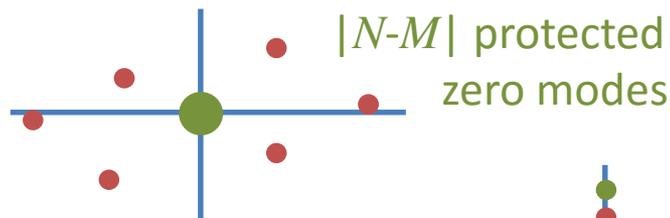
PT $\mathcal{P}H\mathcal{P} = H^* : H = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}$

anti-PT $\mathcal{P}H\mathcal{P} = -H^* : H = \begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}$



topological variant: $\mathcal{Z} = \begin{pmatrix} 1_N & 0 \\ 0 & -1_M \end{pmatrix}$ (\mathbb{Z}_2 gauge transformation)

chiral $\mathcal{Z}H\mathcal{Z} = -H : H = \begin{pmatrix} 0 & B \\ B' & 0 \end{pmatrix}$



charge-conjugate $\mathcal{Z}H\mathcal{Z} = -H^* : H = \begin{pmatrix} iA & B \\ B' & iA' \end{pmatrix}, A, A', B, B' \text{ real}$



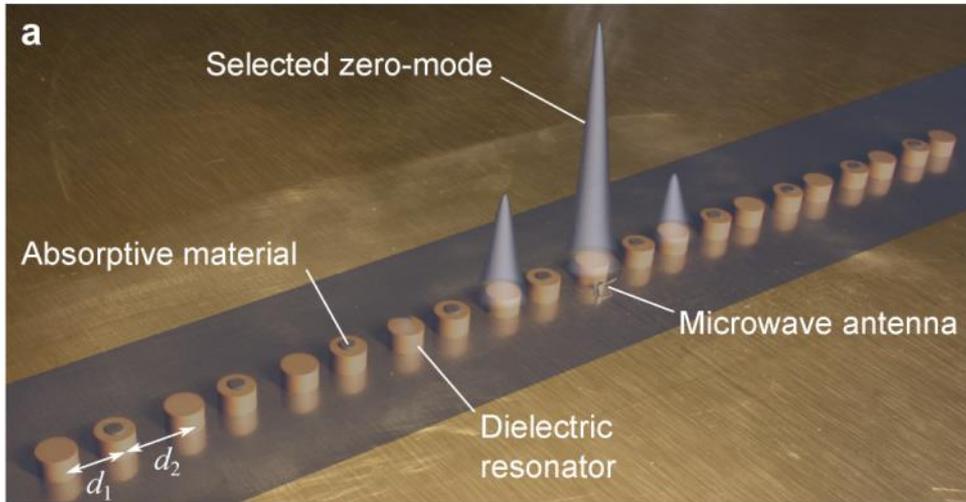
+ reciprocity: $H = H^T, \mathcal{Z}H\mathcal{Z} = -H^\dagger$

$$\langle \psi | \mathcal{Z}H | \psi \rangle = E \langle \mathcal{Z} \rangle = -\langle \psi | H^\dagger \mathcal{Z} | \psi \rangle = -E^* \langle \mathcal{Z} \rangle \Rightarrow \text{Re } E = 0 \text{ or } \langle \mathcal{Z} \rangle = 0$$

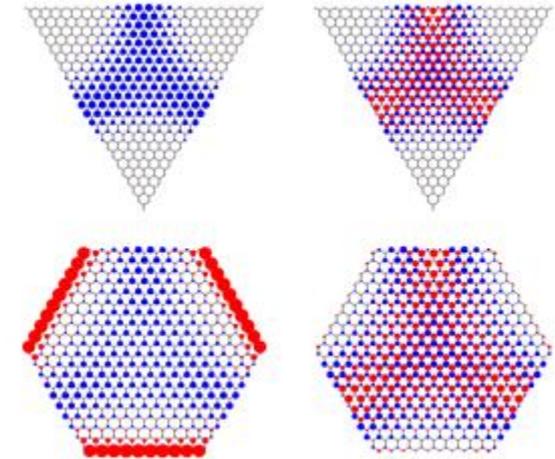
Topological modes have finite $\langle \mathcal{Z} \rangle$: can manipulate selectively

mode selection via topological anomalies in 1D and 2D

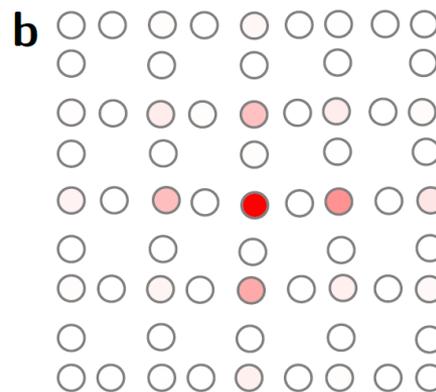
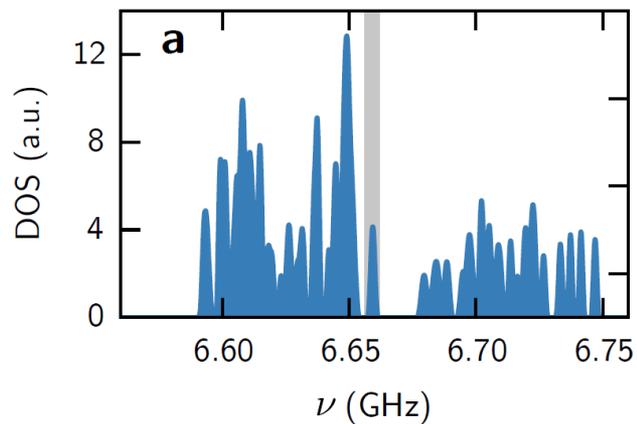
complex SSH chain



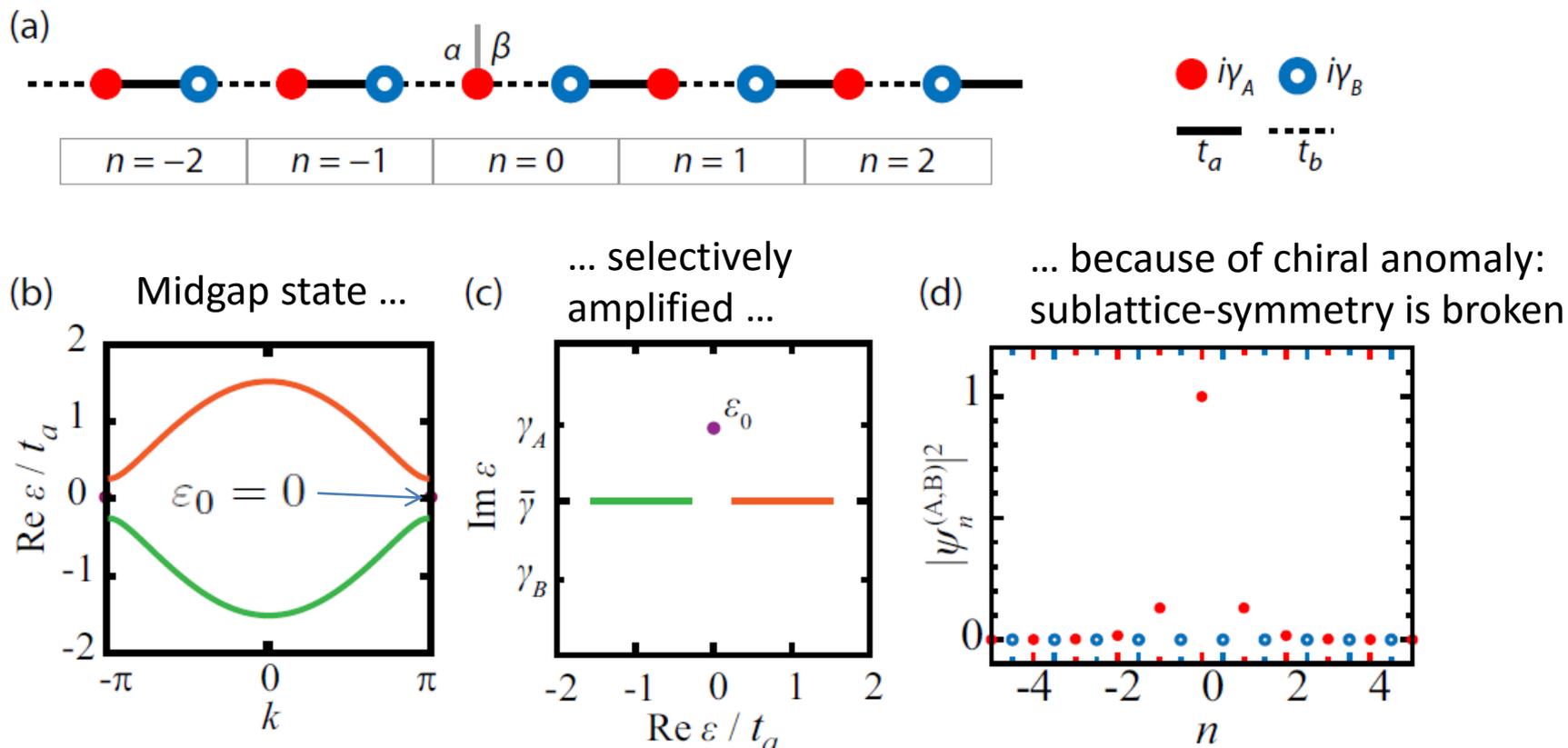
honeycomb lattices



dimerised Lieb lattice



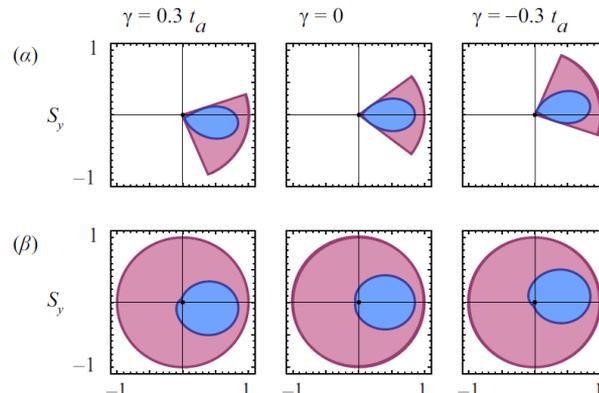
complex SSH chain



**Top. characterization survives
PT-sym. loss and gain**

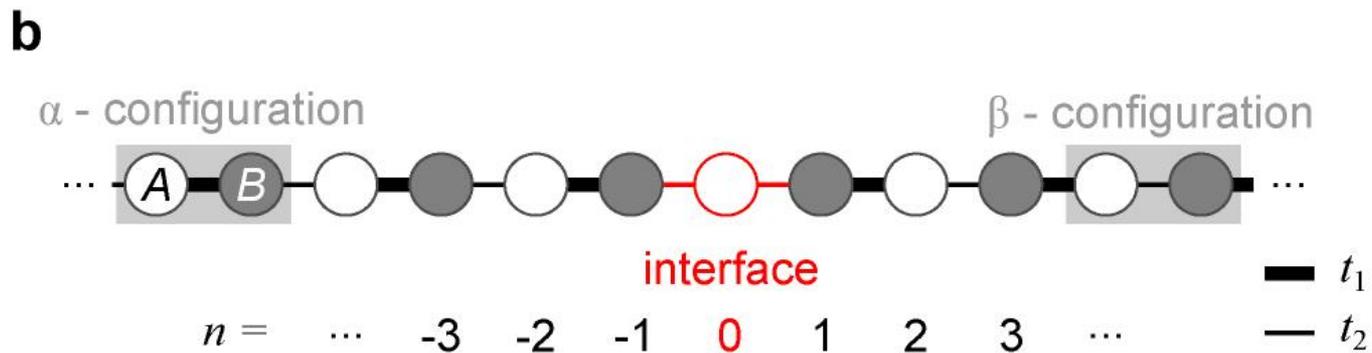
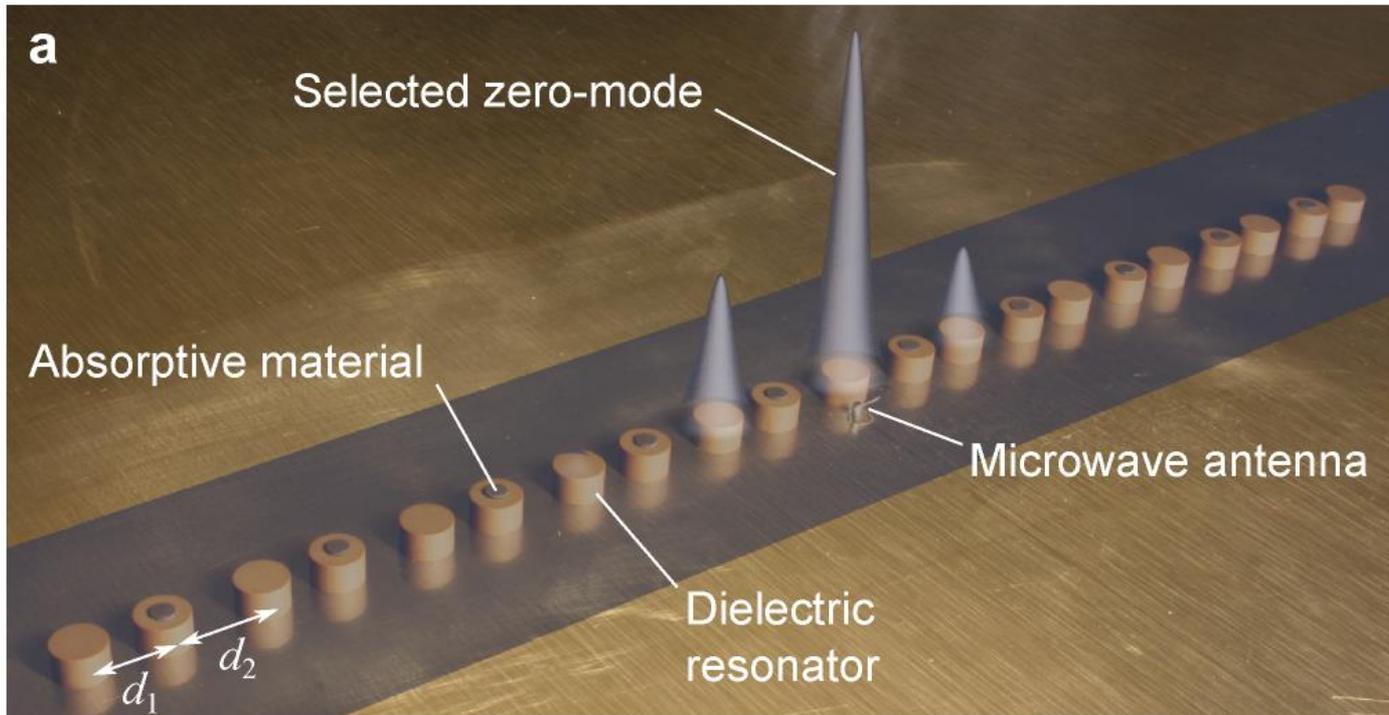
$$\mathbf{S} = \langle (\sigma_x, \sigma_y, \sigma_z) \rangle = (S_x, S_y, S_z)$$

$$|\varphi_A|^2 = |\varphi_B|^2, \text{ thus } S_z = 0$$

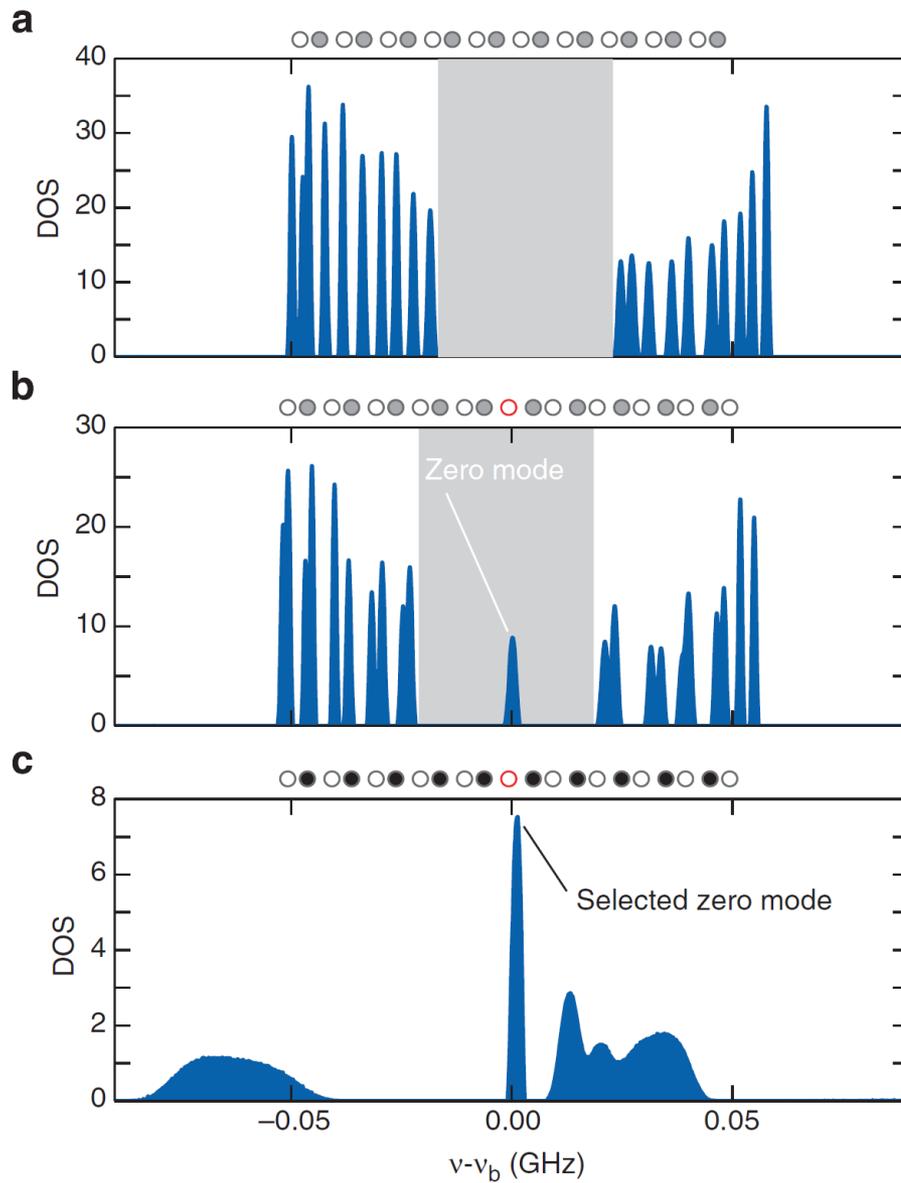


**Midgap state
breaks this
symmetry:
 $S_z = \pm 1$**

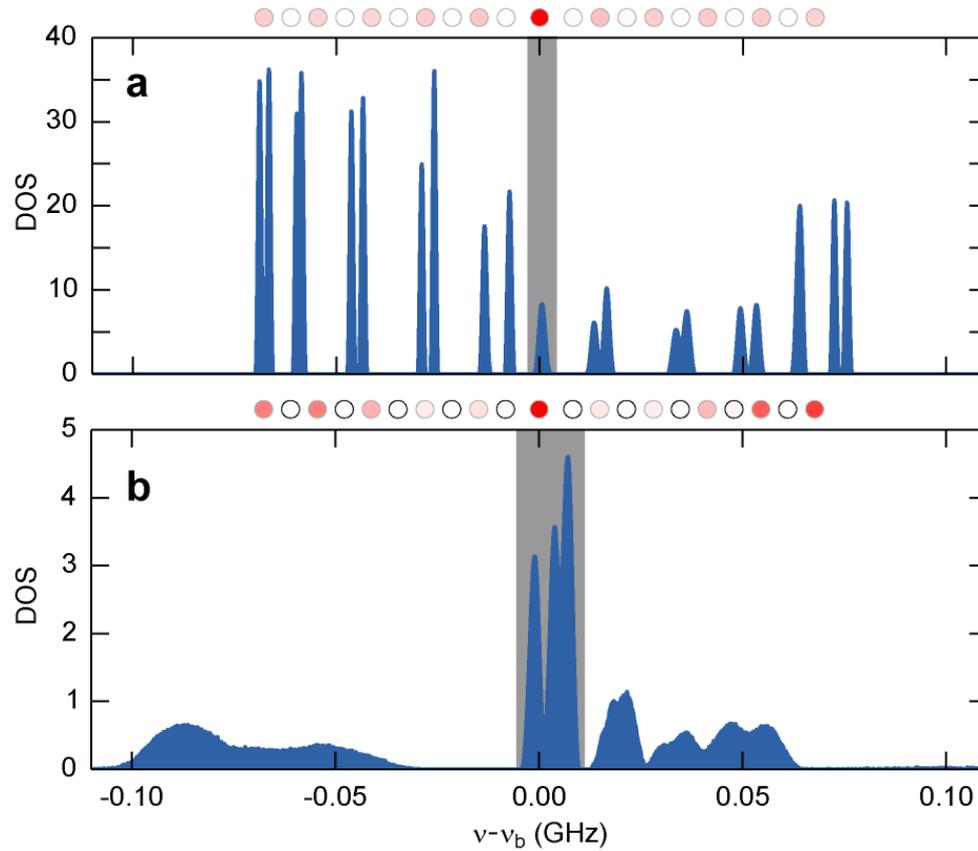
Microwave realization



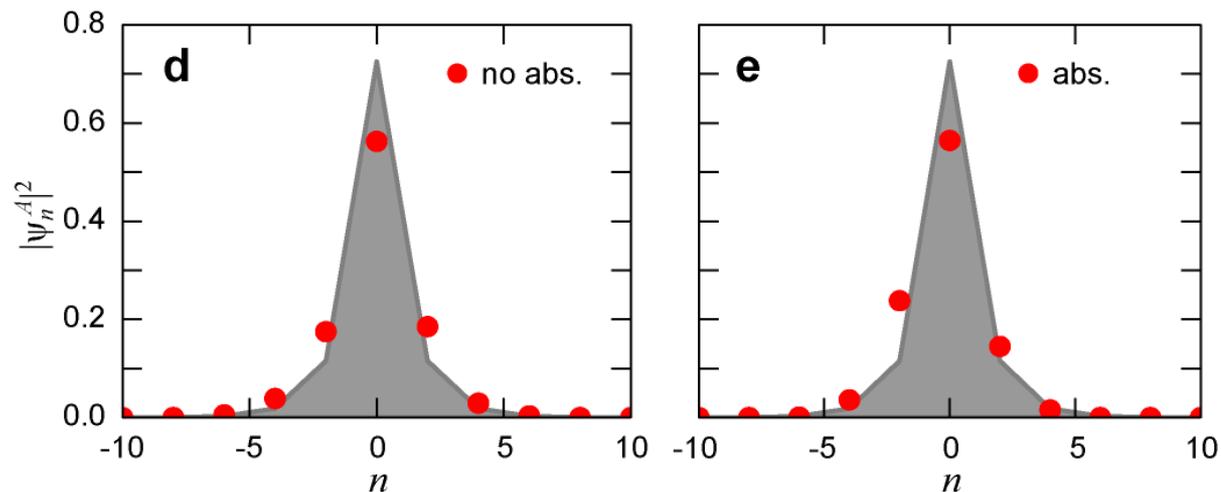
Density of states



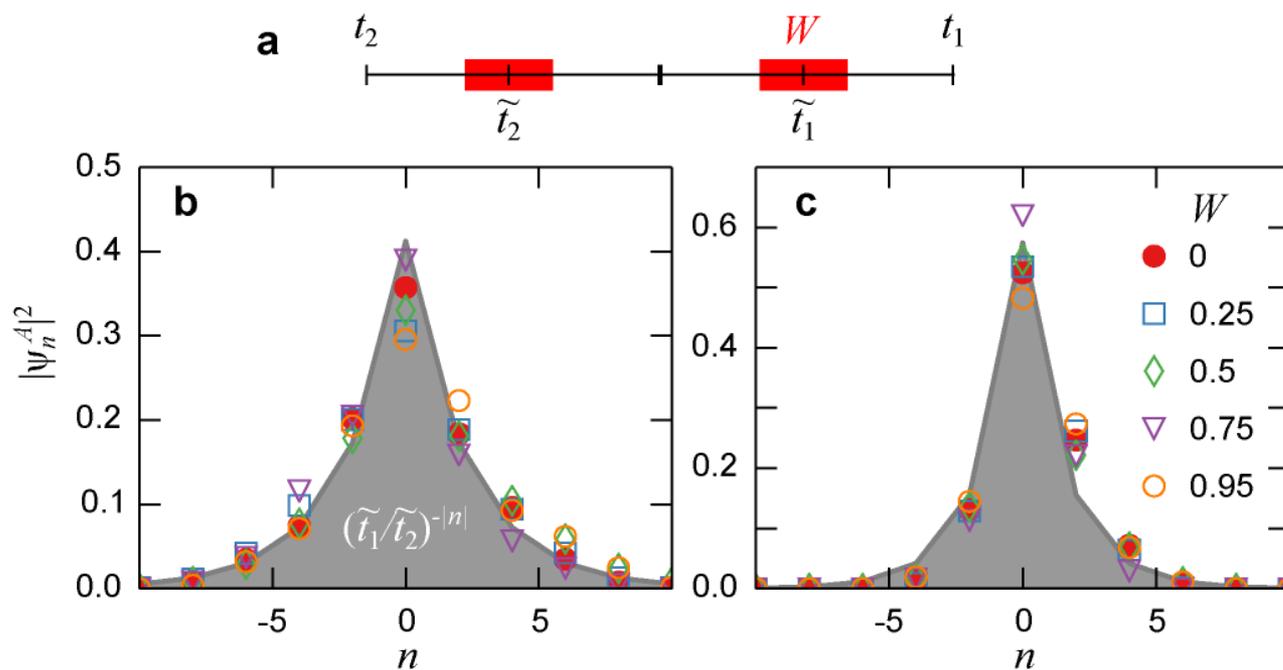
Compare to nontopological defect



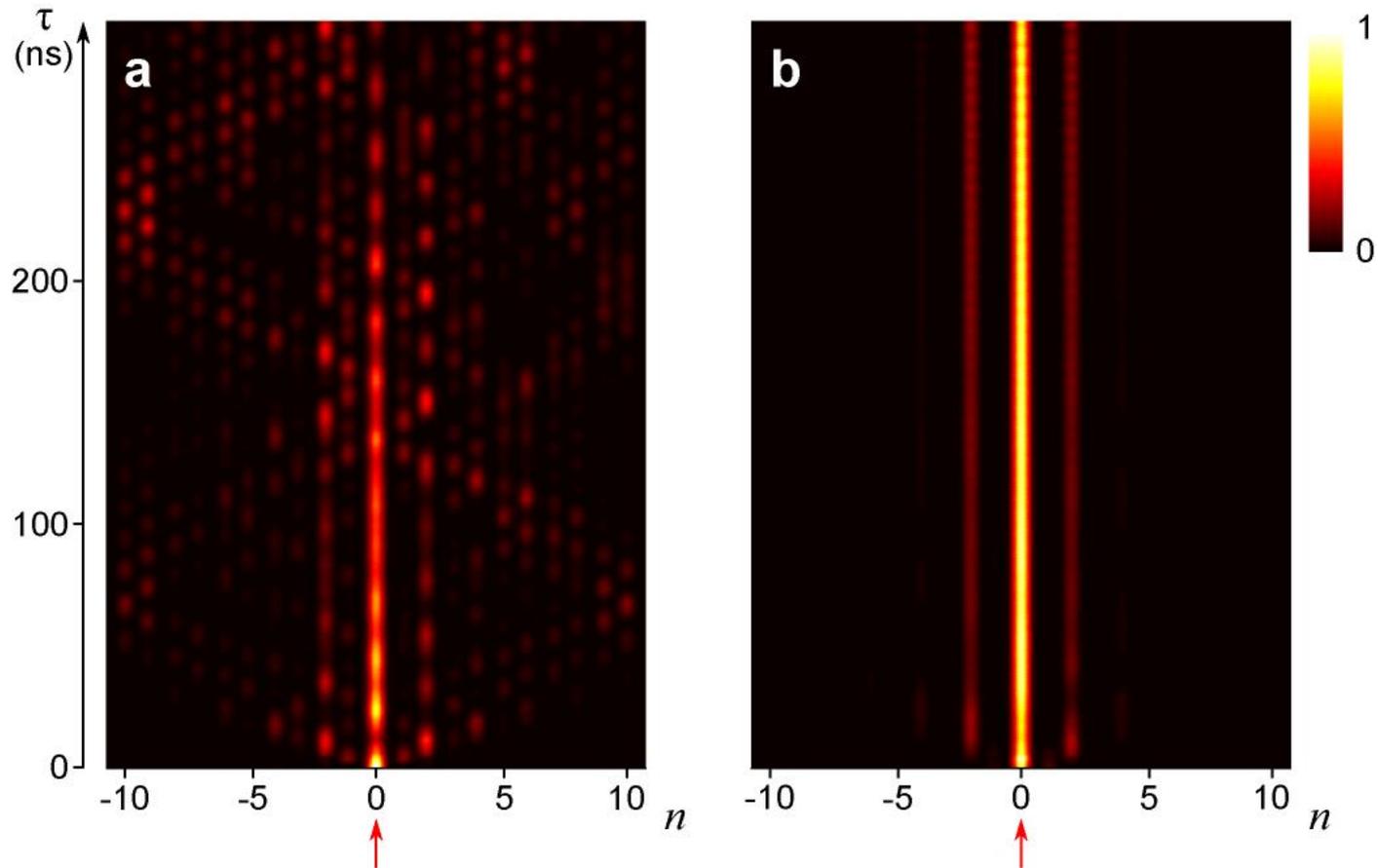
Mode profiles



Robustness to structural disorder



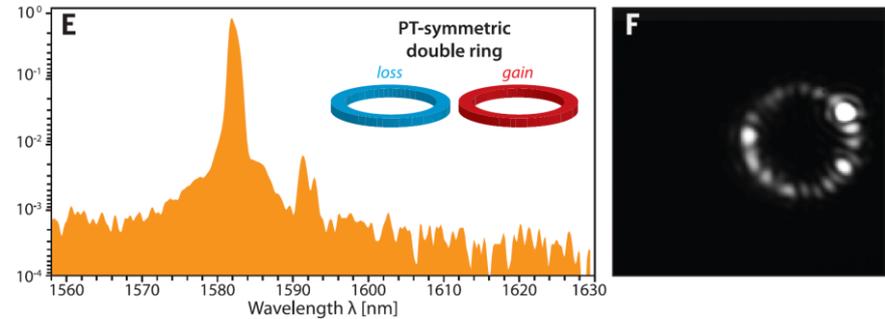
Pulse propagation



PT lasers YD Chong et al PRL 2011; S Longhi PRA 2010; HS, PRL 2010; Guo, Sim, HS PRA 2011

Parity-time-symmetric microring lasers

Hossein Hodaei, Mohammad-Ali Miri, Matthias Heinrich,*
Demetrios N. Christodoulides, Mercedeh Khajavikhan†

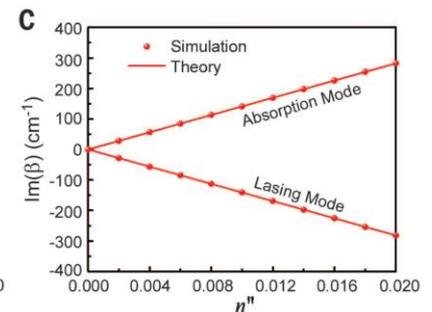
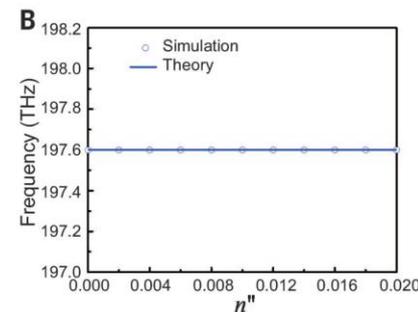
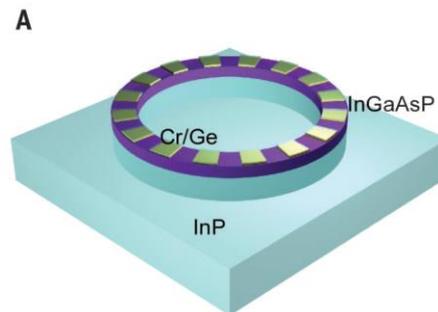


REPORTS

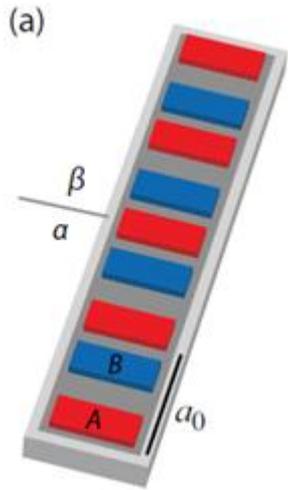
OPTICS

Single-mode laser by parity-time symmetry breaking

Liang Feng,^{1*} Zi Jing Wong,^{1*} Ren-Min Ma,^{1*} Yuan Wang,^{1,2} Xiang Zhang^{1,2,†}



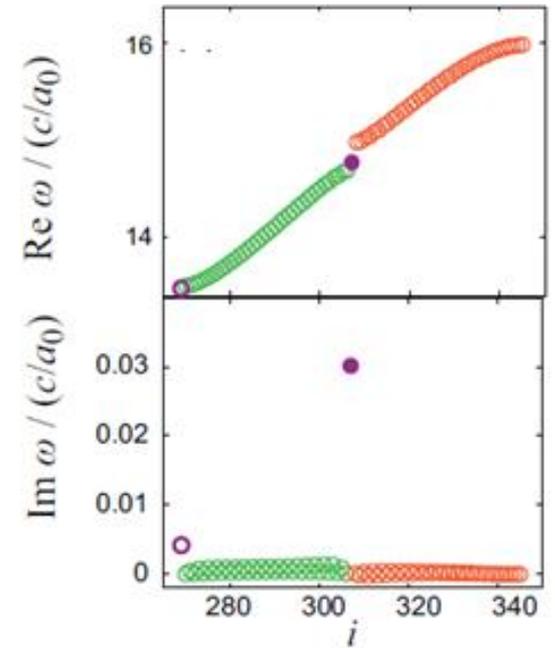
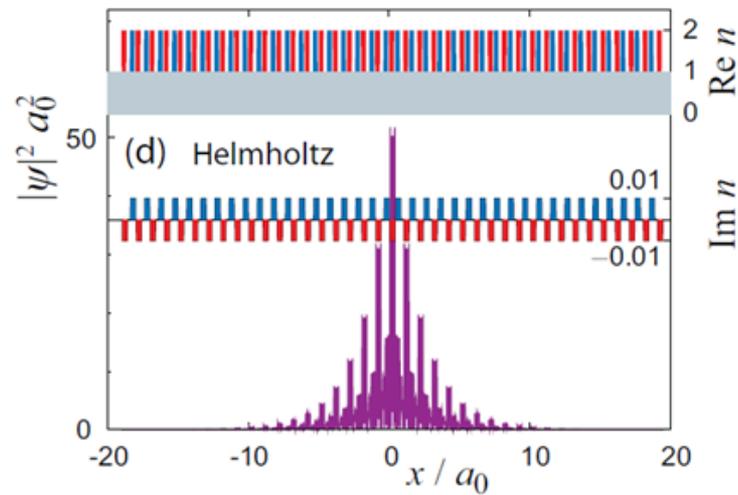
Topological Lasers



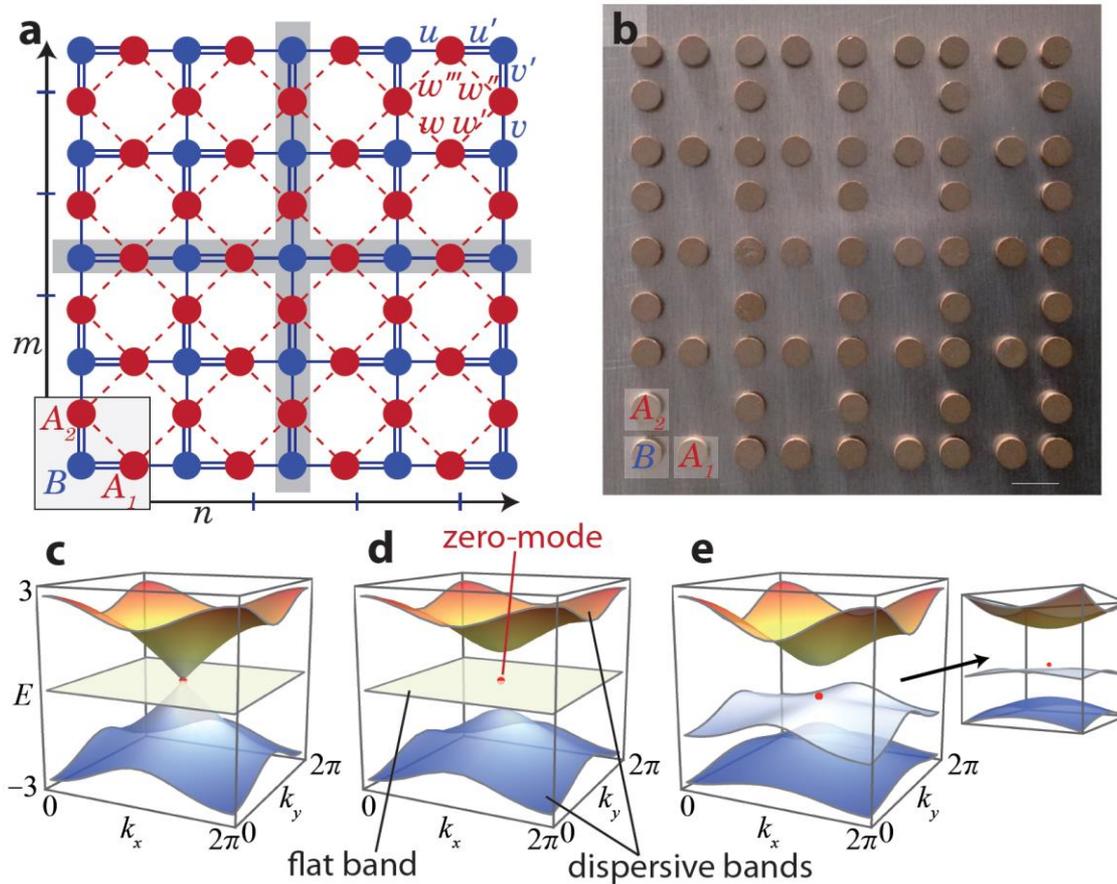
$$n_A = 2 - 0.01i$$

$$n_B = 2 + 0.01i$$

$$(\partial_x^2 + n^2(x)\omega^2/c^2)\psi(x) = 0$$



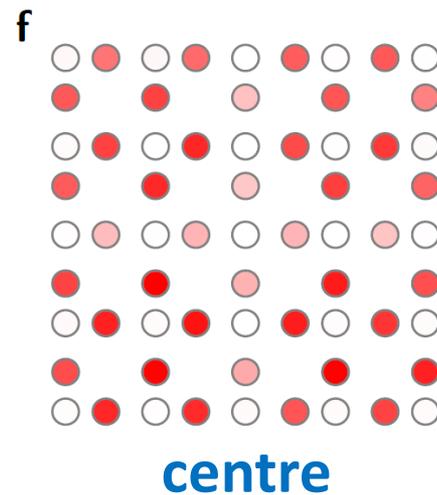
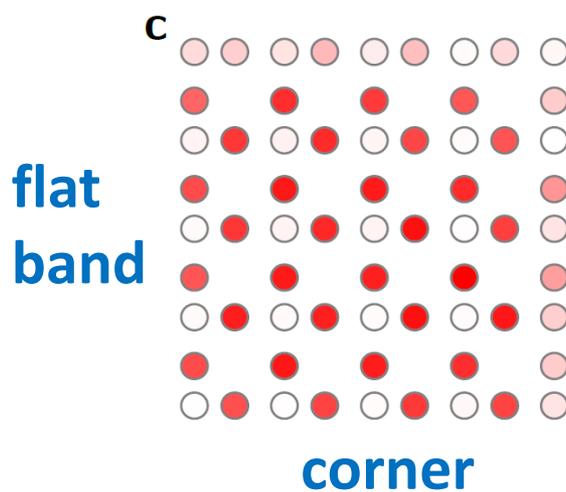
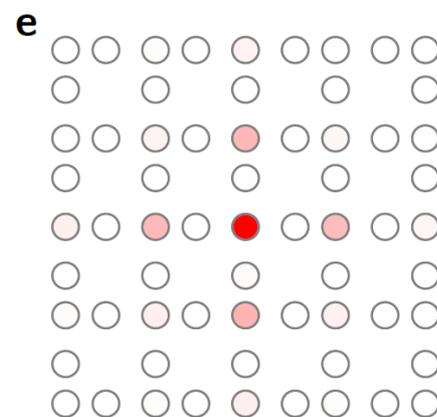
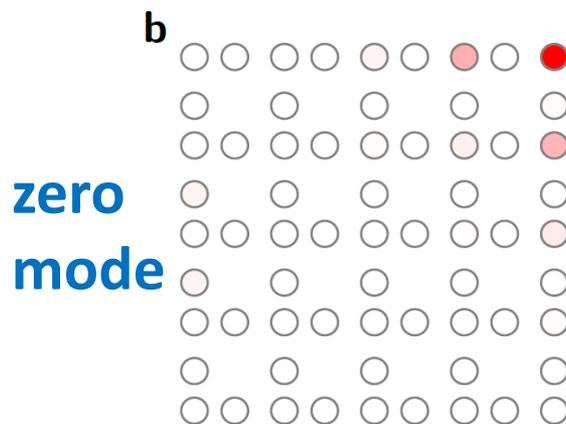
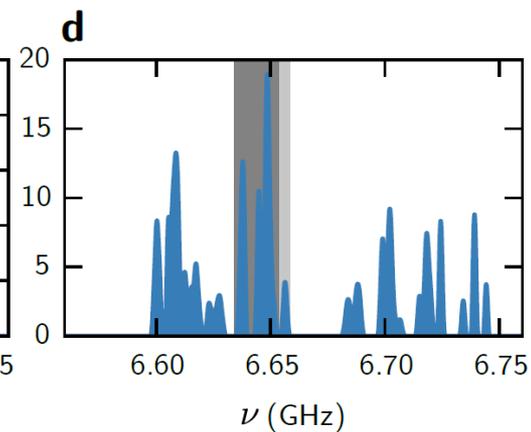
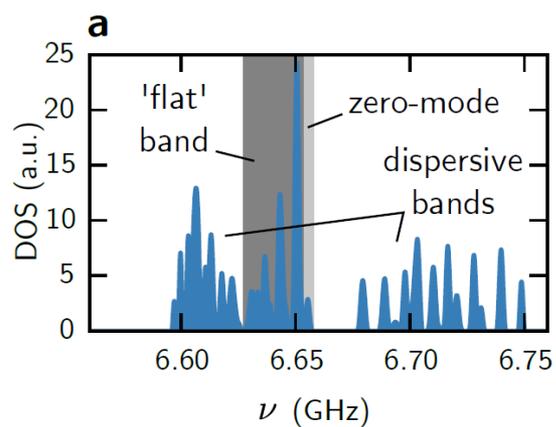
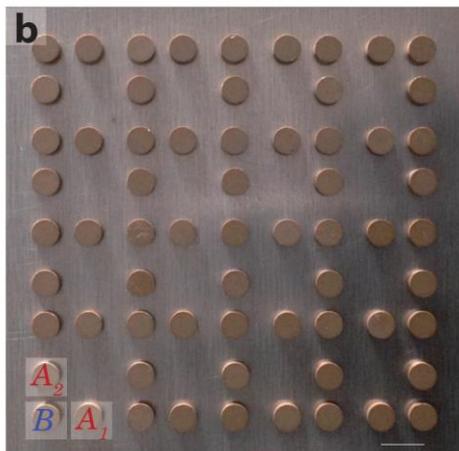
2D version of SSH model: dimerised Lieb lattice



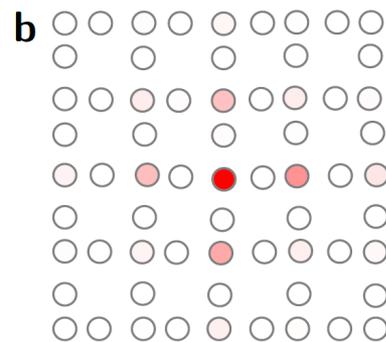
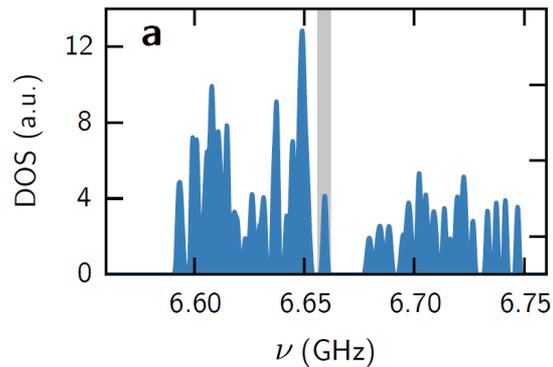
Flat band competes with point-defect states

- generate isolated point defect state
- NNN coupling: remove flat band

Experiment

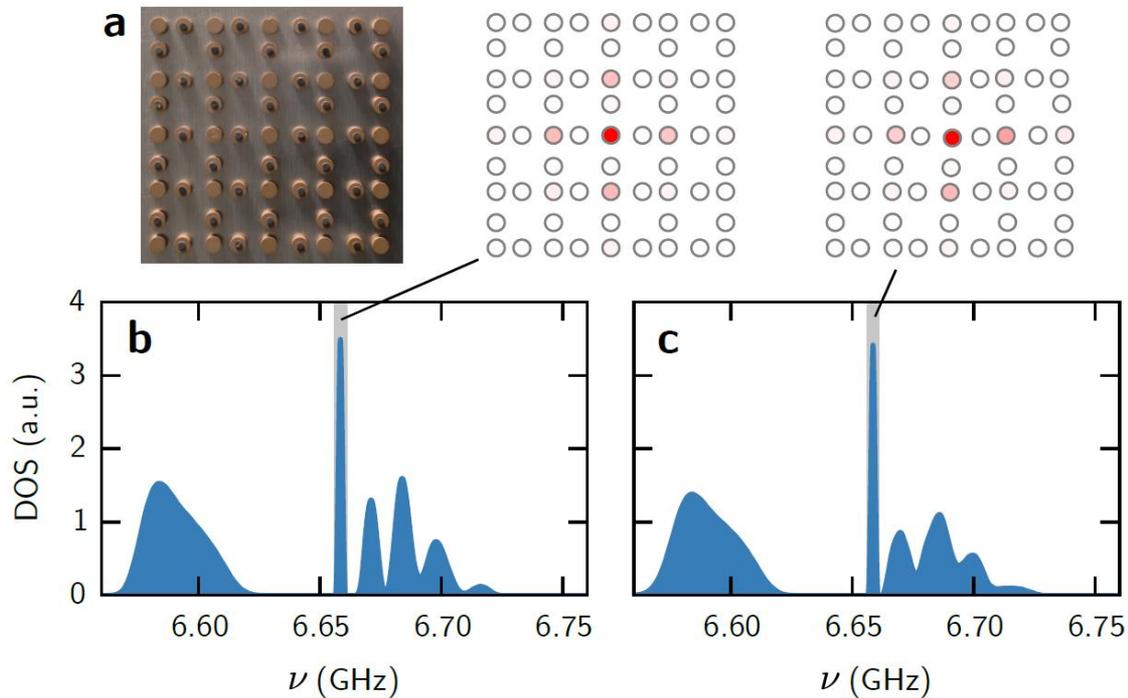


disorder



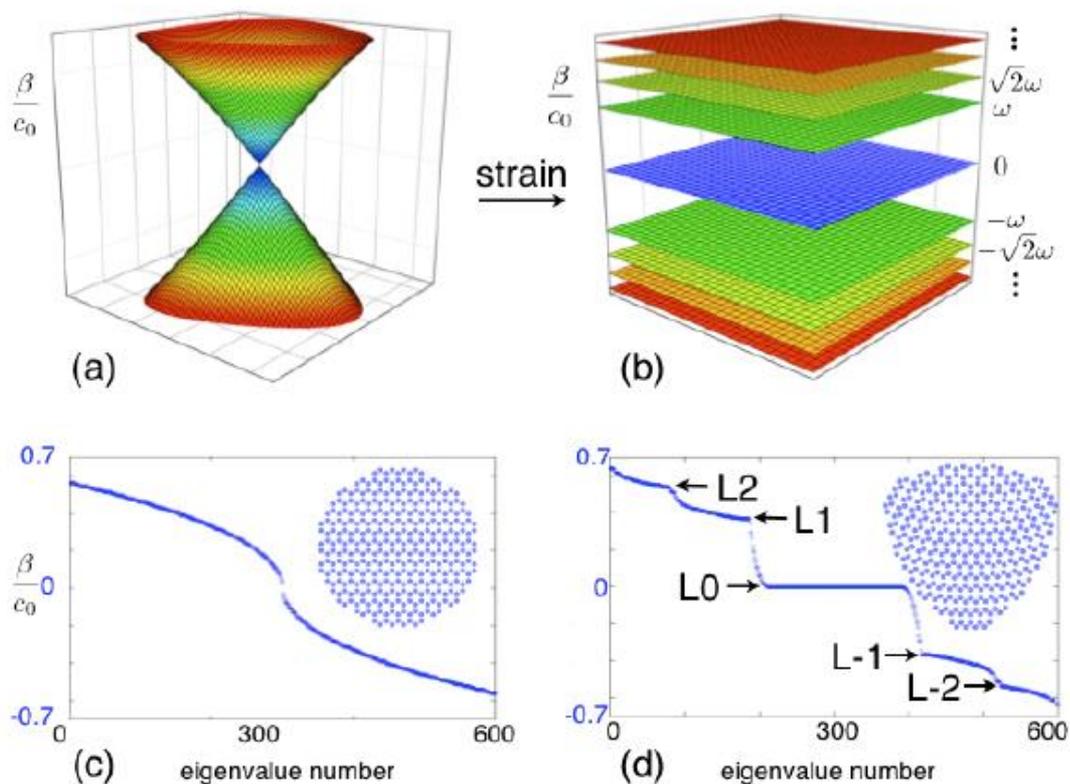
staggered absorption

... + disorder



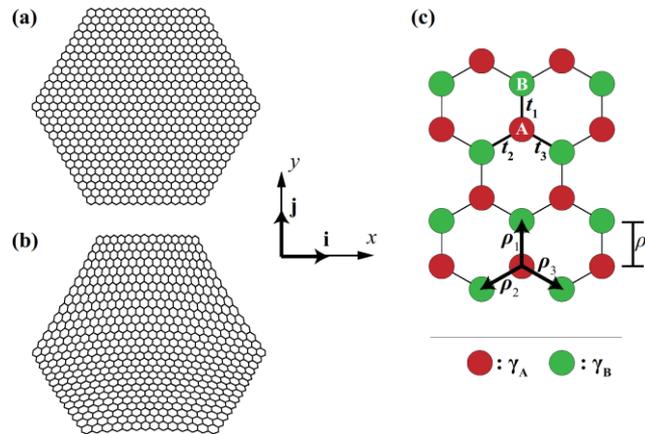
Strain-induced pseudomagnetic field and photonic Landau levels in dielectric structures

Mikael C. Rechtsman^{1†*}, Julia M. Zeuner^{2†}, Andreas Tünnermann², Stefan Nolte², Mordechai Segev¹ and Alexander Szameit²



Parity anomaly: strained honeycomb lattices with gain and loss

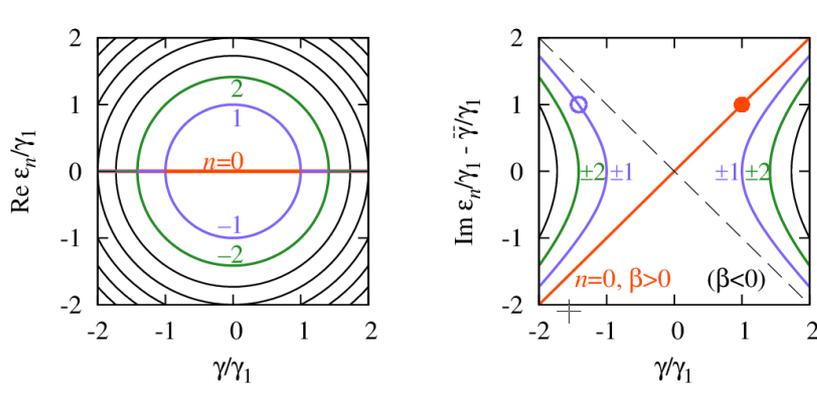
sublattices A and B have amplification rates γ_A and γ_B ,



Low-energy theory: SUSY

$$\mathcal{H} = \begin{pmatrix} i\gamma_A & v\sqrt{2\beta}\Pi^\dagger \\ v\sqrt{2\beta}\Pi & i\gamma_B \end{pmatrix}, \quad [\Pi, \Pi^\dagger] = 1$$

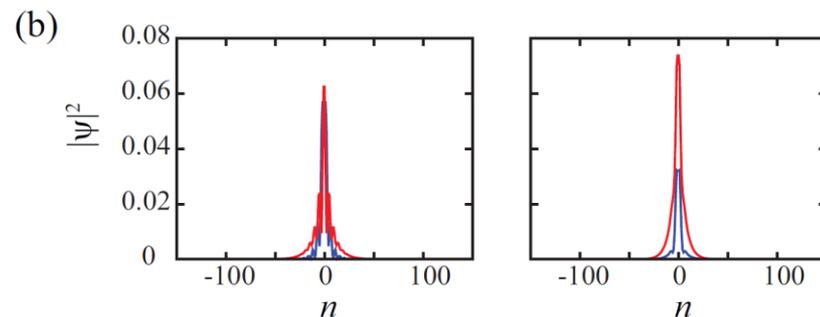
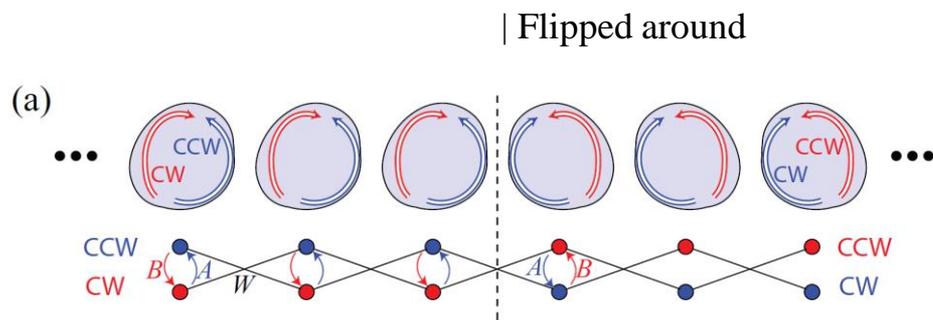
pseudo-Landau level laser



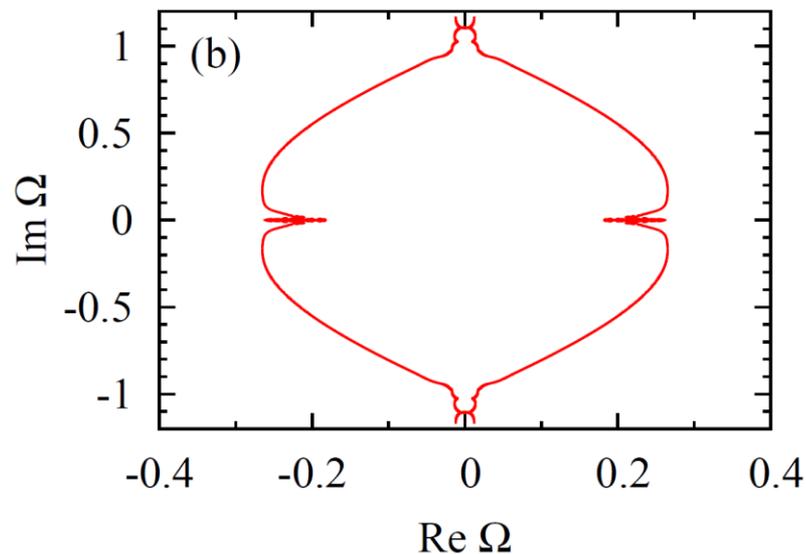
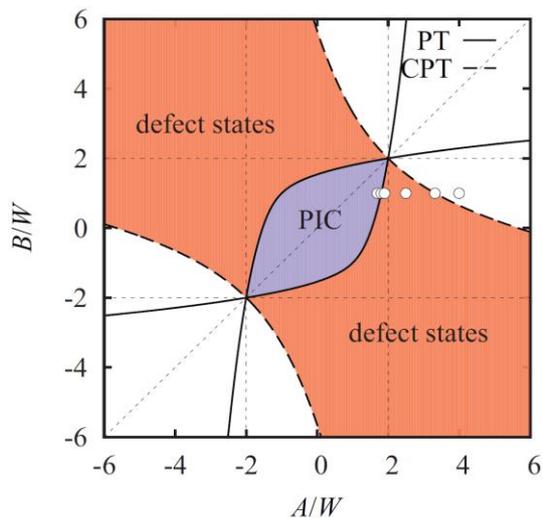
0th LL: sublattice polarized, thresholdless amplification

higher LLs: not polarized and amplified until PT symmetry spontaneously broken

Open sys: 'nontopological' defect... ...becomes topological



...via EPs on real and imaginary axis:



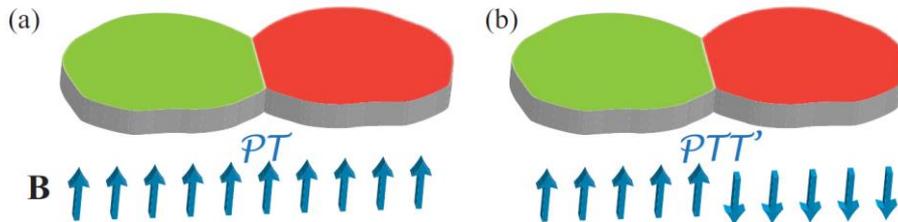
Final notes: beyond reciprocity

Instead of PT:

$$\mathcal{X} H \mathcal{X} = H^* \rightarrow \mathcal{X} S(E)^* \mathcal{X} = [S^{-1}(E^*)]$$

can also do magneto-optics with PTT':

$$\mathcal{X} H \mathcal{X} = H^\dagger \rightarrow \mathcal{X} S(E)^\dagger \mathcal{X} = [S^{-1}(E^*)]$$

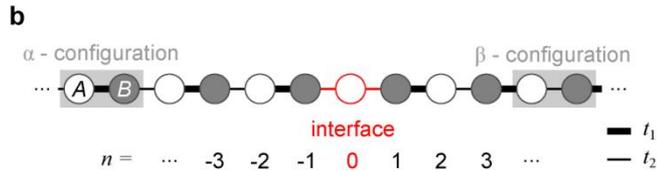
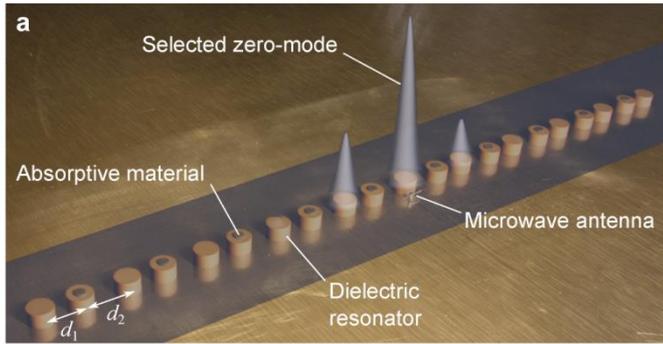


[or even 'skew-Hamiltonian' $J H J = H^\dagger$ where $J^\dagger = -J$]

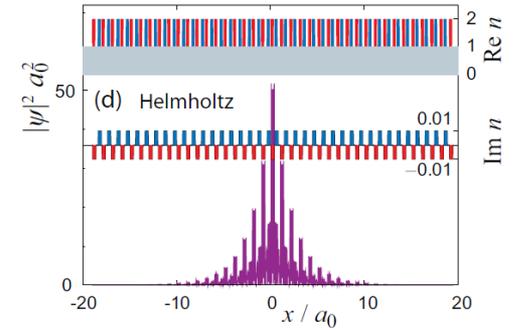
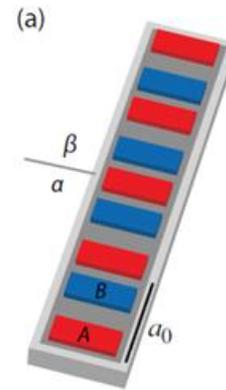
which all have no electronic analogue

Summary: topological anomalies → selectively amplified states

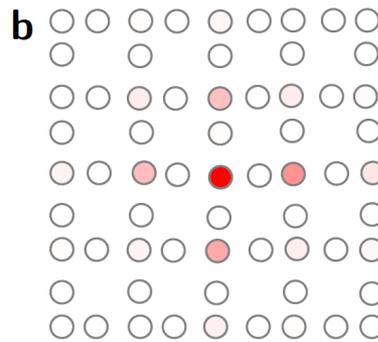
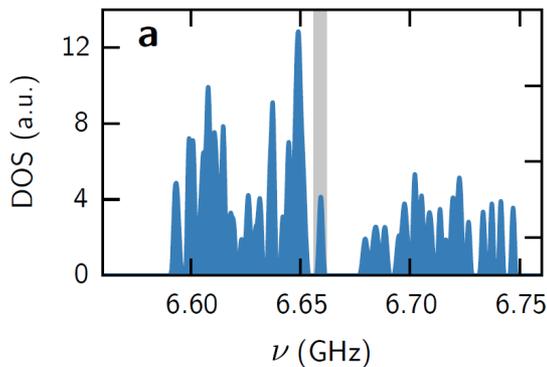
Complex SSH model



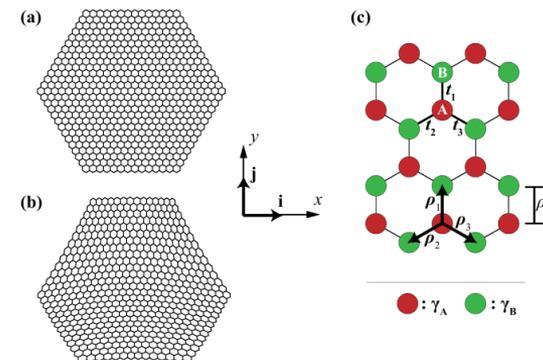
lasers



Point defects



Landau levels



HS, Opt Lett 2013; C Poli, HS, M Bellec, U Kuhl, F Mortessagne, Nat Commun 2015; +subm.
 HS & N Yunger Halpern, PRL 2013; Malzard, Poli, HS, PRL 2015