Asymmetric backscattering of light and exceptional points in optical microcavities

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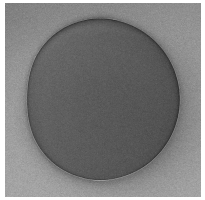
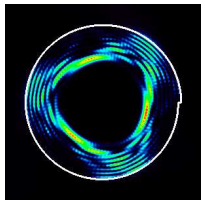
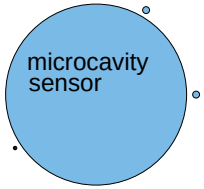
Collaborations: B. Peng + Ş.K. Özdemir + W. Chen + H. Yilmaz + L. Yang (St. Louis)
M. Liertzer + S. Rotter (Vienna)



Introduction

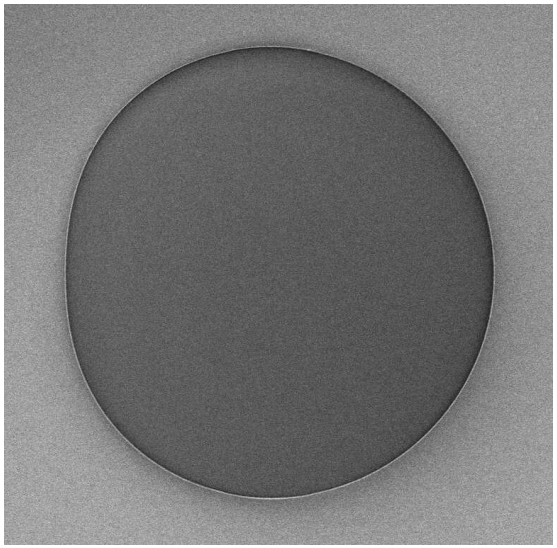


Asymmetric backscattering

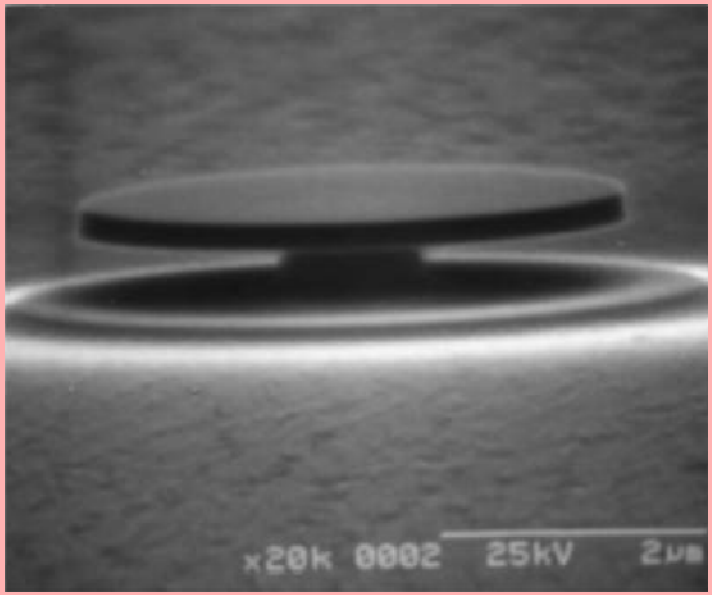


Sensors based on exceptional points

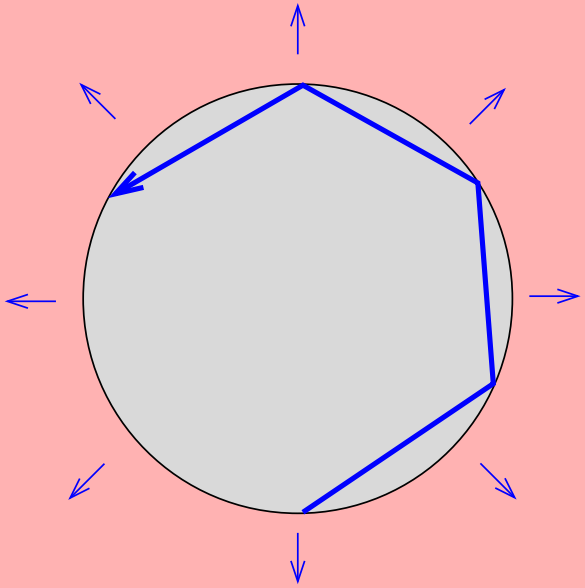
Introduction

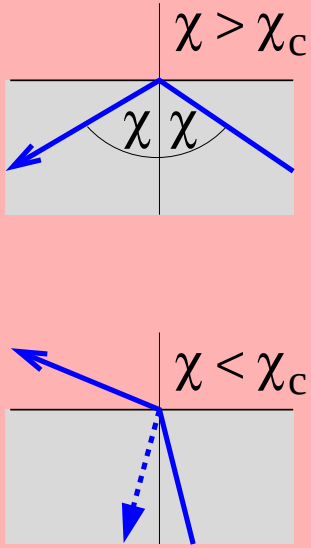


Light confinement by total internal reflection



P. Michler et al.

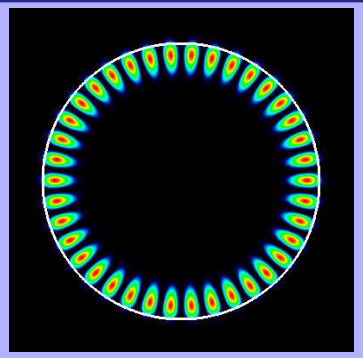


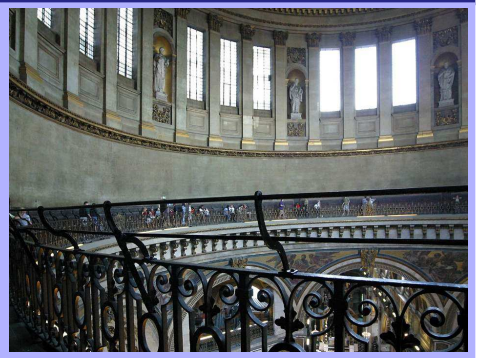


Optical modes: solutions of Maxwell's equations with harmonic time dependence

High $Q = \omega\tau$ with frequency ω and lifetime τ

Applications: microlasers, single-photon sources, sensors, filters, ...

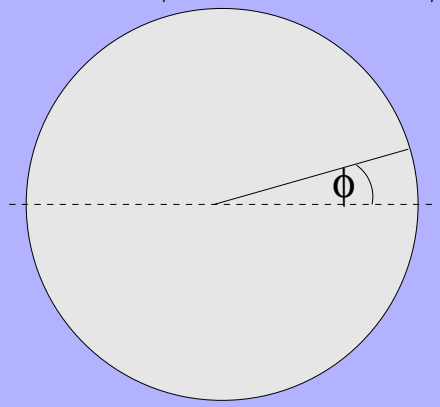




Circle: pairs of degenerate modes

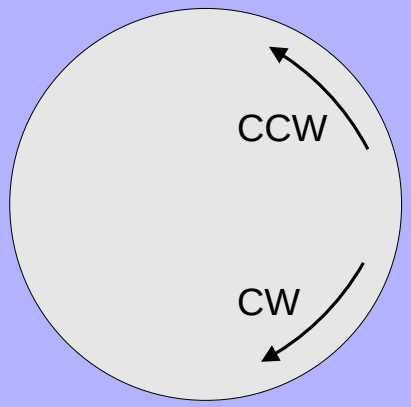
standing-wave modes

$\sin m\phi$ and $\cos m\phi$



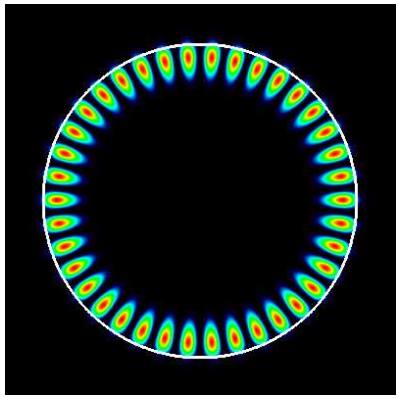
traveling-wave modes

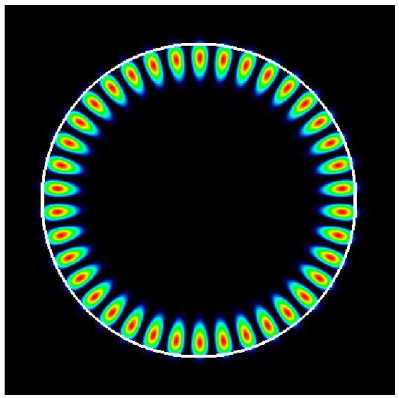
$e^{\pm im\phi}$

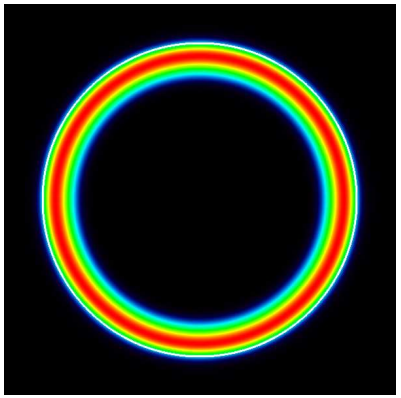


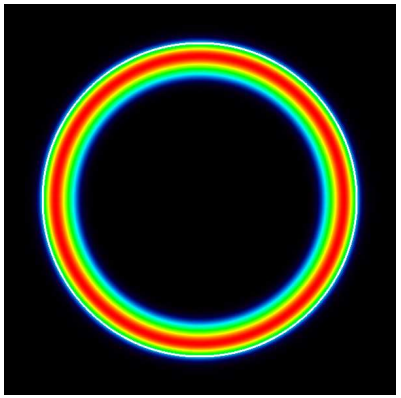
$\omega_1 = \omega_2$

$Q_1 = Q_2$





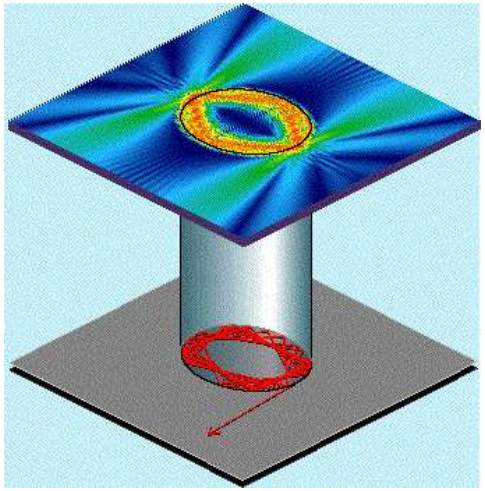




Introduction

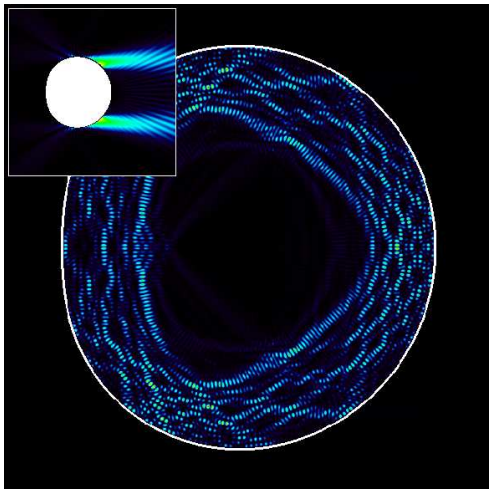
Deformed and perturbed cavities

J.U. Nöckel and A.D. Stone, Nature **385**, 45 (1997)



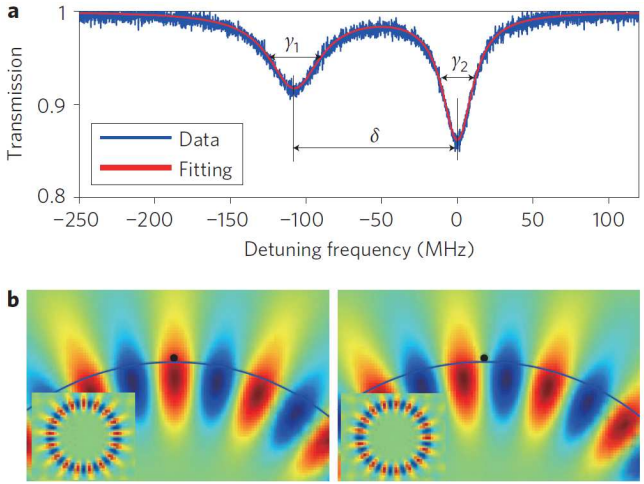
wave chaos

J. Wiersig and M. Hentschel, PRL **100**, 033901 (2008)



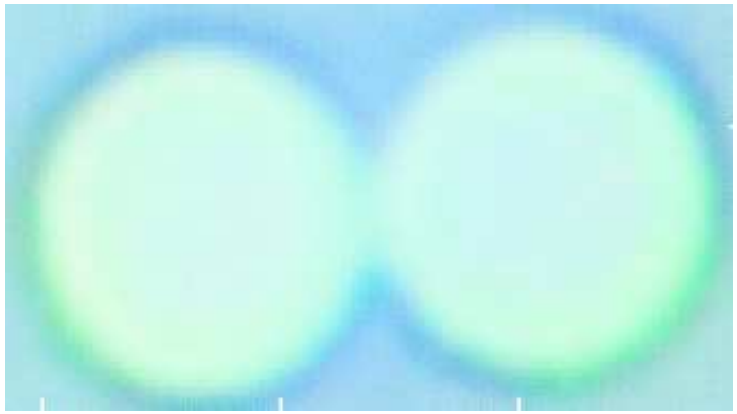
directed emission

J. Zhu *et al.*, Nature Photonics **4**, 46 (2010)



sensors

M. Benyoucef *et al.*, Opt. Lett. **36**, 1317 (2011)

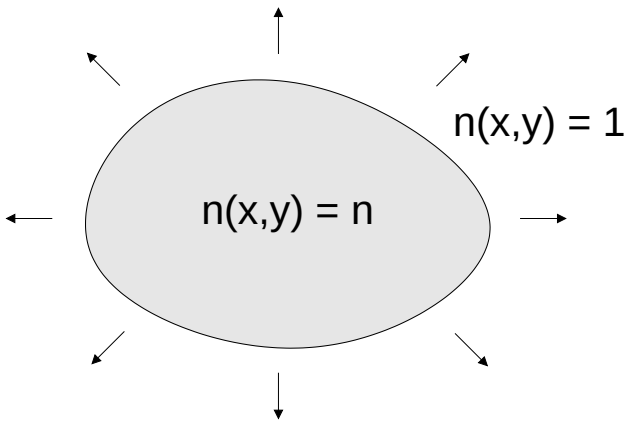


coupled cavities

Effective index approximation

$$\left[\nabla^2 + n(x,y)^2 k^2\right] \psi(x,y) = 0$$

$$\mathrm{Re}[\psi(x,y) e^{-i\omega t}] = \begin{cases} E_z & \mathrm{TM} \\ H_z & \mathrm{TE} \end{cases}$$



Continuity conditions at the cavity's boundary

TM : ψ and $\partial\psi$
TE : ψ and $\frac{1}{n^2}\partial\psi$

Outgoing wave condition at infinity

$\implies \omega \in \mathbb{C}$, quasibound state with lifetime $\tau = -\frac{1}{2\mathrm{Im}(\omega)}$

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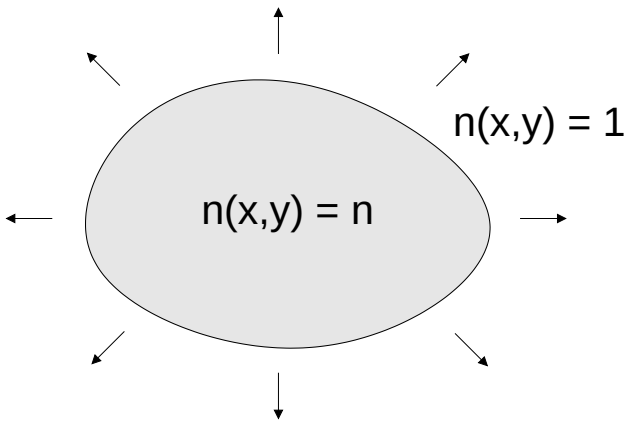

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Boundary element method J. Wiersig, J. Opt. A: Pure Appl. Opt. **5**, 53 (2003)
S-matrix approach/wave matching e.g. M. Hentschel and K. Richter, PRE **66**, 056207 (2002)

Review on deformed and perturbed optical microcavities
H. Cao and J. Wiersig, RMP **87**, 61 (2015)

Exceptional point (EP)

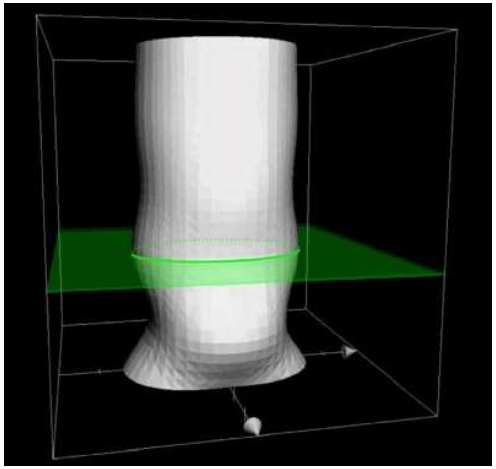
Point in parameter space at which two (or more) eigenvalues <u>and</u> eigenstates of a non-Hermitian linear operator coalesce. EP $\neq$ diabolic point

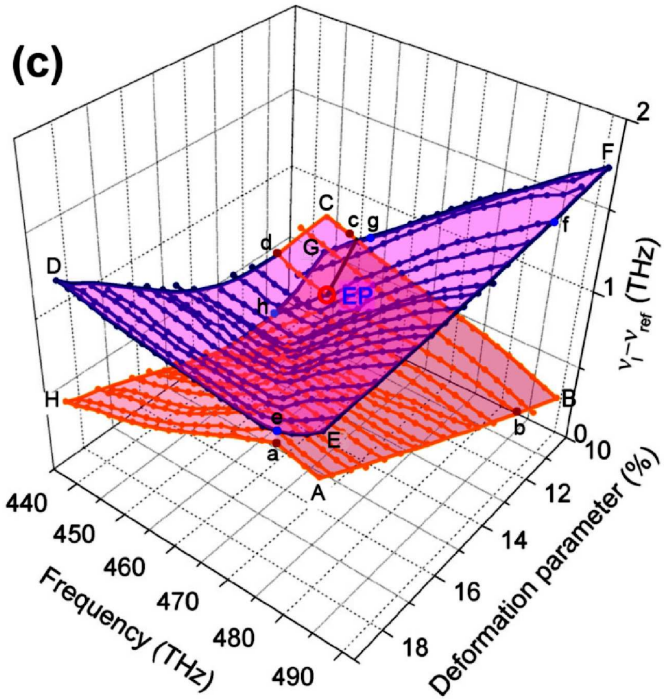
T. Kato, Perturbation Theory for Linear Operators (1966)

microwave cavity C. Dembowski *et al.*, PRL **86**, 787 (2001)

deformed microcavity (liquid jet containing laser dyes)

S.-B. Lee *et al.*, PRL **103**, 134101 (2009)

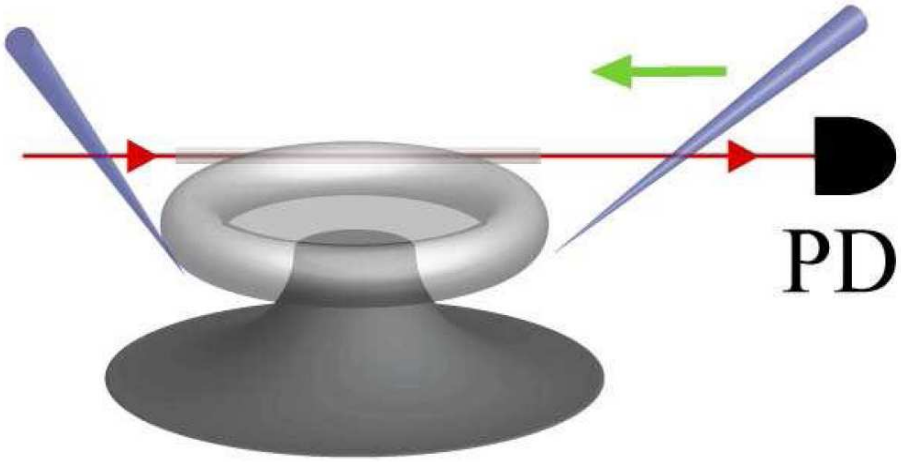


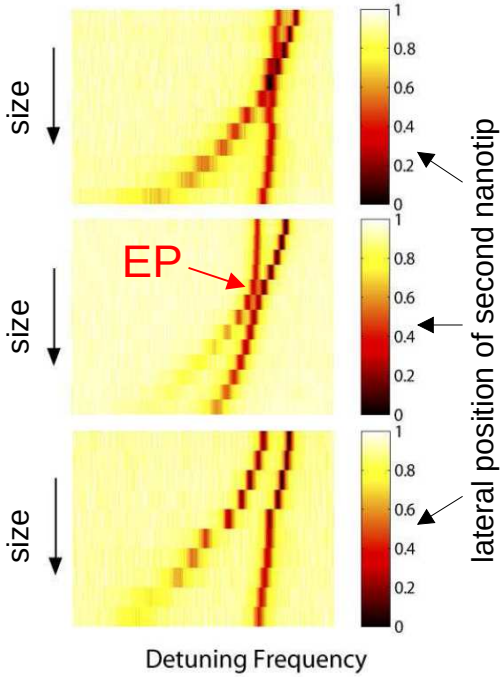


complex-square-root topology at EP

- microtoroid coupled to two nanofiber tips

  J. Zhu *et al.*, Opt. Express **18**, 23535 (2010)





- atom and cavity mode in microcavity Y. Choi *et al.*, PRL **104**, 153601 (2010)
- pump-induced EPs in coupled microlasers

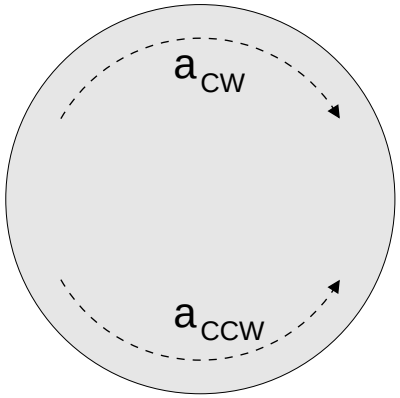
  M. Brandstetter*et al.*, Nat. Commun. **5**, 4034 (2014)
- parity-time symmetric cavities ...

Asymmetric backscattering

Asymmetric backscattering

Coupling of CW and CCW traveling waves

- two-mode approximation
- slowly-varying envelope approximation in time domain



$$
\begin{aligned}
i\frac{d}{dt}a_{\mathrm{CCW}} &= \Omega_0 a_{\mathrm{CCW}} \\
i\frac{d}{dt}a_{\mathrm{CW}} &= \Omega_0 a_{\mathrm{CW}}
\end{aligned}
$$

Frequency and loss $\Omega_0 \in \mathbb{C}$

CW: clockwise
CCW: counterclockwise

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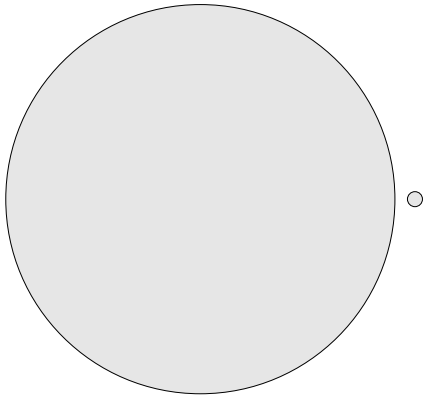

$$
\begin{aligned}
i\frac{d}{dt}a_{\mathrm{CCW}} &= (\Omega_0 + \delta\Omega)a_{\mathrm{CCW}} + s\, a_{\mathrm{CW}} \\
i\frac{d}{dt}a_{\mathrm{CW}} &= s\, a_{\mathrm{CCW}} + (\Omega_0 + \delta\Omega)a_{\mathrm{CW}}
\end{aligned}
$$

frequency shift and additional loss $\delta\Omega \in \mathbb{C}$
backscattering amplitude $s \in \mathbb{C}$

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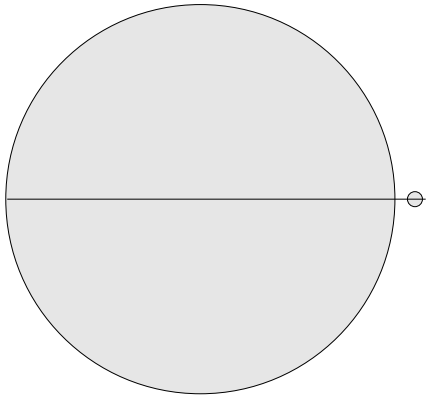

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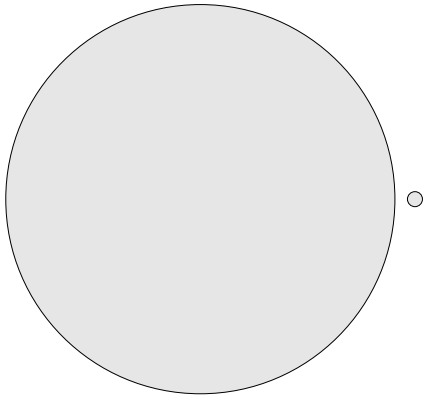

$$i\frac{d}{dt}\begin{pmatrix} a_{\mathrm{CCW}} \\ a_{\mathrm{CW}} \end{pmatrix} = \begin{pmatrix} \Omega_0 + \delta\Omega & s \\ s & \Omega_0 + \delta\Omega \end{pmatrix} \begin{pmatrix} a_{\mathrm{CCW}} \\ a_{\mathrm{CW}} \end{pmatrix}$$

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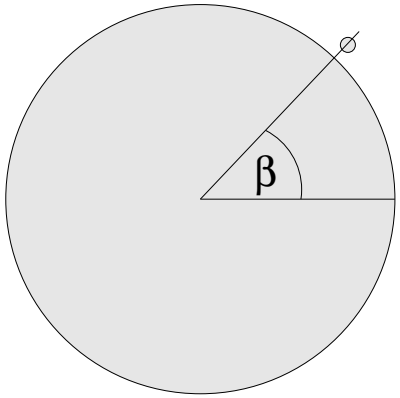

$$i\frac{d}{dt}\begin{pmatrix} a_{\mathrm{CCW}} \\ a_{\mathrm{CW}} \end{pmatrix} = \begin{pmatrix} \Omega_0 + \delta\Omega & s e^{-i2m\beta} \\ s e^{i2m\beta} & \Omega_0 + \delta\Omega \end{pmatrix} \begin{pmatrix} a_{\mathrm{CCW}} \\ a_{\mathrm{CW}} \end{pmatrix}$$

azimuthal mode number $m \in \mathbb{N}$ $(e^{im\phi}, e^{-im\phi})$

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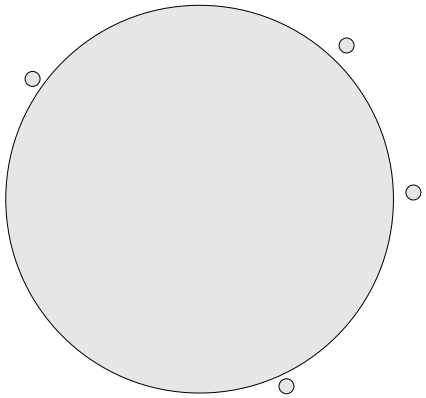

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$\delta\Omega_j, s_j \in \mathbb{C}$

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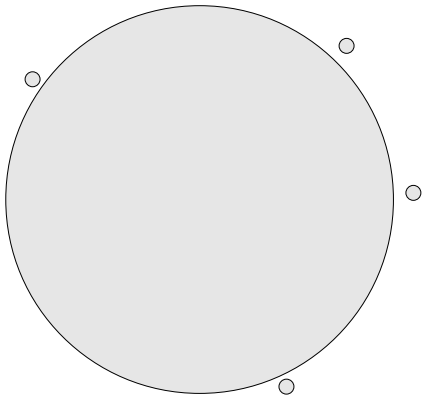

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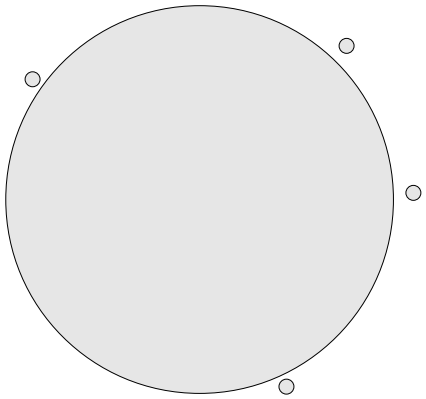

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Asymmetric backscattering

$$s_j \in \mathbb{C} \rightarrow |A| \neq |B|$$

Destructive/constructive interference of the scattered waves

J. Wiersig, PRA **84**, 063828 (2011), see also Y. Yi, Y.-F. Xiao *et al.*, PRA **83**, 023803 (2011)

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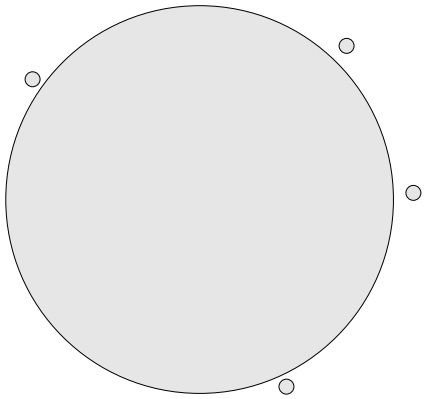

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Special case $|A| = |B|$ if

- the system is closed ($s_j \in \mathbb{R}$) or
- $s_j = s$ or
- a mirror-reflection symmetry is present

Asymmetric backscattering

Properties of the effective Hamiltonian

$$H_{\text{eff}} = \begin{pmatrix} \Omega & A \\ B & \Omega \end{pmatrix} \quad \text{with } |A| \neq |B|$$

Complex eigenvalues and right hand eigenvectors

$$\Omega_{\pm} = \Omega \pm \sqrt{AB}$$

$$\vec{\psi}_{\pm} = \begin{pmatrix} \psi_{\text{CCW},\pm} \\ \psi_{\text{CW},\pm} \end{pmatrix} = \begin{pmatrix} \sqrt{A} \\ \pm\sqrt{B} \end{pmatrix}$$

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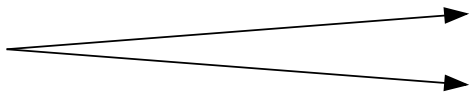
$|A| > |B|$:



- CCW component $>$ CW component
  - $\rightarrow$ chirality: modes are not standing waves
  - $\rightarrow$ copropagation
  - $\rightarrow$ nonorthogonality

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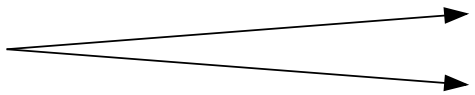

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$|A| < |B|$: CW $\leftrightarrow$ CCW

Asymmetric backscattering

Quantitfy nonorthogonality and chirality

Overlap of two eigenvectors $\vec{\psi}_-$, $\vec{\psi}_+$

$$S = \frac{|\vec{\psi}_- \cdot \vec{\psi}_+|}{|\vec{\psi}_-||\vec{\psi}_+|} \in [0, 1]$$

$S = 0$: modes are orthogonal $\quad$ $S = 1$: modes are collinear

Asymmetric backscattering

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Chirality of an eigenvector $\vec{\psi}$

$$\alpha = 1 - \frac{\min\left(|\psi_{\text{CCW}}|^2, |\psi_{\text{CW}}|^2\right)}{\max\left(|\psi_{\text{CCW}}|^2, |\psi_{\text{CW}}|^2\right)} \in [0, 1]$$

$\alpha = 0$: standing wave $\quad$ $\alpha = 1$: traveling wave

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$\alpha = 0$: standing wave $\quad$ $\alpha = 1$: traveling wave

Relation between overlap and chirality

$$\alpha = \frac{2S}{1 + S}$$

Asymmetric backscattering

Exceptional point

$$H_{\text{eff}} = \begin{pmatrix} \Omega & A \\ B & \Omega \end{pmatrix} \quad ; \quad \Omega_\pm = \Omega \pm \sqrt{AB} \quad ; \quad \vec{\psi}_\pm = \begin{pmatrix} \sqrt{A} \\ \pm\sqrt{B} \end{pmatrix}$$

Fully asymmetric backscattering: $B \to 0$ with $A \neq 0$

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Jordan block

- splitting $\to 0$
- only one linearly independent eigenvector $\hat{=}$ CCW traveling-wave mode
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Asymmetric backscattering

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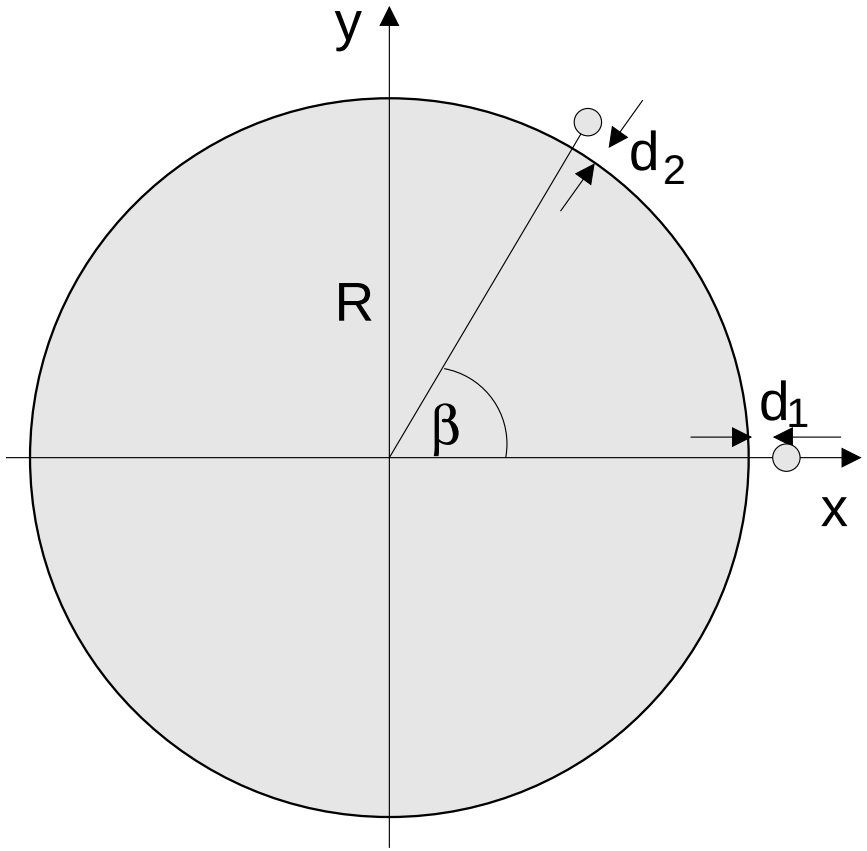
$A \to 0$ with $B \neq 0$: CW $\leftrightarrow$ CCW

Asymmetric backscattering

Microdisk perturbed by two nanoparticles

J. Wiersig, PRA **84**, 063828 (2011)

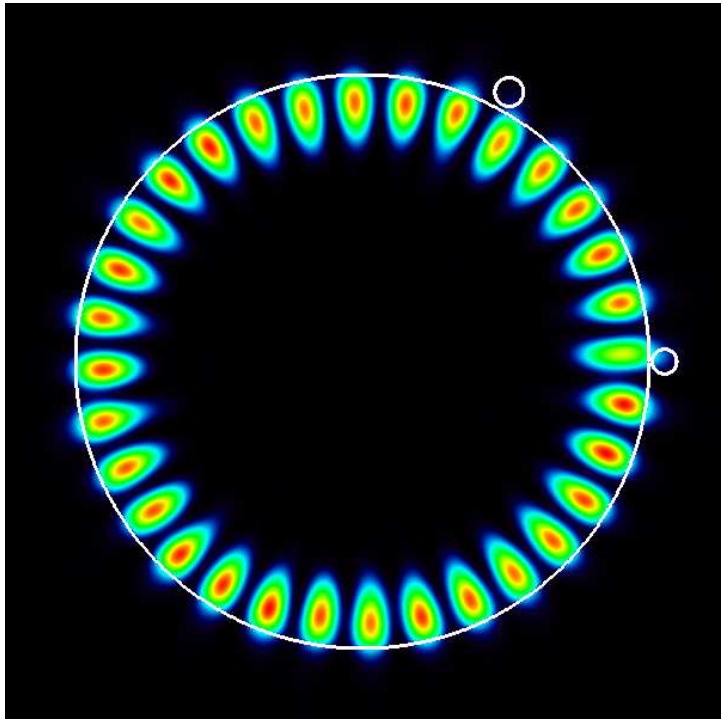
- TM polarization
- $n = n_j = 2$
- $d_1/R = 0.01$, $d_2/R = 0.02$
- $r_1/R = 0.043$, $r_2/R \approx 0.05$
- $\Omega = \frac{\omega}{c} R = kR = \frac{2\pi R}{\lambda} \approx 10$



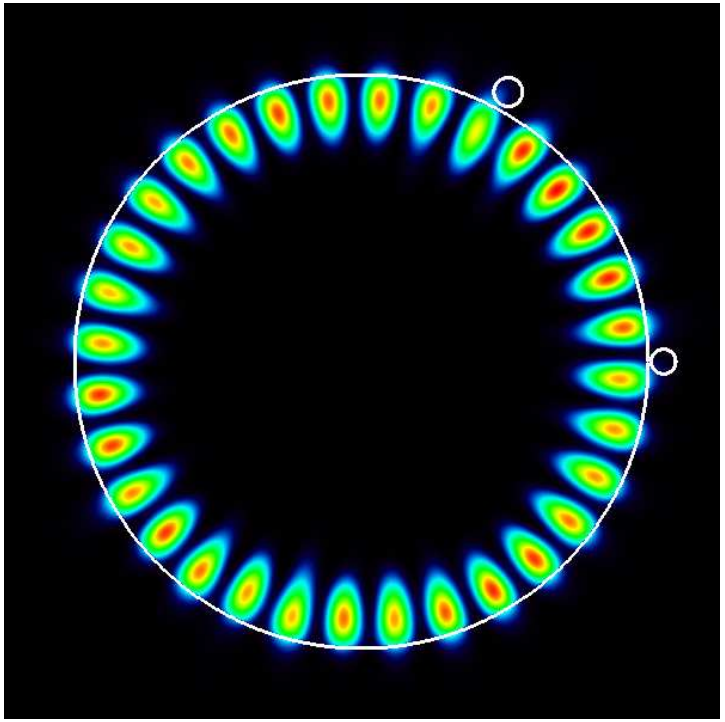
Asymmetric backscattering

A pair of nearly degenerate modes

$\beta = 1.072$ (radian) $\rightarrow$ overlapping resonances



$\Omega_+ = 9.87722 - i0.00243$



$\Omega_- = 9.87869 - i0.00266$

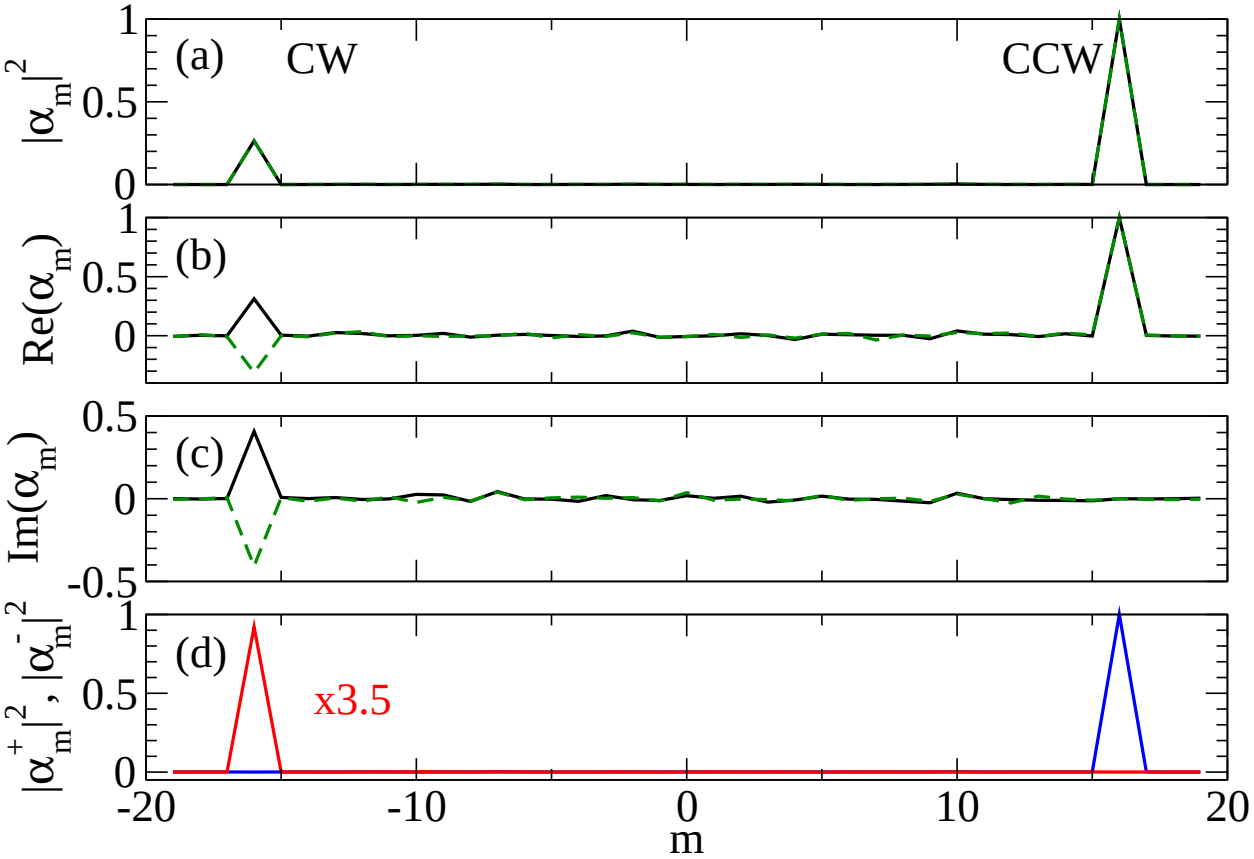
Standing-wave modes?

Asymmetric backscattering

Angular momentum representation

Inside the cavity

$$\psi(r,\phi) = \sum_{m=-\infty}^{\infty} \alpha_m J_m(nkr) e^{im\phi}$$



- 2-mode approx. ✓
- ≈ copropagating traveling waves
- chirality

Asymmetric backscattering

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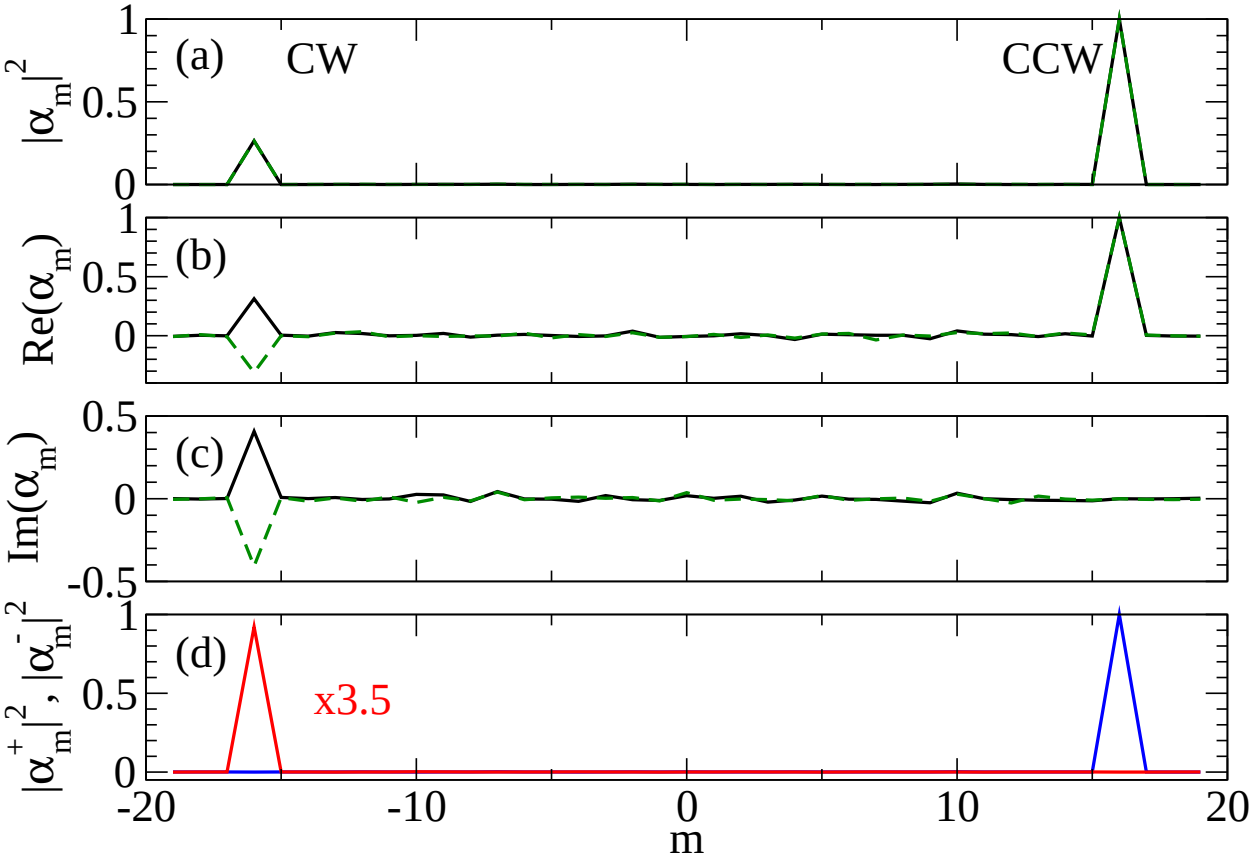

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Asymmetric backscattering

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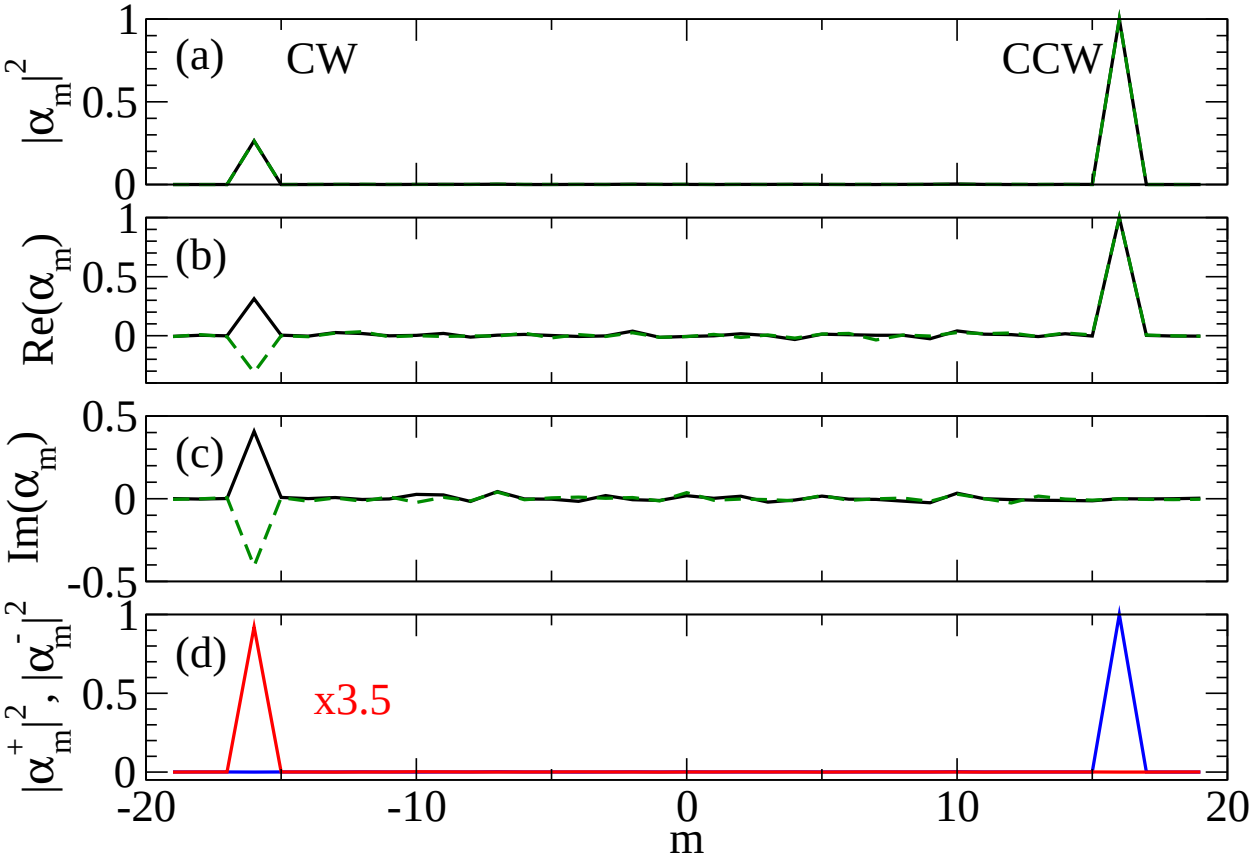

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Asymmetric backscattering

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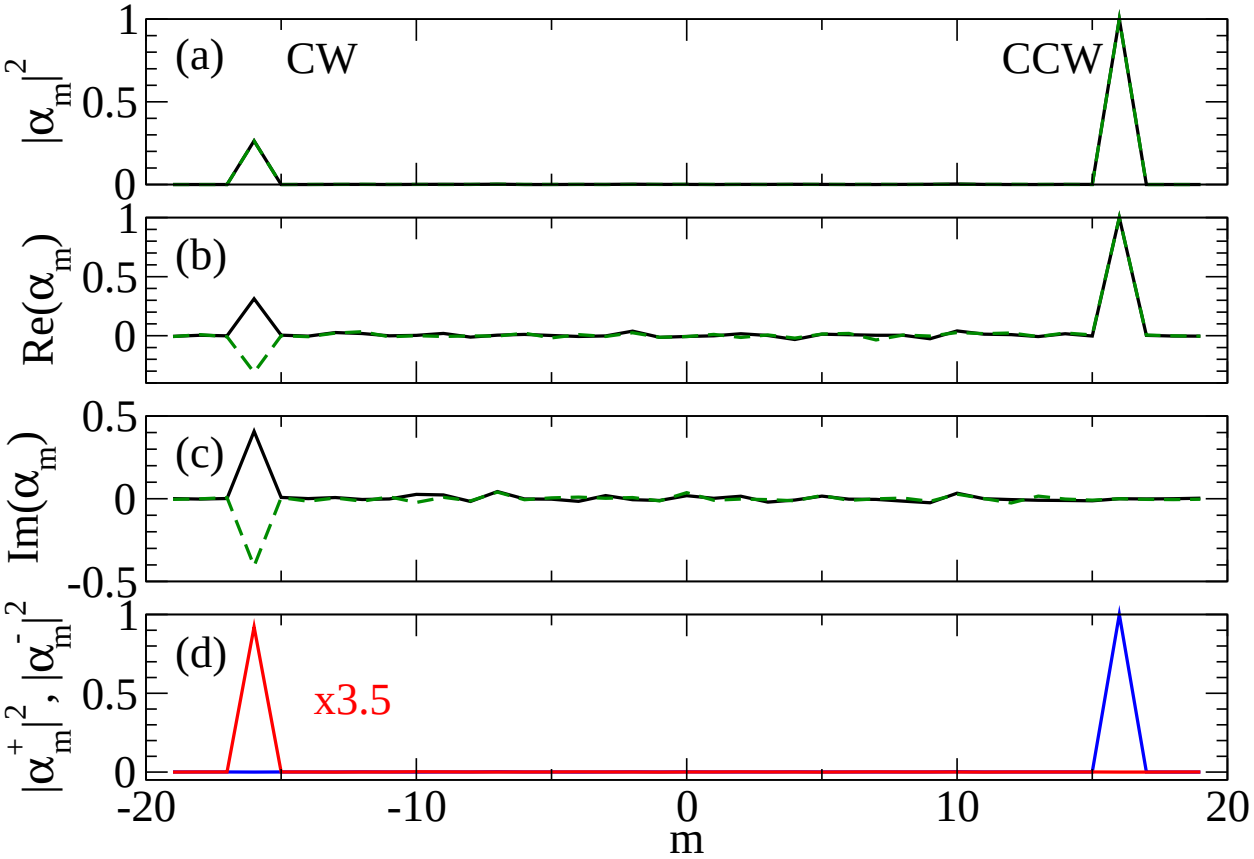

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$$\vec{\psi}_\pm = \begin{pmatrix} \sqrt{A} \\ \pm\sqrt{B} \end{pmatrix}$$

- CW and CCW superpositions

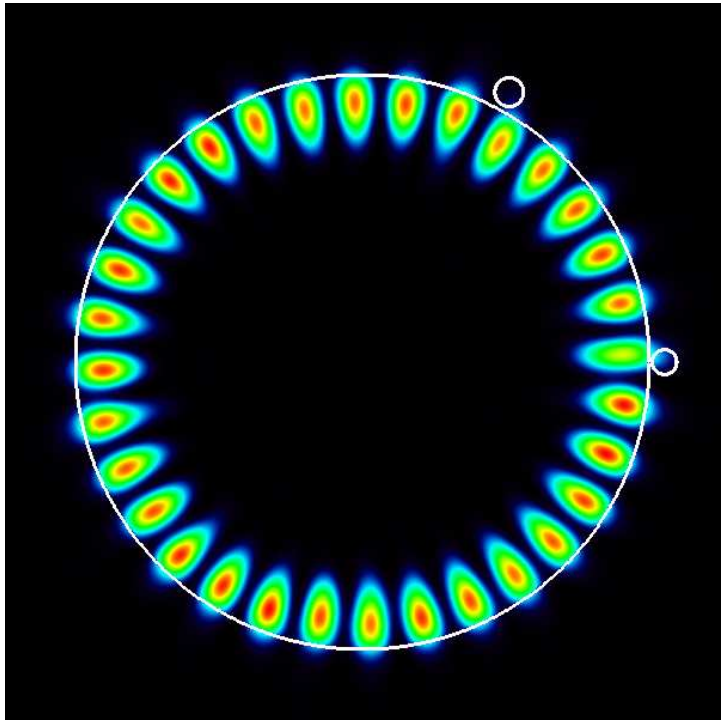
Chirality

$$\alpha = 1 - \frac{\min\left(\sum_{m=-\infty}^{-1} |\alpha_m|^2, \sum_{m=1}^{\infty} |\alpha_m|^2\right)}{\max\left(\sum_{m=-\infty}^{-1} |\alpha_m|^2, \sum_{m=1}^{\infty} |\alpha_m|^2\right)} \approx \begin{cases} 0.73 \\ 0.73 \end{cases}$$

Asymmetric backscattering

A pair of nearly degenerate and nonorthogonal modes

$\beta = 1.072 \rightarrow$ overlapping resonances



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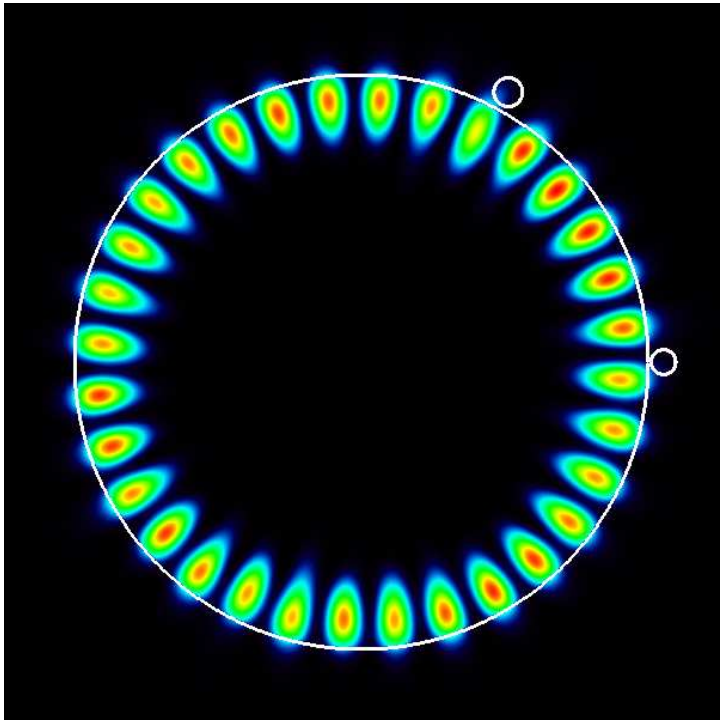


$\Omega_- = 9.87869 - i0.00266$

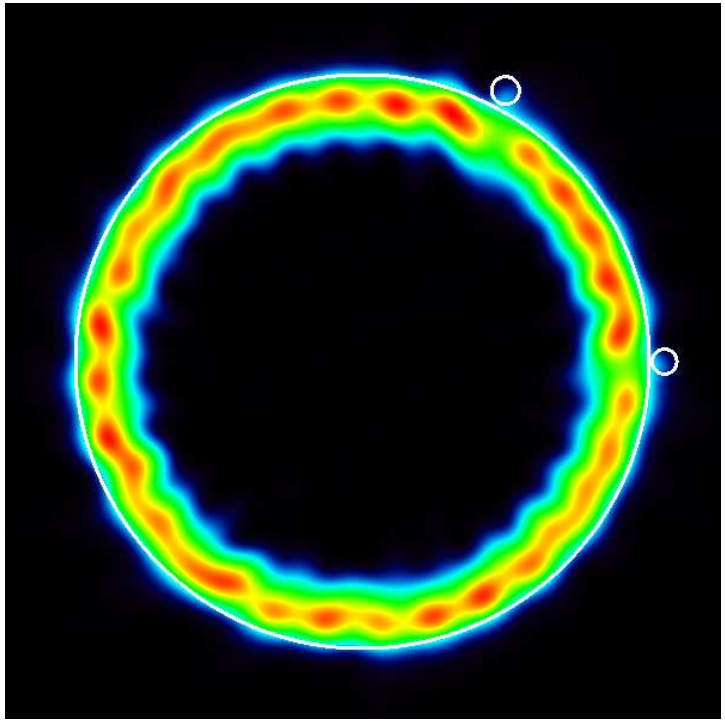
Normalized overlap integral

$$S = \frac{|\int_C dxdy\ \psi_1^* \psi_2|}{\sqrt{\int_C dxdy\ \psi_1^* \psi_1}\sqrt{\int_C dxdy\ \psi_2^* \psi_2}} \approx 0.583 \quad \text{significant nonorthogonality}$$

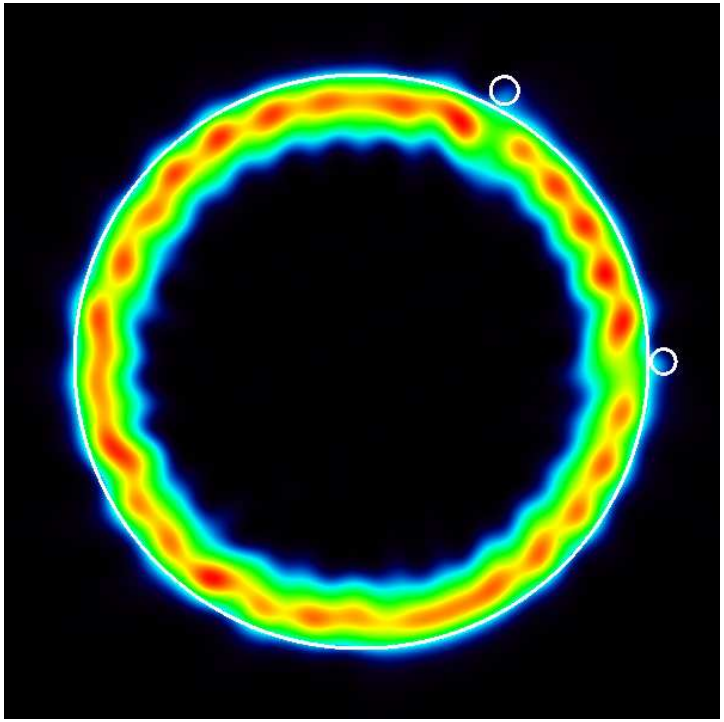
Asymmetric backscattering

A pair of nearly degenerate modes in the regime of strongly overlapping resonances

$r_2/R = 0.04838$ and $\beta = 1.08468 \rightarrow$ strongly overlapping resonances



$\Omega_+ = 9.87807 - i0.002427$



$\Omega_- = 9.87803 - i0.002428$

Chirality

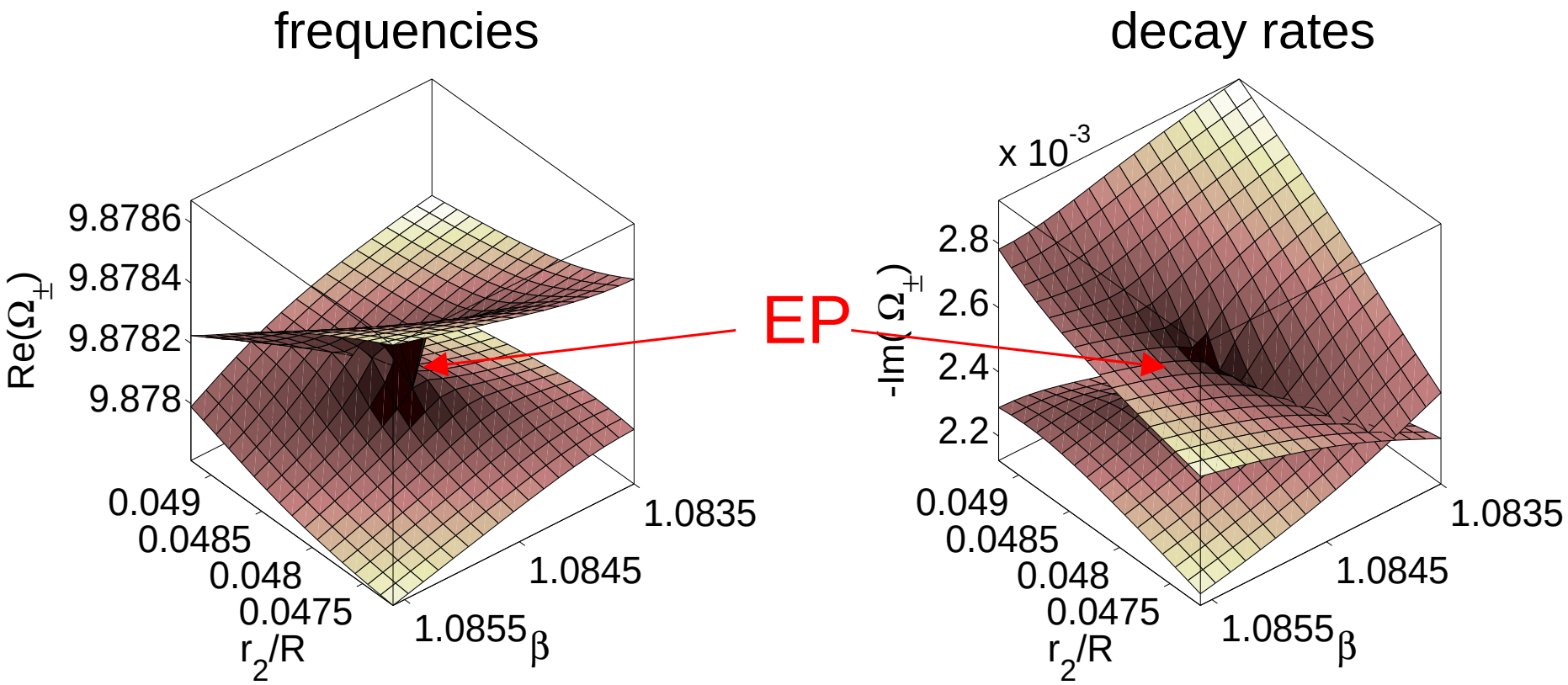
$$\alpha \approx \begin{cases} 1 \\ 1 \end{cases}$$ copropagating CW traveling-wave modes

Normalized overlap integral

$S \approx 1$ almost collinear!

Asymmetric backscattering

Complex-square-root topology with a branch point singularity at the EP



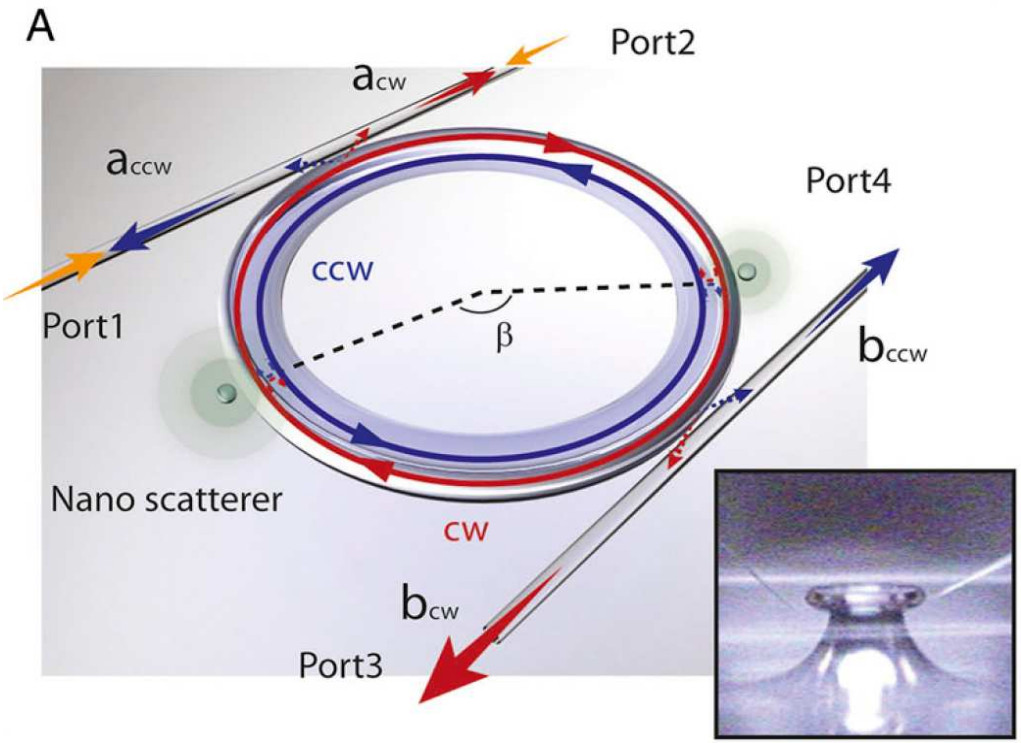
$\alpha \approx 0.8$ to 1 when moving from light to dark areas

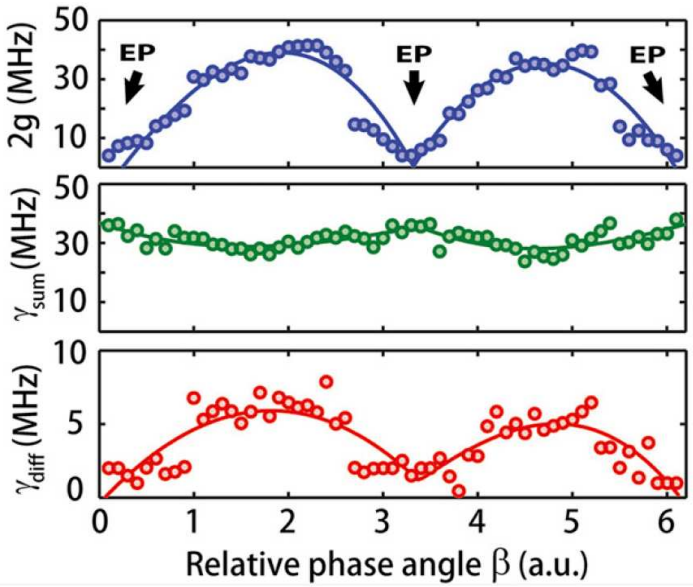
- EP: frequency and decay rate splitting $\to$ 0 and chirality $\to$ 1

Asymmetric backscattering

Experimental confirmation of asymmetric backscattering and chirality

B. Peng, Ş. K. Özdemir, M. Liertzer, W. Chen, J. Kramer, H. Yilmaz, JW, S. Rotter, and L. Yang, PNAS (2016)



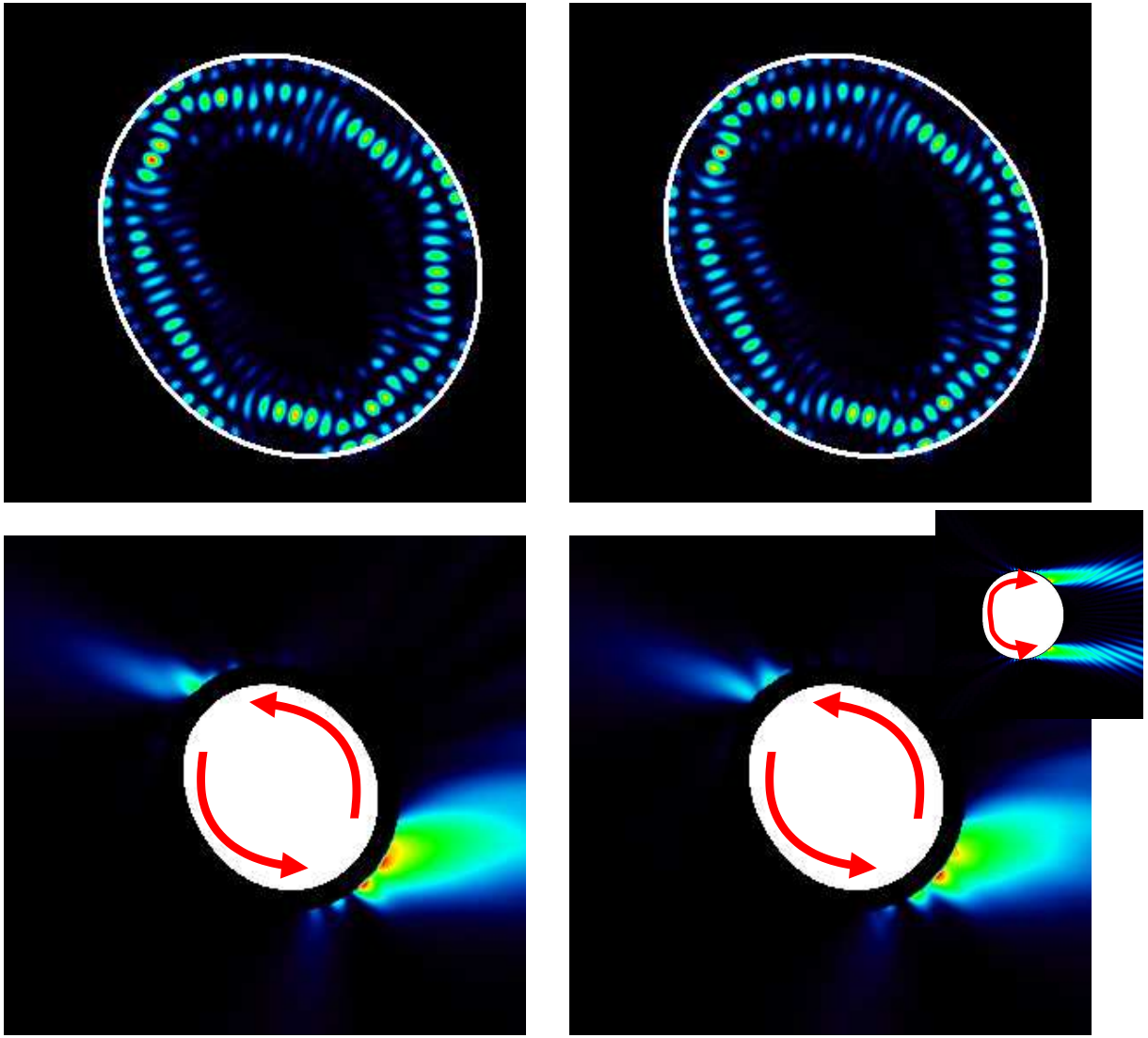


→ talk by Şahin Özdemir

Asymmetric backscattering

Asymmetric Limaçon cavity

$\rho = R\,[1 + \varepsilon_1 \cos\phi + \varepsilon_2 \cos(2\phi + \delta)]$ J. Wiersig *et al.*, PRA **84**, 023845 (2011)



Overlap $S \approx 0.72$

Field inside

Field outside

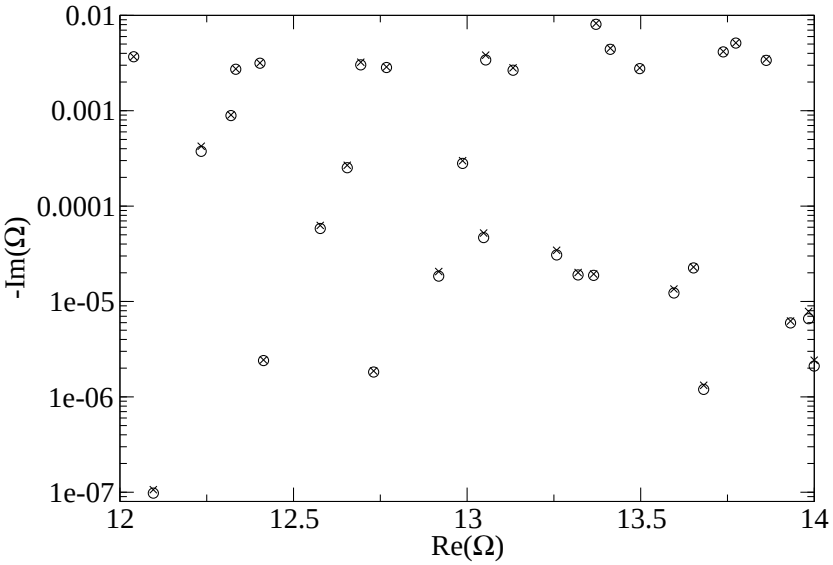
Chirality $\alpha \approx 0.84$

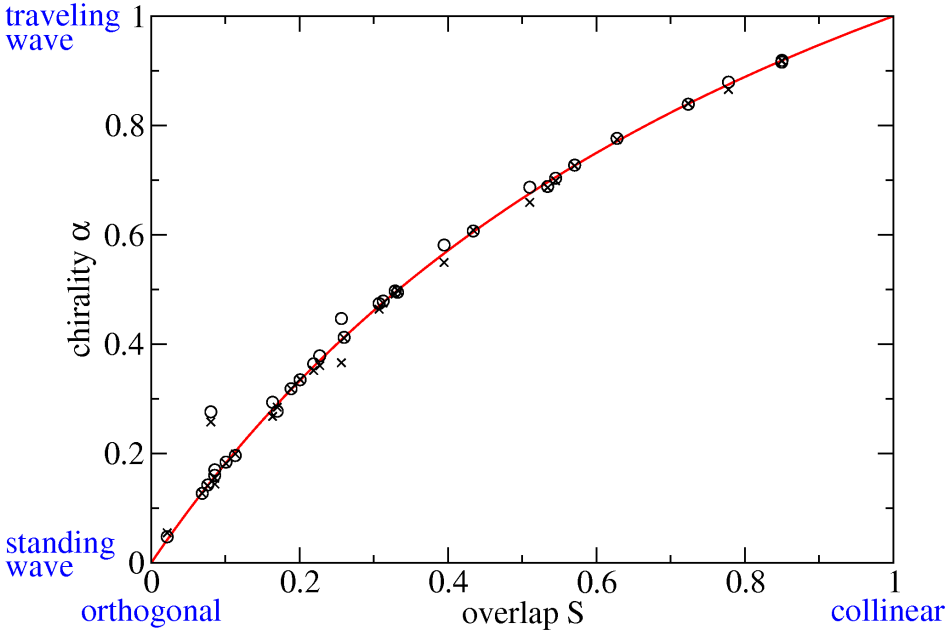
$\Omega_+ = 12.31981 - i0.00089$ $\Omega_- = 12.31985 - i0.0009$

Effective Hamiltonian predicts

$$\alpha = \frac{2S}{1+S}$$

Asymmetric Limaçon cavity



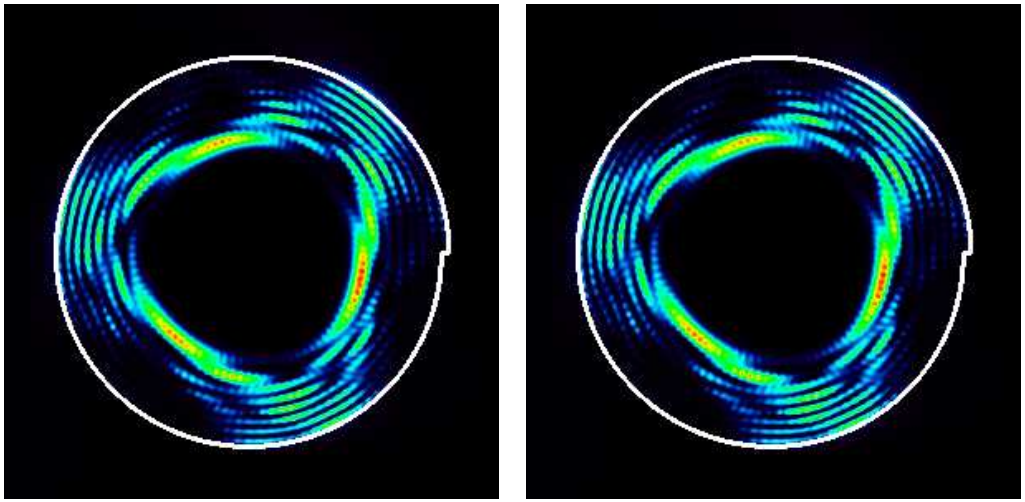


Effective Hamiltonian explains the relation between chirality and overlap

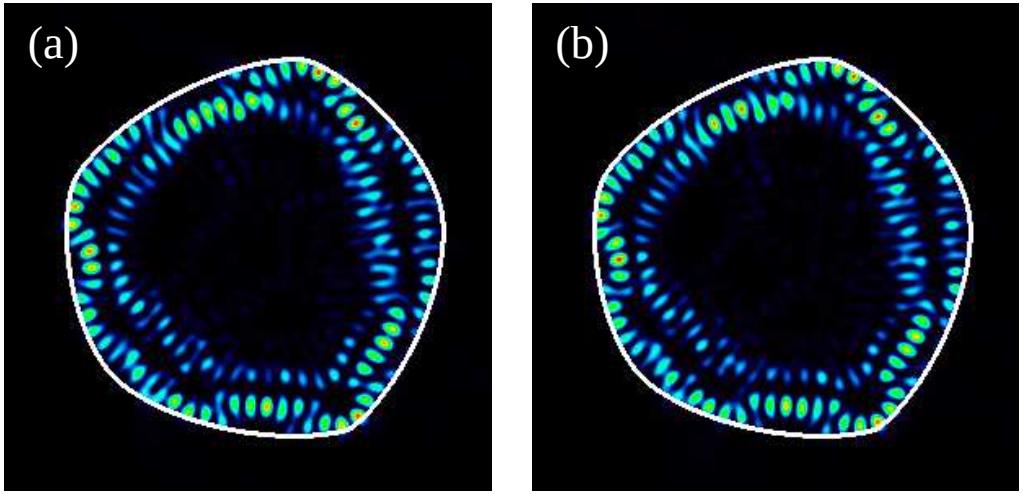
Asymmetric backscattering

Other systems

Spiral cavity J. Wiersig, S.W. Kim, and M. Hentschel, PRA **78**, 053809 (2008)



Cavities of constant width J. Wiersig *et al.*, PRA **84**, 023845 (2011)



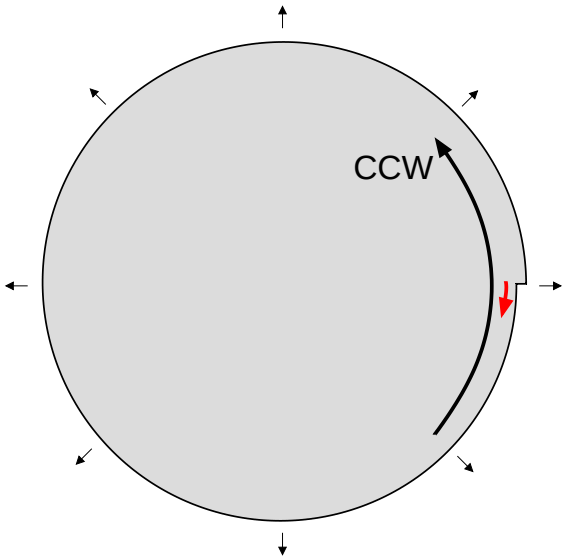
Open quantum and wave systems J. Wiersig, PRA **89**, 012119 (2014)

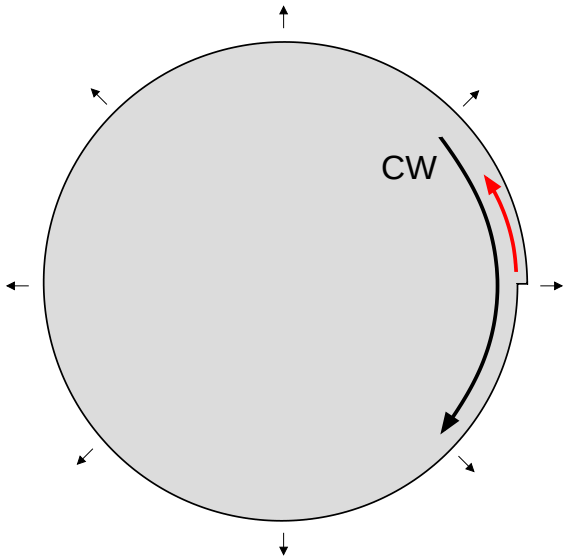
Asymmetric backscattering

Frobenius-Perron operator for deformed microdisks

asymmetric backscattering in ray dynamics $\overset{?}{\Longrightarrow}$ chirality, copropagation, and nonorthogonality?

J. Kullig and J. Wiersig, New J. Phys. **18**, 015005 (2016)





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discrete time evolution of phase-space density ρ with Frobenius-Perron operator $\mathcal{F}$

$$\rho(n+1) = \mathcal{F}\rho(n)$$

for maps see e.g. J. Weber *et al.*, PRL **85**, 3620 (2000), K. Frahm and D. Shepelyansky, EPL **75**, 299 (2010)

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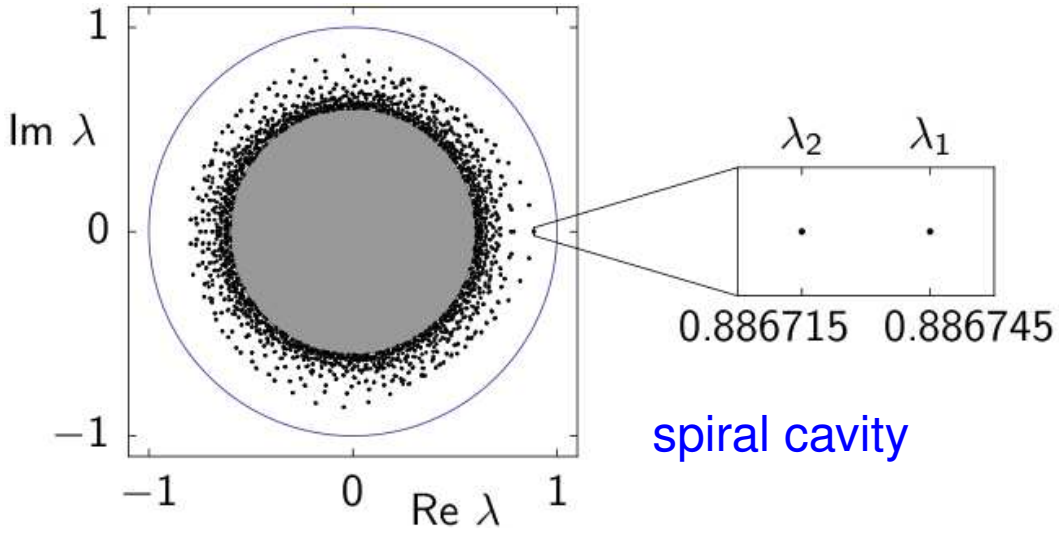
$$\mathcal{F}\rho_j = \lambda_j \rho_j$$

- weight to incorporate reflectivity
  $\Longrightarrow$ $\mathcal{F}$ is sub-unitary



spiral cavity

the two largest eigenvalues are nearly degenerate (eigenstate pair)

Asymmetric backscattering

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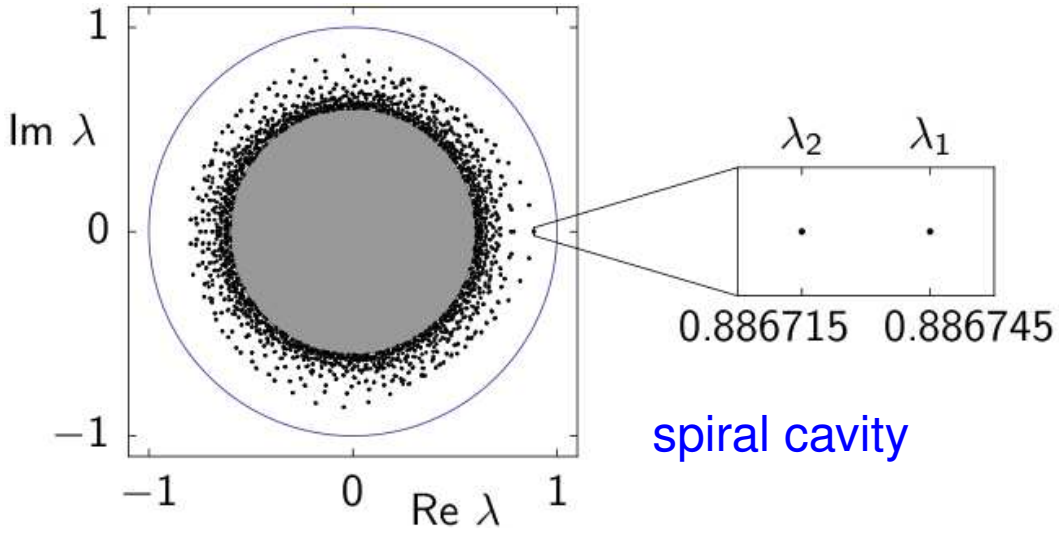

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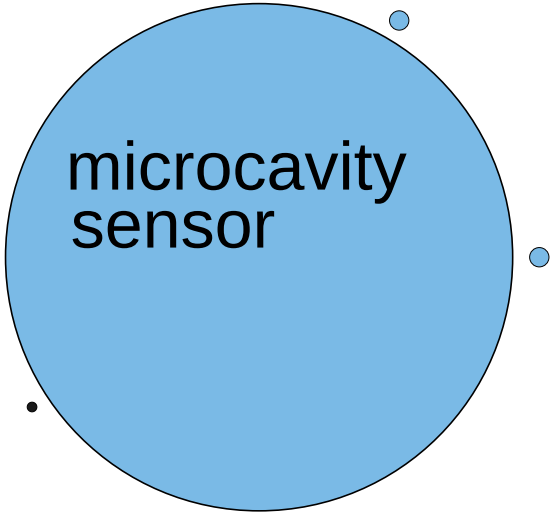
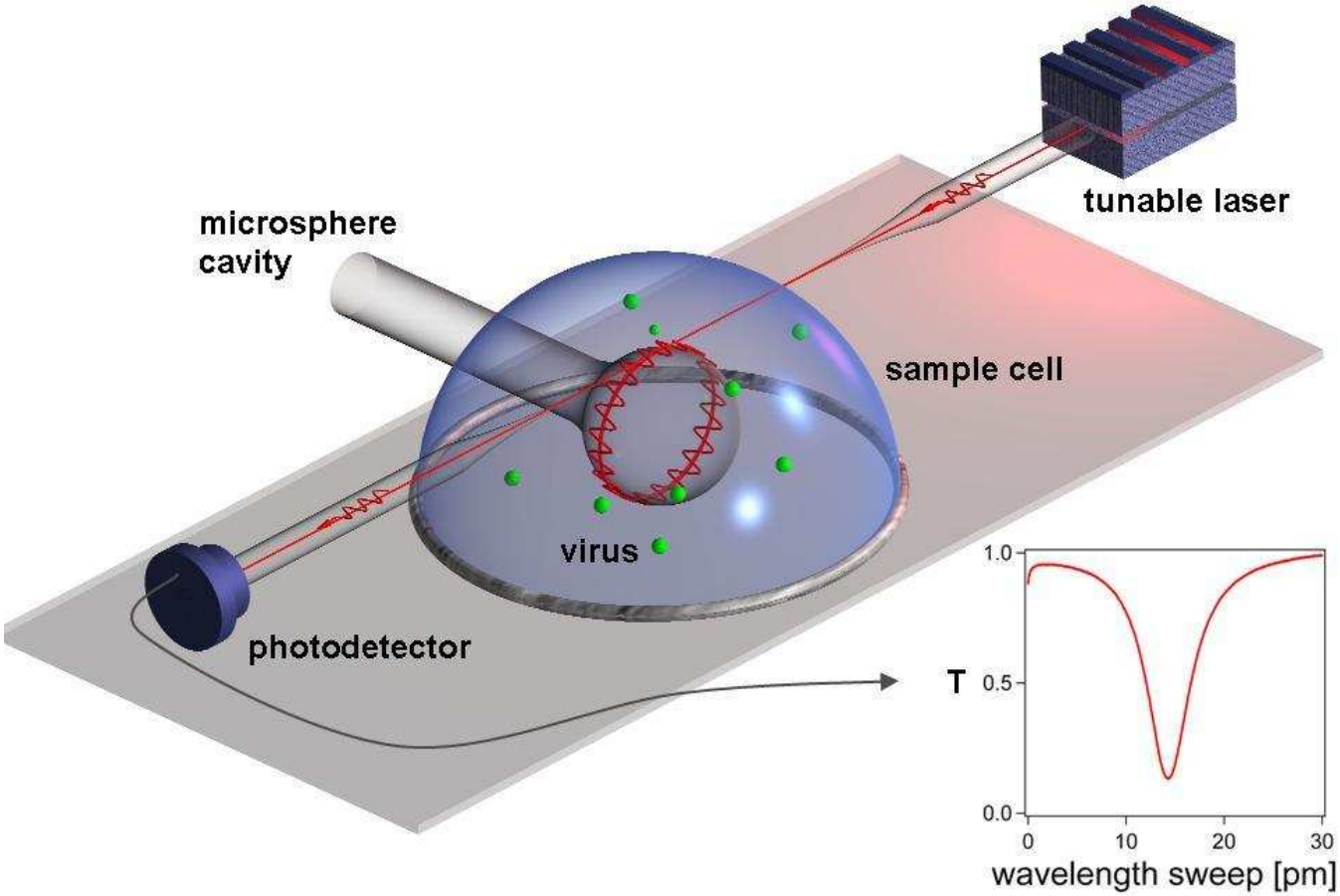
Sensors based on exceptional points



Sensors based on exceptional points

Microcavity sensor for single-particle detection



F. Vollmer *et al.*, PNAS **105**, 20701 (2008)

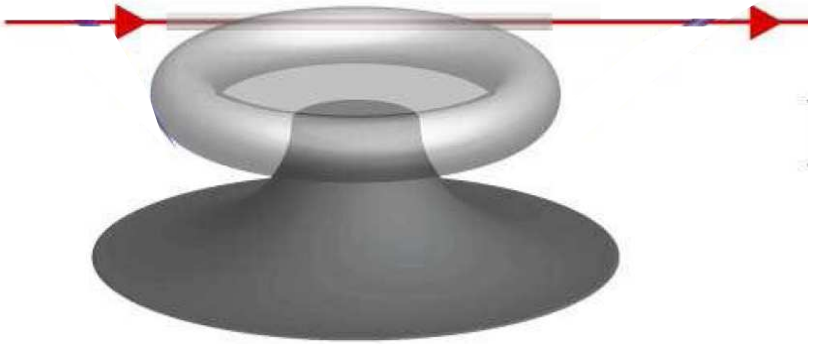
Measure frequency shift $\Longrightarrow$ particle detection

Sensors based on exceptional points

Microcavity sensor based on frequency-splitting detection

Measure frequency splitting of initially degenerate modes (diabolic point)

J. Zhu *et al.*, Nature Photonics **4**, 46 (2010)



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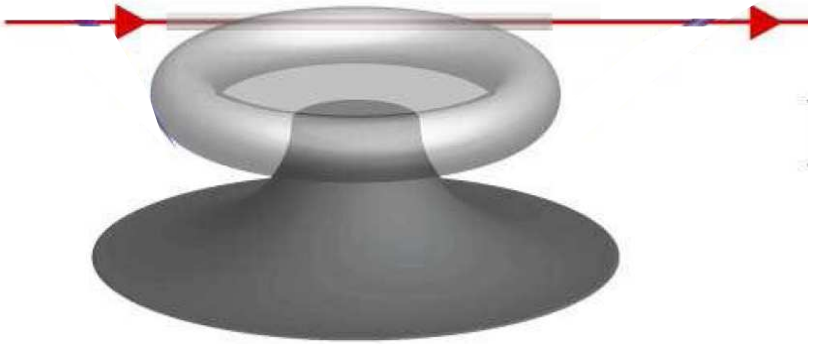

Problem: initial splitting due to fabrication imperfections

Sensors based on exceptional points

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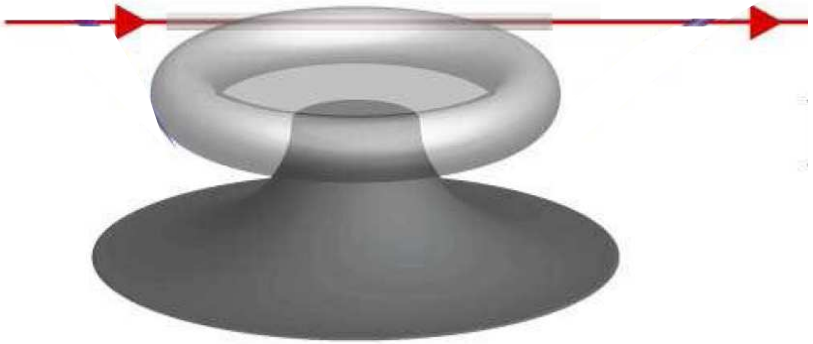

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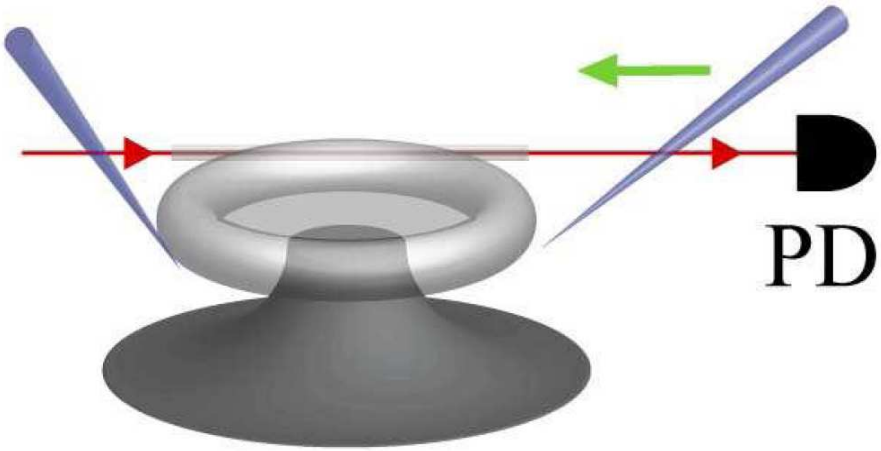
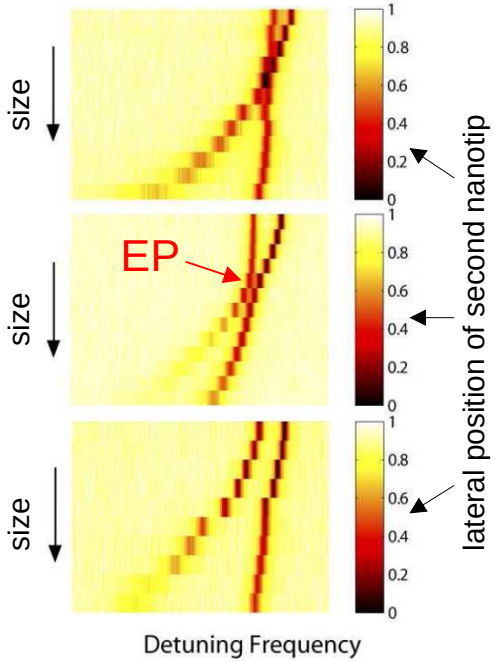


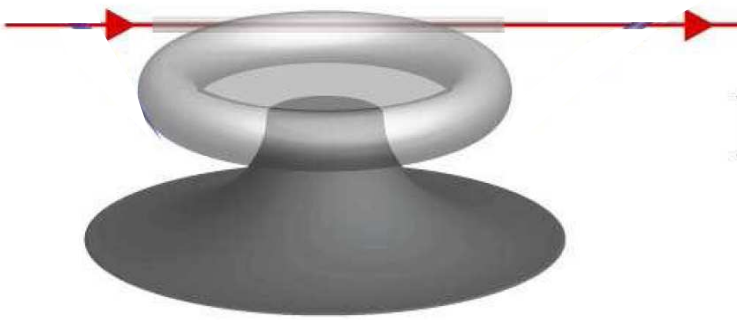


J. Zhu *et al.*, Opt. Express **18**, 23535 (2010)

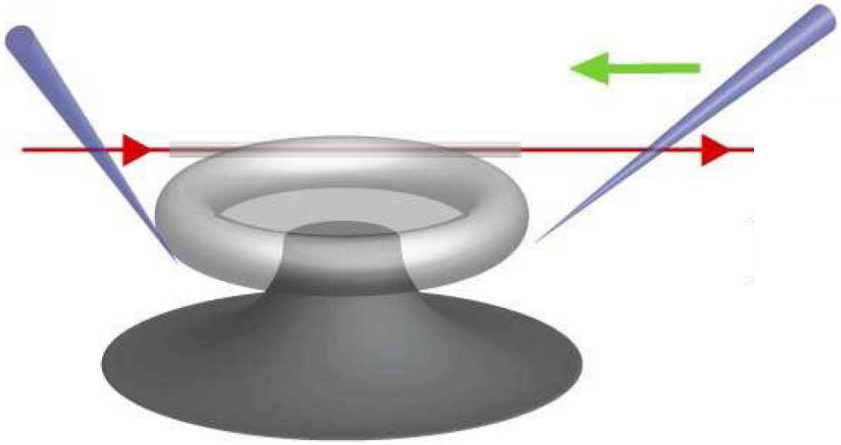
Sensors based on exceptional points

Conventional degeneracy vs exceptional point

J. Wiersig, PRL **112**, 203901 (2014)



conventional (DP)



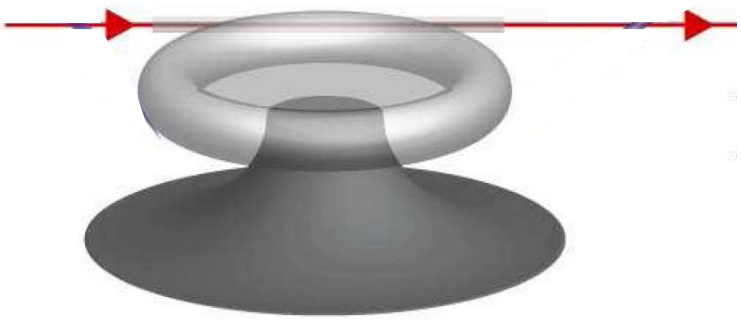
EP

Which one is better for sensing?

Sensors based on exceptional points

Conventional degeneracy vs exceptional point

J. Wiersig, PRL **112**, 203901 (2014)



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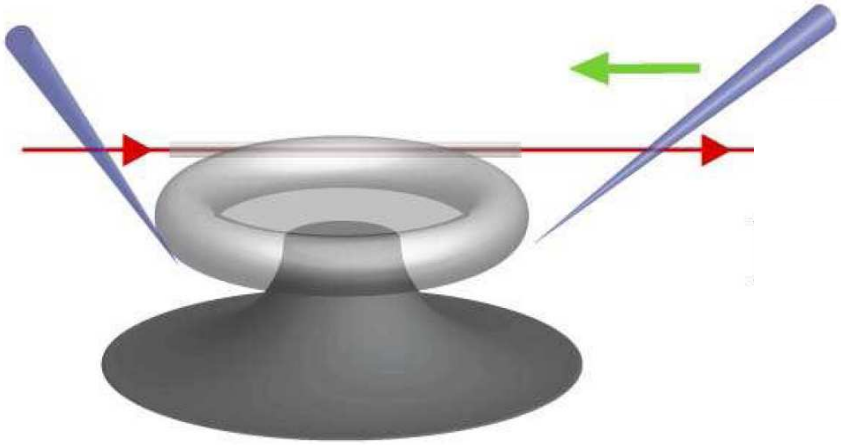


EP

Which one is better for sensing?

Apply a perturbation of strength ϵ to a (two-fold) degeneracy

$$\Delta\Omega_{\mathrm{DP}} = \mathcal{O}(\varepsilon)$$

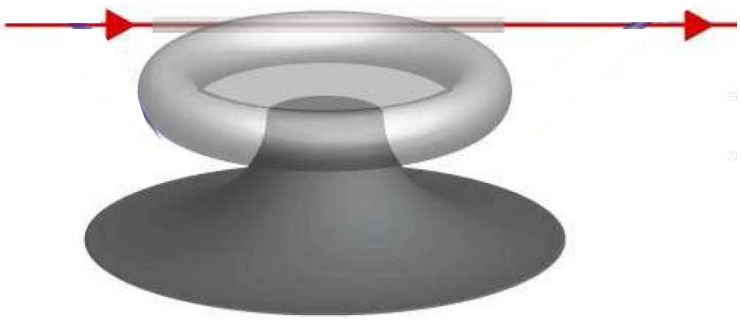
$$\Delta\Omega_{\mathrm{EP}} = \mathcal{O}(\sqrt{\varepsilon})$$

T. Kato (1966)

Sensors based on exceptional points

Conventional degeneracy vs exceptional point

J. Wiersig, PRL **112**, 203901 (2014)



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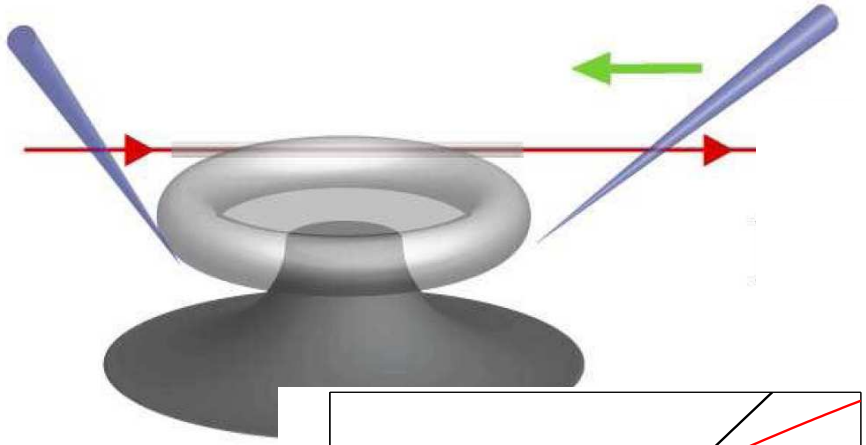
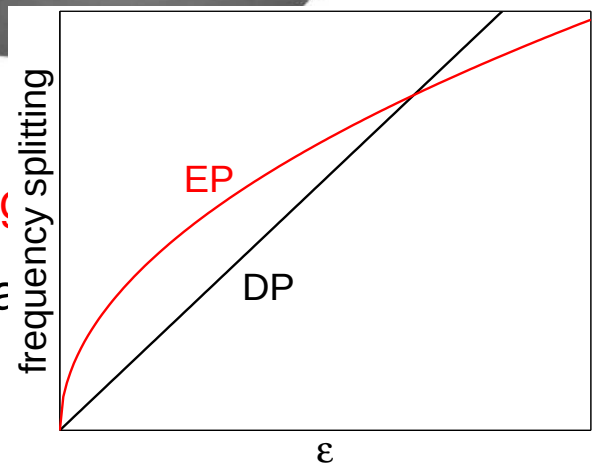


Which one is better for sensing

Apply a perturbation of strength ϵ to a (two-fold) degenera

$$\Delta\Omega_{\mathrm{DP}} = \mathcal{O}(\varepsilon)$$

$$\Delta\Omega_{\mathrm{EP}}$$

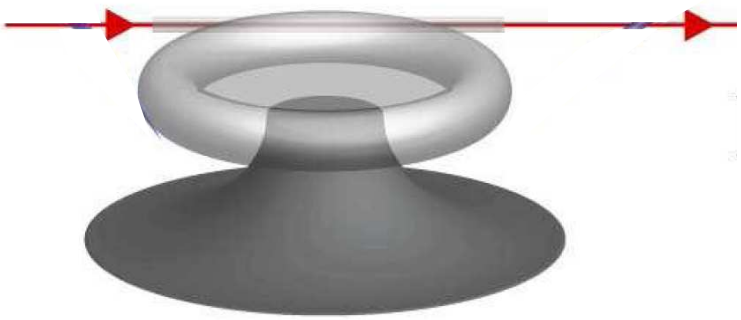


T. Kato (1966)

Sensors based on exceptional points

Conventional degeneracy vs exceptional point

J. Wiersig, PRL **112**, 203901 (2014)



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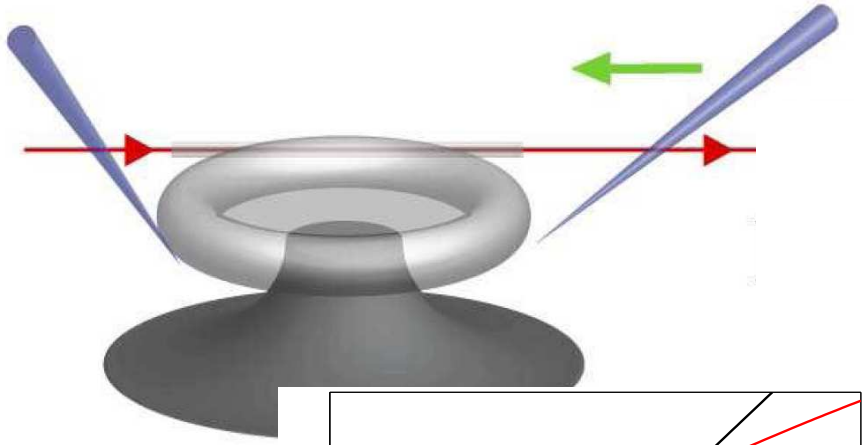

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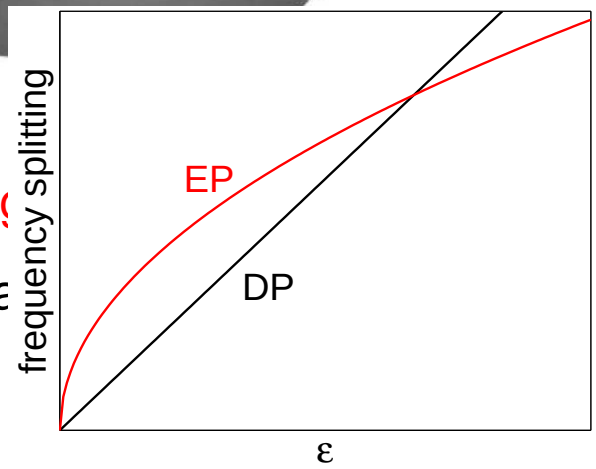


T. Kato (1966)

Enhancement factor of sensitivity for splitting detection

$$\frac{\Delta\Omega_{\mathrm{EP}}}{\Delta\Omega_{\mathrm{DP}}} = \mathcal{O}\left(\frac{1}{\sqrt{\varepsilon}}\right) \quad \text{for sufficiently small } \varepsilon$$

Sensors based on exceptional points

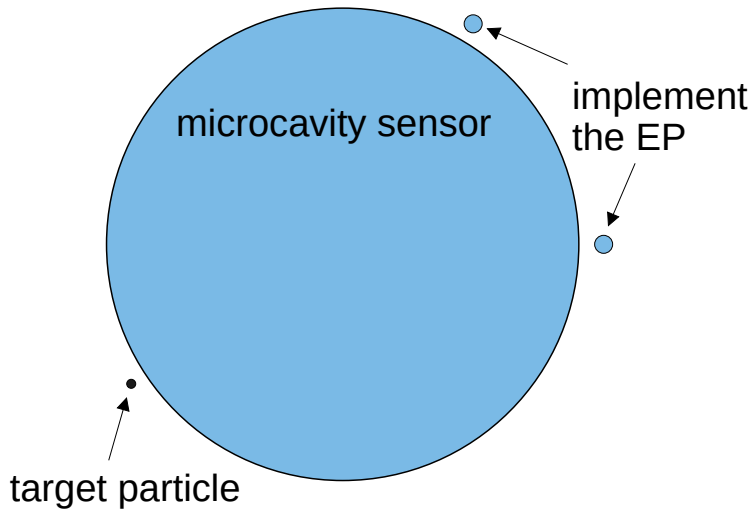
Price to pay

- $\Delta\Omega_{\mathrm{EP}} \in \mathbb{C}$
  $\Longrightarrow$ frequency and linewidth splitting has to be measured

- peaks strongly overlap near EP
  $\Longrightarrow$ gain helps to resolve peaks

Sensors based on exceptional points

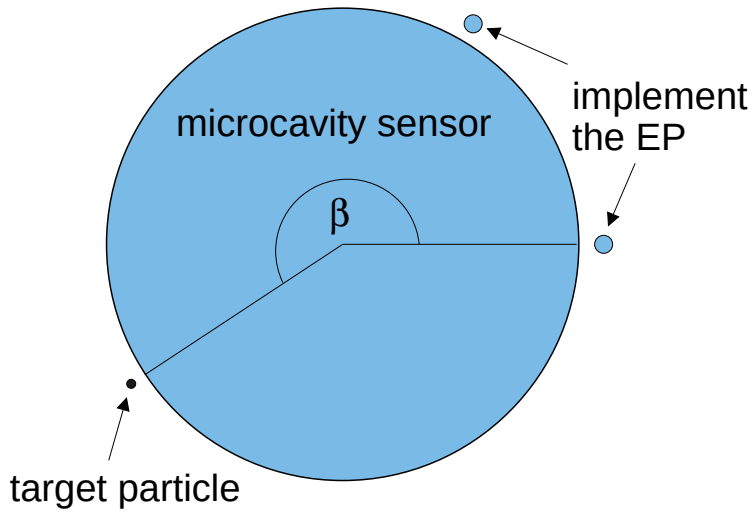
Results for a microcavity sensor at an EP

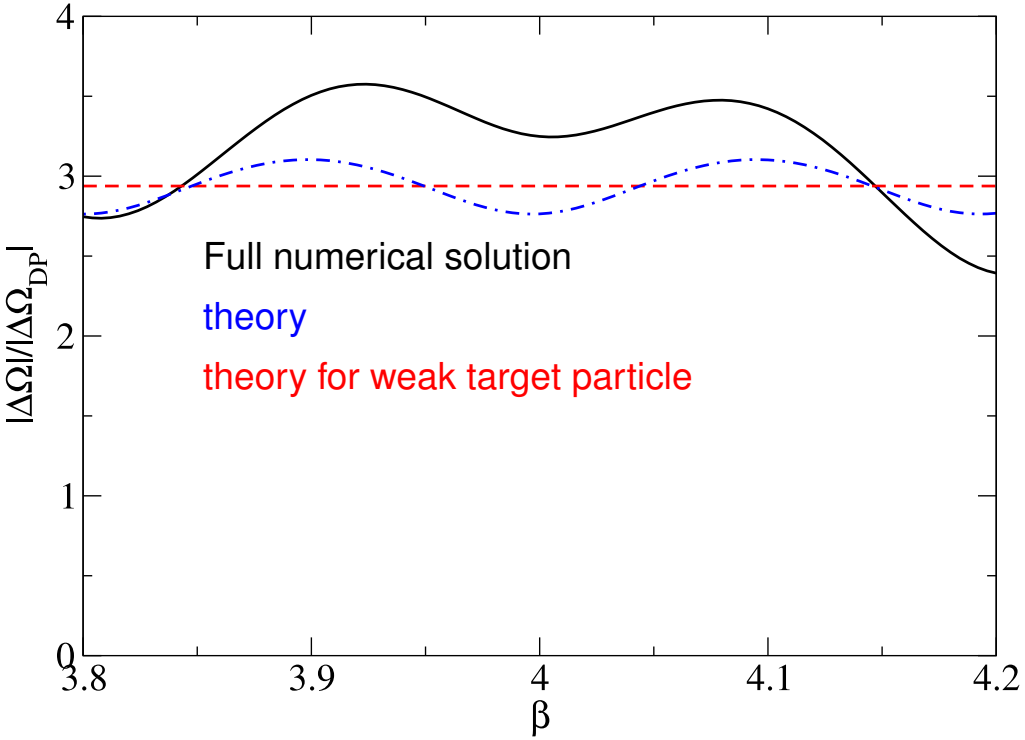


EP is due to fully asymmetric backscattering

Sensors based on exceptional points

Results for a microcavity sensor at an EP



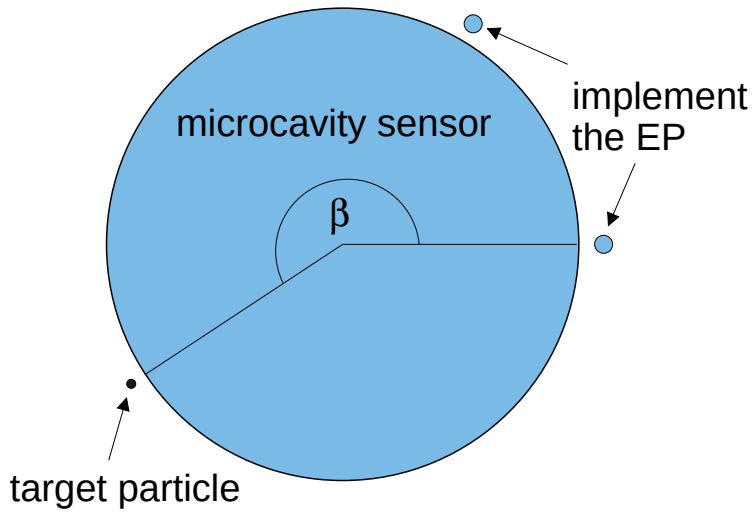


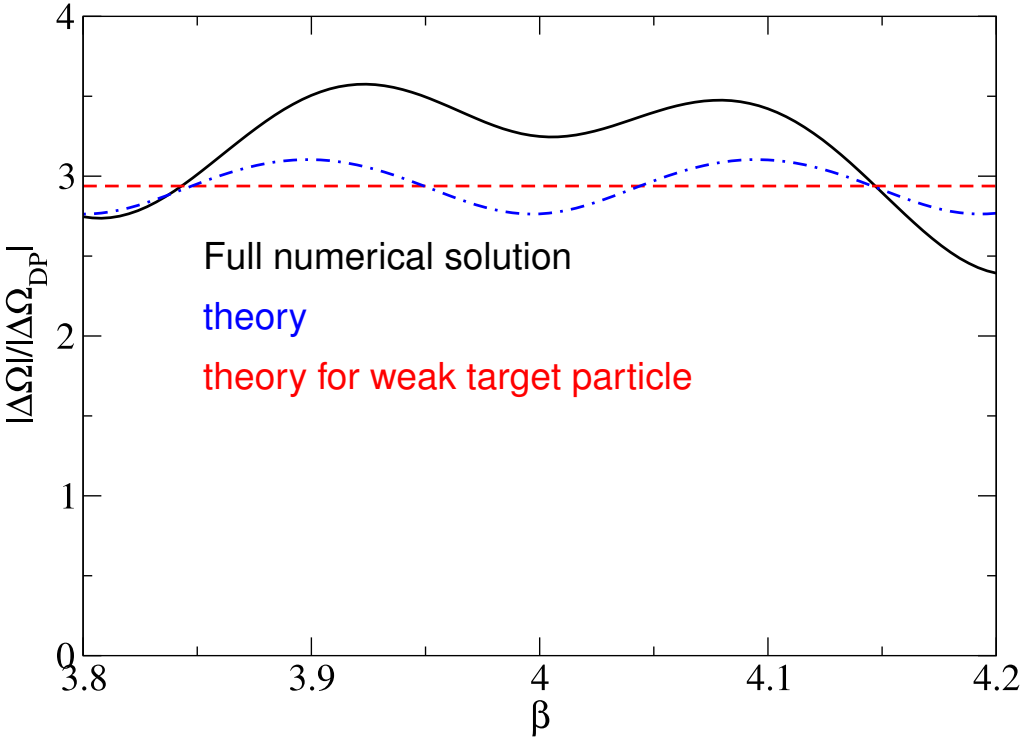
EP is due to fully asymmetric backscattering

- 3 to 3.5 fold enhancement of sensitivity
- Splitting $|\Delta\Omega|$ is nearly independent on β

Sensors based on exceptional points

Results for a microcavity sensor at an EP





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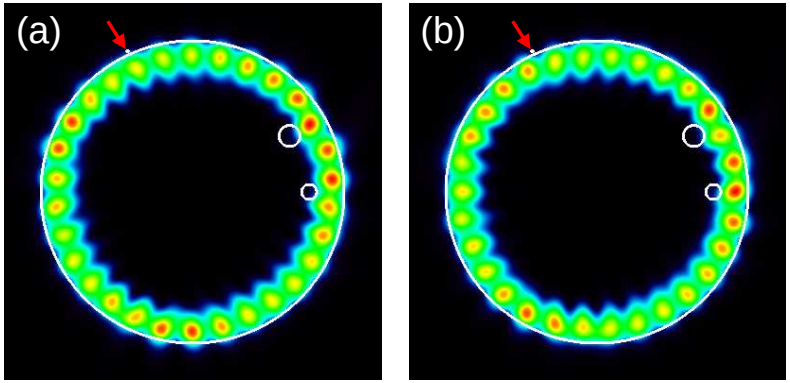
Sensitivity of sensors based on frequency splitting detection can be enhanced at an EP

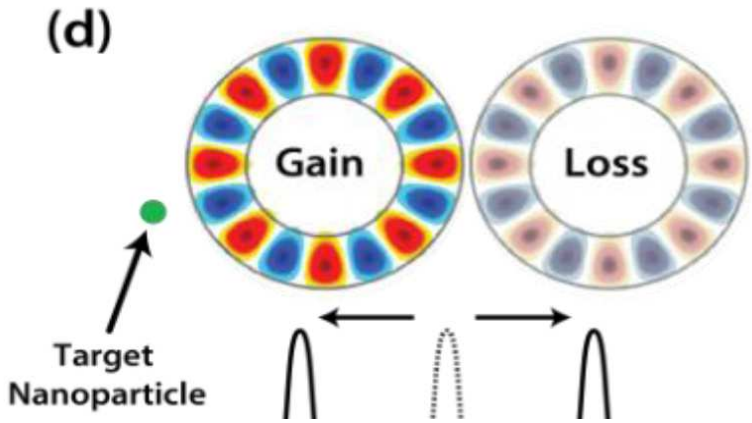
Sensors based on exceptional points

Other theoretical studies

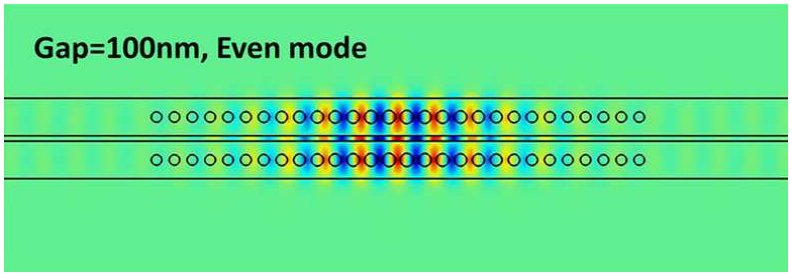


J. Wiersig, PRA **93**, 033809 (2016)

7 fold enhancement of sensitivity

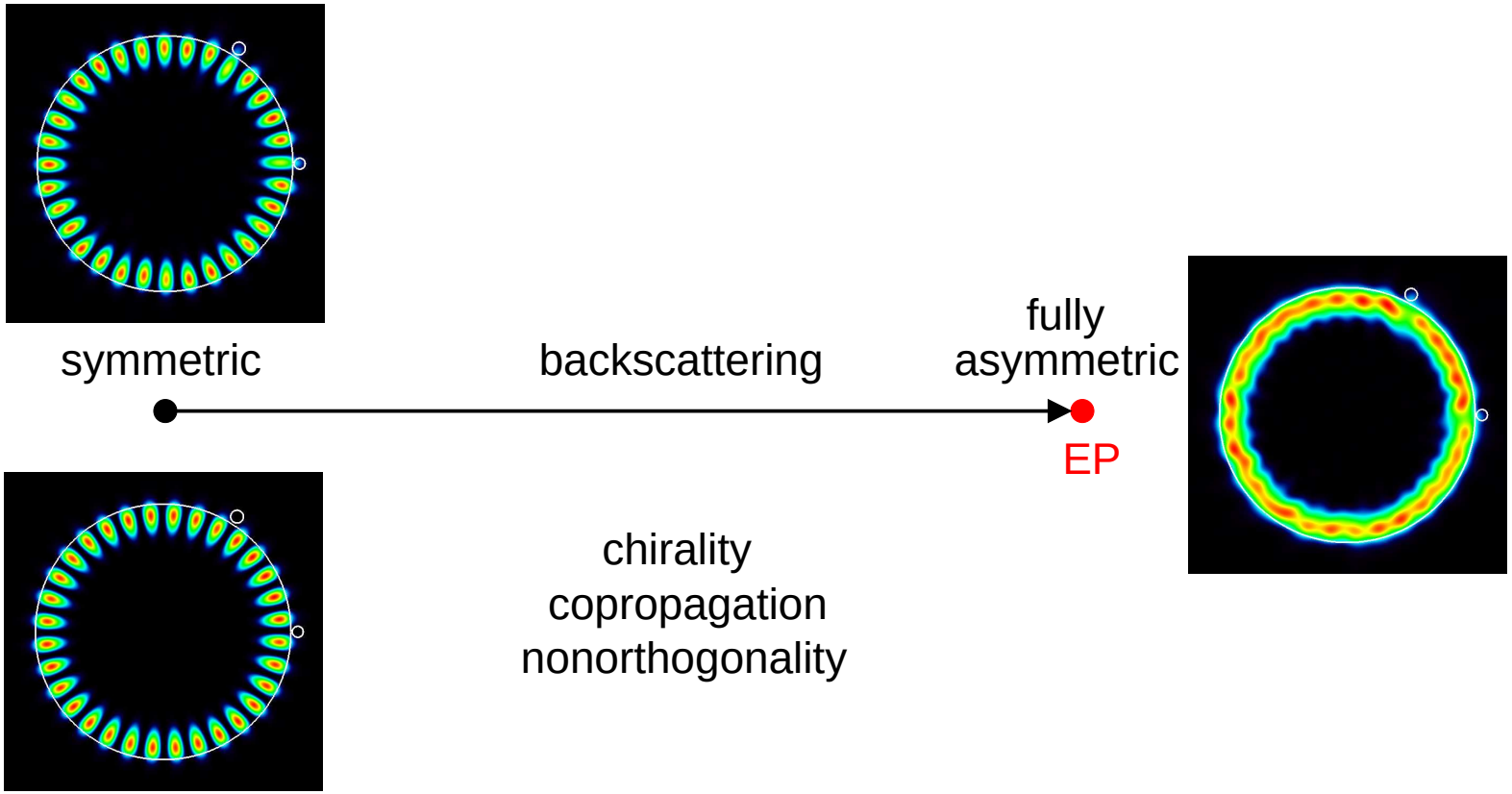


A. Hassan *et al.*, Advanced Photonics 2015, SeT4C.3

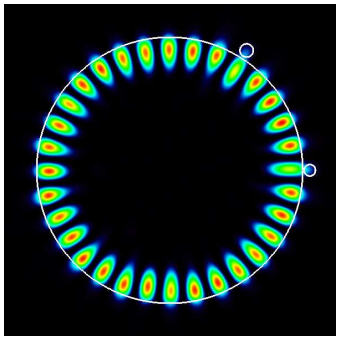


S. Zhang *et al.*, Sci. Rep. **6**, 24487 (2016)

Summary


symmetric
backscattering
fully
asymmetric
EP
chirality
copropagation
nonorthogonality

Summary

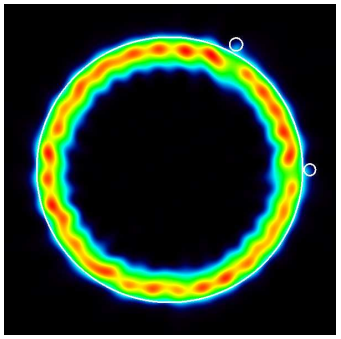


symmetric

backscattering

L. Yang's group

fully asymmetric



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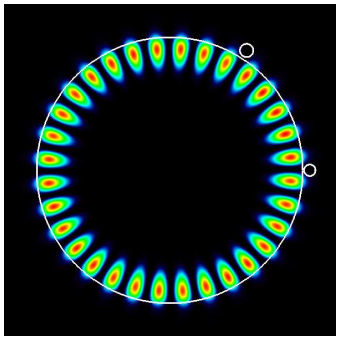

chirality
copropagation
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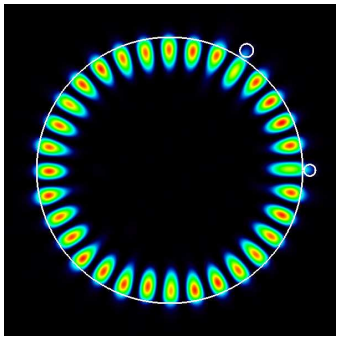
Summary



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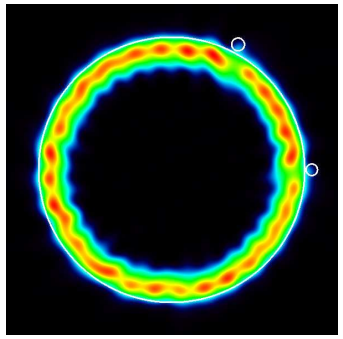

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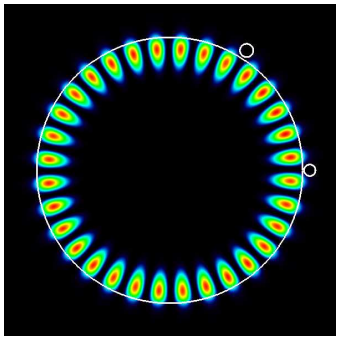

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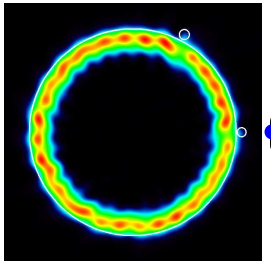
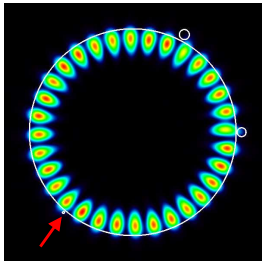
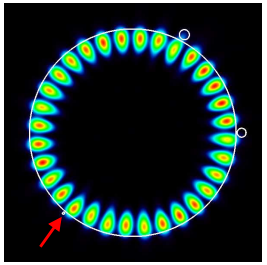


EP





enhanced sensitivity of sensors based on frequency splitting detection