Asymmetric backscattering of light and exceptional points in optical microcavities

Jan Wiersig

Introduction

Asymmetric backscattering

Sensors based on exceptional points
Introduction
Introduction

Microdisk

Light confinement by total internal reflection

Optical modes: solutions of Maxwell’s equations with harmonic time dependence

High $Q = \omega \tau$ with frequency $\omega$ and lifetime $\tau$

Applications: microlasers, single-photon sources, sensors, filters, ...
Circle: pairs of degenerate modes

**standing-wave modes**
\[ \sin m\phi \text{ and } \cos m\phi \]

**traveling-wave modes**
\[ e^{\pm im\phi} \]

**Properties**
- \( Q_1 = Q_2 \)
- \( \omega_1 = \omega_2 \)

**Modes**
- CCW (Clockwise)
- CW (Counter-Clockwise)
Introduction
Deformed and perturbed cavities


wave chaos

sensors

J. Wiersig and M. Hentschel, PRL **100**, 033901 (2008)
directed emission

coupled cavities
Effective index approximation

\[
\left[ \nabla^2 + n(x, y)^2 k^2 \right] \psi(x, y) = 0
\]

\[
\text{Re}[\psi(x, y)e^{-i\omega t}] = \begin{cases} 
E_z & \text{TM} \\
H_z & \text{TE}
\end{cases}
\]

Continuity conditions at the cavity’s boundary

- TM: $$\psi$$ and $$\partial \psi$$
- TE: $$\psi$$ and $$\frac{1}{n^2} \partial \psi$$

Outgoing wave condition at infinity

$$\implies \omega \in \mathbb{C}, \text{ quasibound state with lifetime}$$

$$\tau = -\frac{1}{2\text{Im}(\omega)}$$
**Introduction**

**2D mode equation**

**Effective index approximation**

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\tau = -\frac{1}{2\text{Im}(\omega)}
\]


**S-matrix approach/wave matching** e.g. M. Hentschel and K. Richter, PRE 66, 056207 (2002)

**Review on deformed and perturbed optical microcavities**

Exceptional point (EP)

Point in parameter space at which two (or more) eigenvalues and eigenstates of a non-Hermitian linear operator coalesce. \( EP \neq \text{diabolic point} \)


**microwave cavity** C. Dembowski *et al.*, PRL 86, 787 (2001)

**deformed microcavity** (liquid jet containing laser dyes)

S.-B. Lee *et al.*, PRL 103, 134101 (2009)

complex-square-root topology at EP
Exceptional points in microcavities

- **microtoroid coupled to two nanofiber tips**

- **atom and cavity mode in microcavity**
  Y. Choi *et al.*, PRL **104**, 153601 (2010)

- **pump-induced EPs in coupled microlasers**

- **parity-time symmetric cavities** ...
Asymmetric backscattering
Asymmetric backscattering
Coupling of CW and CCW traveling waves

- two-mode approximation
- slowly-varying envelope approximation in time domain

\[
i \frac{d}{dt} a_{\text{CCW}} = \Omega_0 a_{\text{CCW}}
\]
\[
i \frac{d}{dt} a_{\text{CW}} = \Omega_0 a_{\text{CW}}
\]

Frequency and loss \( \Omega_0 \in \mathbb{C} \)

CW: clockwise
CCW: counterclockwise
Asymmetric backscattering
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\[
\begin{align*}
\frac{id}{dt} a_{\text{CCW}} &= (\Omega_0 + \delta \Omega) a_{\text{CCW}} + s a_{\text{CW}} \\
\frac{id}{dt} a_{\text{CW}} &= s a_{\text{CCW}} + (\Omega_0 + \delta \Omega) a_{\text{CW}}
\end{align*}
\]

frequency shift and additional loss $\delta \Omega \in \mathbb{C}$
backscattering amplitude $s \in \mathbb{C}$
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\[
i \frac{d}{dt} \begin{pmatrix} a_{CCW} \\ a_{CW} \end{pmatrix} = \begin{pmatrix} \Omega_0 + \delta \Omega & s \\ s & \Omega_0 + \delta \Omega \end{pmatrix} \begin{pmatrix} a_{CCW} \\ a_{CW} \end{pmatrix}
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\[
\begin{align*}
    i \frac{d}{dt} \begin{pmatrix} a_{CCW} \\ a_{CW} \end{pmatrix} &= \begin{pmatrix} \Omega_0 + \delta\Omega & se^{-i2m\beta} \\ se^{i2m\beta} & \Omega_0 + \delta\Omega \end{pmatrix} \begin{pmatrix} a_{CCW} \\ a_{CW} \end{pmatrix} \\
    \text{azimuthal mode number } m &\in \mathbb{N} (e^{im\phi}, e^{-im\phi})
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i \frac{d}{dt} \begin{pmatrix} a_{\text{CCW}} \\ a_{\text{CW}} \end{pmatrix} = \begin{pmatrix} \Omega_0 + \sum_j \delta \Omega_j & \sum_j s_j e^{-i2m\beta_j} \\ \sum_j s_j e^{i2m\beta_j} & \Omega_0 + \sum_j \delta \Omega_j \end{pmatrix} \begin{pmatrix} a_{\text{CCW}} \\ a_{\text{CW}} \end{pmatrix}
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\[\delta \Omega_j, s_j \in \mathbb{C}\]
Asymmetric backscattering
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\sum_j s_j e^{i2m\beta_j} & \Omega_0 + \sum_j \delta \Omega_j \end{pmatrix} \begin{pmatrix} a_{CCW} \\ a_{CW} \end{pmatrix} \\
H_{eff} &= \begin{pmatrix} \Omega & A \\
B & \Omega \end{pmatrix}
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Asymmetric backscattering

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Asymmetric backscattering

\[ s_j \in \mathbb{C} \rightarrow |A| \neq |B| \]

Destructive/constructive interference of the scattered waves

J. Wiersig, PRA 84, 063828 (2011), see also Y. Yi, Y.-F. Xiao et al., PRA 83, 023803 (2011)
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Special case \( |A| = |B| \) if
- the system is closed \( s_j \in \mathbb{R} \) or
- \( s_j = s \) or
- a mirror-reflection symmetry is present
Asymmetric backscattering
Properties of the effective Hamiltonian

\[ H_{\text{eff}} = \begin{pmatrix} \Omega & A \\ B & \Omega \end{pmatrix} \quad \text{with} \quad |A| \neq |B| \]

Complex eigenvalues and right hand eigenvectors

\[ \Omega_{\pm} = \Omega \pm \sqrt{AB} \]
\[ \vec{\psi}_{\pm} = \begin{pmatrix} \psi_{\text{CCW}, \pm} \\ \psi_{\text{CW}, \pm} \end{pmatrix} = \begin{pmatrix} \sqrt{A} \\ \pm \sqrt{B} \end{pmatrix} \]
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|A| > |B|:

- CCW component > CW component
  - chirality: modes are not standing waves
  - copropagation
  - nonorthogonality
Asymmetric backscattering
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|A| < |B|: CW ↔ CCW
Asymmetric backscattering
Quantify nonorthogonality and chirality

Overlap of two eigenvectors $\vec{\psi}_-, \vec{\psi}_+$

$$S = \frac{|\vec{\psi}_- \cdot \vec{\psi}_+|}{||\vec{\psi}_-|| ||\vec{\psi}_+||} \in [0, 1]$$

$S = 0$: modes are orthogonal  \quad S = 1$: modes are collinear
Asymmetric backscattering
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Chirality of an eigenvector $\vec{\psi}$

$$\alpha = 1 - \frac{\min (|\psi_{ccw}|^2, |\psi_{cw}|^2)}{\max (|\psi_{ccw}|^2, |\psi_{cw}|^2)} \in [0, 1]$$

$\alpha = 0$: standing wave $\alpha = 1$: traveling wave
Asymmetric backscattering
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Relation between overlap and chirality

$$\alpha = \frac{2S}{1 + S}$$
Asymmetric backscattering

Exceptional point

\[ H_{\text{eff}} = \begin{pmatrix} \Omega & A \\ B & \Omega \end{pmatrix} ; \quad \Omega_\pm = \Omega \pm \sqrt{AB} ; \quad \vec{\psi}_\pm = \begin{pmatrix} \sqrt{A} \\ \pm \sqrt{B} \end{pmatrix} \]

Fully asymmetric backscattering: \( B \to 0 \) with \( A \neq 0 \)

\[ H_{\text{eff}} = \begin{pmatrix} \Omega & A \\ 0 & \Omega \end{pmatrix} ; \quad \Omega_\pm = \Omega ; \quad \vec{\psi} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]

- splitting \( \to 0 \)
- only one linearly independent eigenvector \( \triangleq \) CCW traveling-wave mode
- exceptional point
Asymmetric backscattering

Exceptional point

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\( A \to 0 \) with \( B \neq 0 \): CW \( \leftrightarrow \) CCW
Asymmetric backscattering
Microdisk perturbed by two nanoparticles

J. Wiersig, PRA 84, 063828 (2011)

- TM polarization
- $n = n_j = 2$
- $d_1 / R = 0.01, d_2 / R = 0.02$
- $r_1 / R = 0.043, r_2 / R \approx 0.05$
- $\Omega = \frac{\omega}{c} R = kR = \frac{2\pi R}{\lambda} \approx 10$
Asymmetric backscattering
A pair of nearly degenerate modes

\[ \beta = 1.072 \text{ (radian)} \rightarrow \text{overlapping resonances} \]

\[ \Omega_+ = 9.87722 - i0.00243 \]
\[ \Omega_- = 9.87869 - i0.00266 \]

Standing-wave modes?
Asymmetric backscattering
Angular momentum representation

Inside the cavity

$$\psi(r, \phi) = \sum_{m=-\infty}^{\infty} \alpha_m J_m(nkr)e^{im\phi}$$

- **2-mode approx.** ✓
- ≈ copropagating traveling waves
- chirality
Asymmetric backscattering
Angular momentum representation

Inside the cavity

\[ \psi(r, \phi) = \sum_{m=-\infty}^{\infty} \alpha_m J_m(nkr)e^{im\phi} \]

- 2-mode approx. ✓
- \( \approx \) copropagating traveling waves
- chirality

\[ \vec{\psi}_\pm = \left( \begin{array}{c} \sqrt{A} \\ \pm \sqrt{B} \end{array} \right) \]
Asymmetric backscattering
Angular momentum representation

Inside the cavity

\[ \psi(r, \phi) = \sum_{m=-\infty}^{\infty} \alpha_m J_m(nkr)e^{im\phi} \]

- 2-mode approx. ✓
- ± copropagating traveling waves
- chirality

\[ \vec{\psi}_\pm = \begin{pmatrix} \sqrt{A} \\ \pm \sqrt{B} \end{pmatrix} \]

- CW and CCW superpositions
Inside the cavity

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- CW and CCW superpositions

Chirality

\[
\alpha = 1 - \frac{\min \left( \sum_{m=-\infty}^{-1} |\alpha_m|^2, \sum_{m=1}^{\infty} |\alpha_m|^2 \right)}{\max \left( \sum_{m=-\infty}^{-1} |\alpha_m|^2, \sum_{m=1}^{\infty} |\alpha_m|^2 \right)} \approx \begin{cases} 0.73 \\ 0.73 \end{cases}
\]
Asymmetric backscattering
A pair of nearly degenerate and nonorthogonal modes

\[ \beta = 1.072 \rightarrow \text{overlapping resonances} \]

\[ \Omega_+ = 9.87722 - i0.00243 \]

\[ \Omega_- = 9.87869 - i0.00266 \]

Normalized overlap integral

\[ S = \frac{\left| \int_C dx\,dy \, \psi_1^* \psi_2 \right|}{\sqrt{\int_C dx\,dy \, \psi_1^* \psi_1} \sqrt{\int_C dx\,dy \, \psi_2^* \psi_2}} \approx 0.583 \quad \text{significant nonorthogonality} \]
Asymmetric backscattering
A pair of nearly degenerate modes in the regime of strongly overlapping resonances

\[ \frac{r_2}{R} = 0.04838 \text{ and } \beta = 1.08468 \rightarrow \text{strongly overlapping resonances} \]

\[ \Omega_+ = 9.87807 - i0.002427 \]
\[ \Omega_- = 9.87803 - i0.002428 \]

Chirality

\[ \alpha \approx \begin{cases} 1 \\ 1 \end{cases} \text{ copropagating CW traveling-wave modes} \]

Normalized overlap integral

\[ S \approx 1 \text{ almost collinear!} \]
Asymmetric backscattering
Complex-square-root topology with a branch point singularity at the EP

\[ \alpha \approx 0.8 \text{ to } 1 \text{ when moving from light to dark areas} \]

- EP: frequency and decay rate splitting $\rightarrow 0$ and chirality $\rightarrow 1$
Asymmetric backscattering
Experimental confirmation of asymmetric backscattering and chirality


→ talk by Şahin Özdemir
Asymmetric backscattering
Asymmetric Limaçon cavity

\[ \rho = R \left[ 1 + \varepsilon_1 \cos \phi + \varepsilon_2 \cos(2\phi + \delta) \right] \]

J. Wiersig et al., PRA 84, 023845 (2011)

Overlap \( S \approx 0.72 \)

Field inside

Field outside

Chirality \( \alpha \approx 0.84 \)

\[ \Omega_+ = 12.31981 - i0.00089 \]

\[ \Omega_- = 12.31985 - i0.0009 \]
Asymmetric backscattering
Asymmetric Limaçon cavity: chirality vs. nonorthogonality

Effective Hamiltonian predicts

\[ \alpha = \frac{2S}{1 + S} \]

Asymmetric Limaçon cavity

Effective Hamiltonian explains the relation between chirality and overlap
Asymmetric backscattering

Other systems

Spiral cavity J. Wiersig, S.W. Kim, and M. Hentschel, PRA 78, 053809 (2008)

Cavities of constant width J. Wiersig et al., PRA 84, 023845 (2011)

Open quantum and wave systems J. Wiersig, PRA 89, 012119 (2014)
Asymmetric backscattering
Frobenius-Perron operator for deformed microdisks

asymmetric backscattering in ray dynamics \Rightarrow \text{chirality, copropagation, and nonorthogonality?}

Asymmetric backscattering
Frobenius-Perron operator for deformed microdisks

asymmetric backscattering in ray dynamics \( \Rightarrow \) chirality, copropagation, and nonorthogonality?


discrete time evolution of phase-space density \( \rho \) with Frobenius-Perron operator \( \mathcal{F} \)

\[
\rho(n + 1) = \mathcal{F} \rho(n)
\]

for maps see e.g. J. Weber et al., PRL 85, 3620 (2000), K. Frahm and D. Shepelyansky, EPL 75, 299 (2010)
Asymmetric backscattering
Frobenius-Perron operator for deformed microdisks

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$\mathcal{F}\rho_j = \lambda_j \rho_j$

- weight to incorporate reflectivity
  $\Rightarrow$ $\mathcal{F}$ is sub-unitary

the two largest eigenvalues are nearly degenerate (eigenstate pair)
Asymmetric backscattering
Frobenius-Perron operator for deformed microdisks

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Sensors based on exceptional points

microcavity sensor
Sensors based on exceptional points

Microcavity sensor for single-particle detection

F. Vollmer et al., PNAS 105, 20701 (2008)

Measure frequency shift $\rightarrow$ particle detection
Measure frequency splitting of initially degenerate modes (diabolic point)

Sensors based on exceptional points
Microcavity sensor based on frequency-splitting detection

Measure frequency splitting of initially degenerate modes (diabolic point)

Problem: initial splitting due to fabrication imperfections
Sensors based on exceptional points
Microcavity sensor based on frequency-splitting detection

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Which one is better for sensing?
Which one is better for sensing?

Apply a perturbation of strength $\epsilon$ to a (two-fold) degeneracy

$$\Delta \Omega_{\text{DP}} = O(\epsilon)$$

$$\Delta \Omega_{\text{EP}} = O(\sqrt{\epsilon})$$

T. Kato (1966)
Sensors based on exceptional points

Conventional degeneracy vs exceptional point

J. Wiersig, PRL 112, 203901 (2014)

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Apply a perturbation of strength $\epsilon$ to a (two-fold) degeneracy

$$\Delta \Omega_{\text{DP}} = O(\epsilon)$$

T. Kato (1966)

Enhancement factor of sensitivity for splitting detection

$$\frac{\Delta \Omega_{\text{EP}}}{\Delta \Omega_{\text{DP}}} = O\left(\frac{1}{\sqrt{\epsilon}}\right) \quad \text{for sufficiently small } \epsilon$$
Sensors based on exceptional points

Price to pay

- \( \Delta \Omega_{\text{EP}} \in \mathbb{C} \)
  \[ \implies \] frequency and linewidth splitting has to be measured

- peaks strongly overlap near EP
  \[ \implies \] gain helps to resolve peaks
Sensors based on exceptional points

Results for a microcavity sensor at an EP

EP is due to fully asymmetric backscattering
Sensors based on exceptional points

Results for a microcavity sensor at an EP

EP is due to fully asymmetric backscattering

- 3 to 3.5 fold enhancement of sensitivity
- Splitting $|\Delta \Omega|$ is nearly independent on $\beta$

![Diagram showing a microcavity sensor and target particle with annotations](image)
Sensors based on exceptional points

Results for a microcavity sensor at an EP

EP is due to fully asymmetric backscattering

- 3 to 3.5 fold enhancement of sensitivity
- Splitting $|\Delta \Omega|$ is nearly independent on $\beta$

Sensitivity of sensors based on frequency splitting detection can be enhanced at an EP
Sensors based on exceptional points

Other theoretical studies

J. Wiersig, PRA 93, 033809 (2016)
7 fold enhancement of sensitivity

A. Hassan et al., Advanced Photonics 2015, SeT4C.3

Summary

- symmetric
- backscattering
- fully asymmetric EP
- chirality
- copropagation
- nonorthogonality
Summary

- symmetric
- backscattering
- fully asymmetric
- L. Yang's group
- chirality
- copropagation
- nonorthogonality
- EP
Summary

- Symmetric
- Backscattering
- Fully asymmetric
- Chirality
- Copropagation
- Nonorthogonality

Enhanced sensitivity of sensors based on frequency splitting detection

L. Yang's group

EP