Dynamics and relaxation in integrable quantum spin chains

Quantum magnets workshop, Kolymbari, 15 September 2015

Jean-Sébastien Caux
Universiteit van Amsterdam

Work done in collaboration with (among others):

A'dam gang: S. Eliens, J. De Nardis, B. Wouters, R. van den Berg, R. Vlijm,
M. Brockmann, D. Fioretto, O. El Araby, F.H.L. Essler, R. Konik, T. Prosen, …

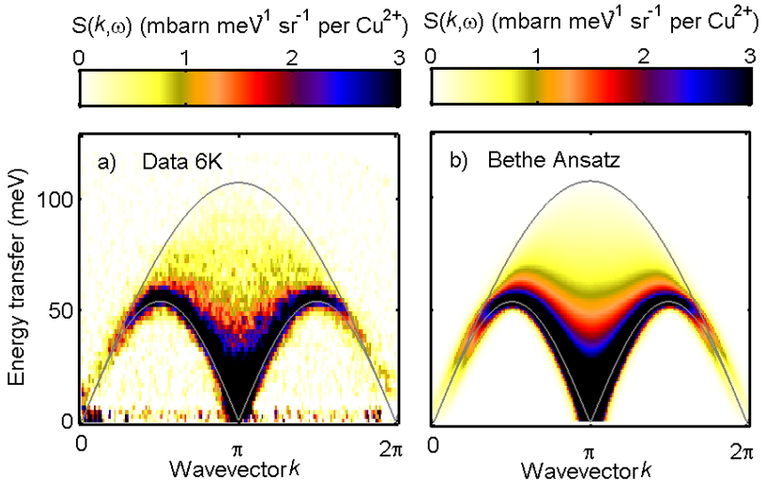
Plan of the talk

- Equilibrium dynamics
  - *Spinons in Heisenberg*
  - *Spin-1 integrable chain*
- Out-of-equilibrium dynamics
  - *Quasisoliton dynamics in XXZ*
  - *Interaction quench in Lieb-Liniger*
  - *Anisotropy quench in XXZ*

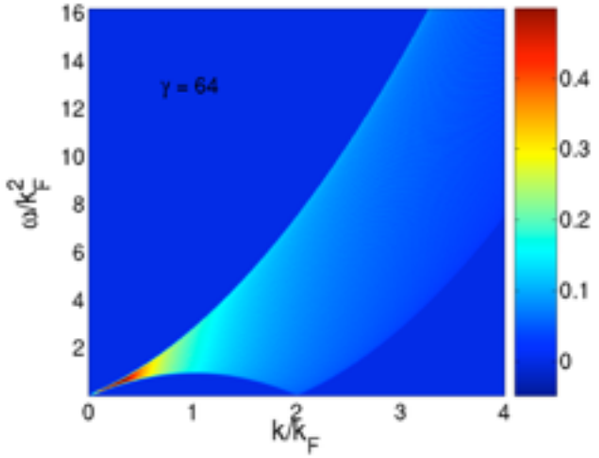
The Quench Action

- Summary & perspectives

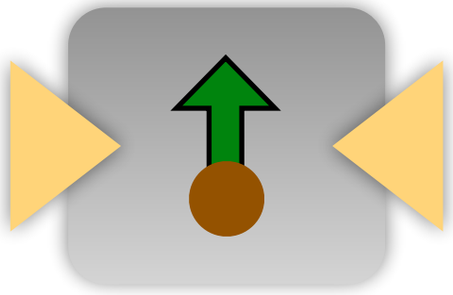
Applications of integrability in many-body physics



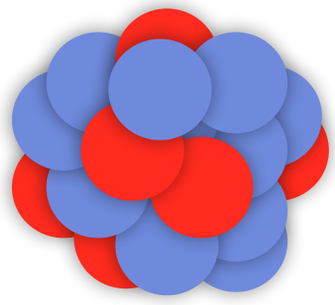
Quantum magnetism



Ultracold atoms

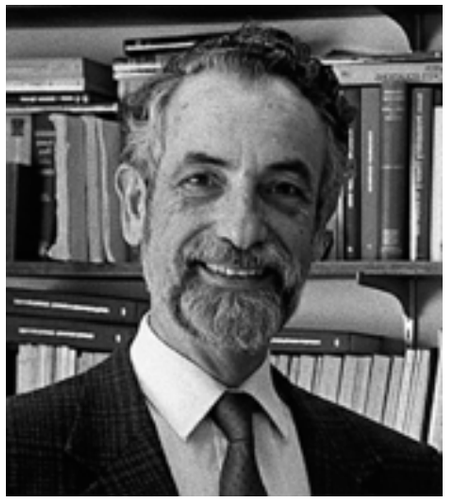


Quantum dots, NV centers



Atomic nuclei

Models discussed in this talk:



Interacting Bose gas (Lieb-Liniger)

$$\mathcal{H}_N = -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + 2c \sum_{1\le j<l\le N} \delta(x_j - x_l)$$

Heisenberg spin-1/2 chain

$$H = \sum_{j=1}^{N} \left[J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z) - H_z S_j^z \right]$$

July 2, 1906 – March 6, 2005

Bethe Ansatz (1931)

Integrable Hamiltonian:

$$H = \int_0^L dx\ \mathcal{H}(x)$$

'Reference state': vacuum, FM state,...

'Particles': atoms, down spins, ...

Exact many-body wavefunctions (in N-particle sector):

$$\Psi_N(\{x\}\{\lambda\}) = \sum_P (-1)^{[P]} A_P(\{\lambda\}) e^{ix_j k(\lambda_{P_j})}$$

... parametrized by rapidities, made up of free waves ...
... with specified relative amplitudes ...
... and obeying some form of Pauli principle

The Bethe Wavefunction

Michel Gaudin's book *La fonction d'onde de Bethe* is a uniquely influential masterpiece on exactly solvable models of quantum mechanics and statistical physics. Available in English for the first time, this translation brings his classic work to a new generation of graduate students and researchers in physics. It presents a mixture of mathematics interspersed with powerful physical intuition, retaining the author's unmistakably honest tone.

The book begins with the Heisenberg spin chain, starting from the coordinate Bethe Ansatz and culminating in a discussion of its thermodynamic properties. Delta-interacting bosons (the Lieb-Liniger model) are then explored, and extended to exactly solvable models associated with a reflection group. After discussing the continuum limit of spin chains, the book covers six- and eight-vertex models in extensive detail, from their lattice definition to their thermodynamics. Later chapters examine advanced topics such as multicomponent delta-interacting systems, Gaudin magnets and the Toda chain.

MICHEL GAUDIN is recognized as one of the foremost experts in this field, and has worked at Commissariat à l'énergie atomique (CEA) and the Service de Physique Théorique, Saclay. His numerous scientific contributions to the theory of exactly solvable models are well known, including his famous formula for the norm of Bethe wavefunctions.

JEAN-SÉBASTIEN CAUX is a Professor in the theory of low-dimensional quantum condensed matter at the University of Amsterdam. He has made significant contributions to the calculation of experimentally observable dynamical properties of these systems.

Cover illustration: a representation of the Yang-Baxter relation by John Collingwood.

CAMBRIDGE
UNIVERSITY PRESS
www.cambridge.org

ISBN 978-1-107-04585-9

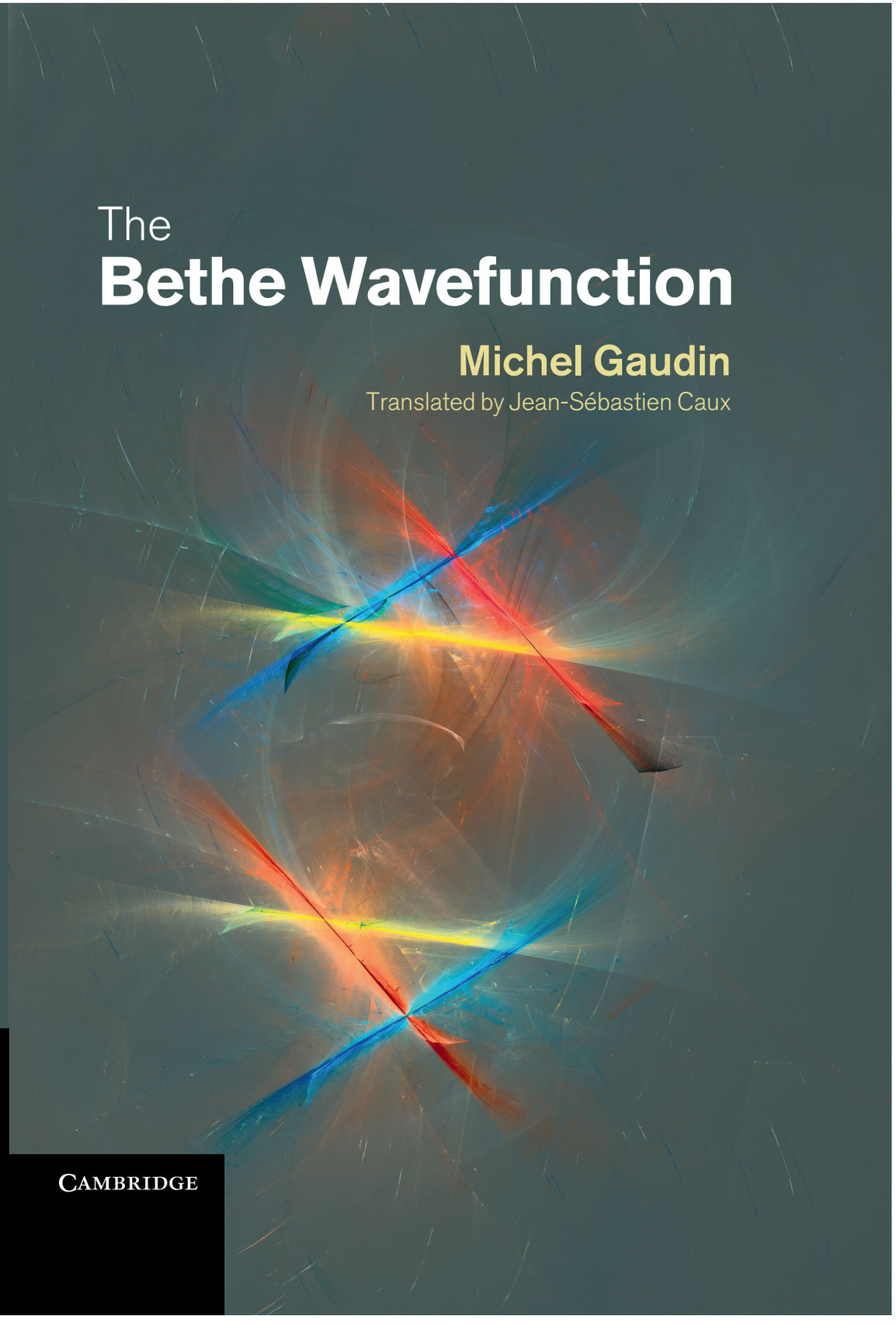
Cover designed by Hart McLeod Ltd

Gaudin and Caux The Bethe Wavefunction

CAMBRIDGE

The
Bethe Wavefunction

Michel Gaudin
Translated by Jean-Sébastien Caux

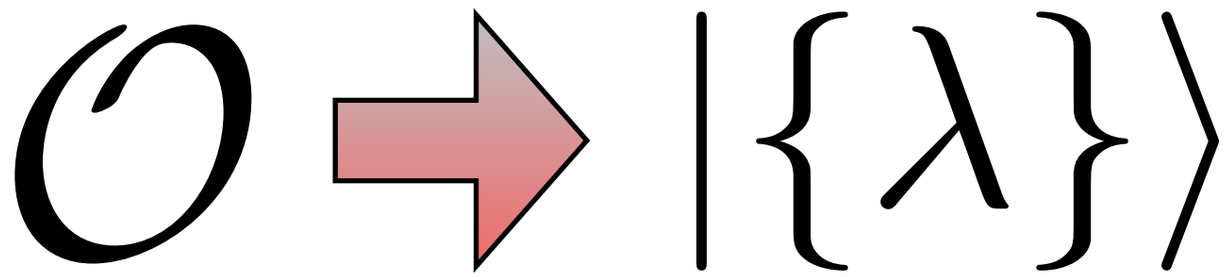


CAMBRIDGE

The general idea, simply stated:

Start with your favourite quantum state

(expressed in terms of Bethe states)



Apply some operator on it

Reexpress the result in the basis of Bethe states:

$$\mathcal{O}|\{\lambda\}\rangle = \sum_{\{\mu\}} F^{\mathcal{O}}_{\{\mu\},\{\lambda\}} |\{\mu\}\rangle$$

using 'matrix elements' $F^{\mathcal{O}}_{\{\mu\},\{\lambda\}} = \langle\{\mu\}|\mathcal{O}|\{\lambda\}\rangle$

Equilibrium dynamics from integrability

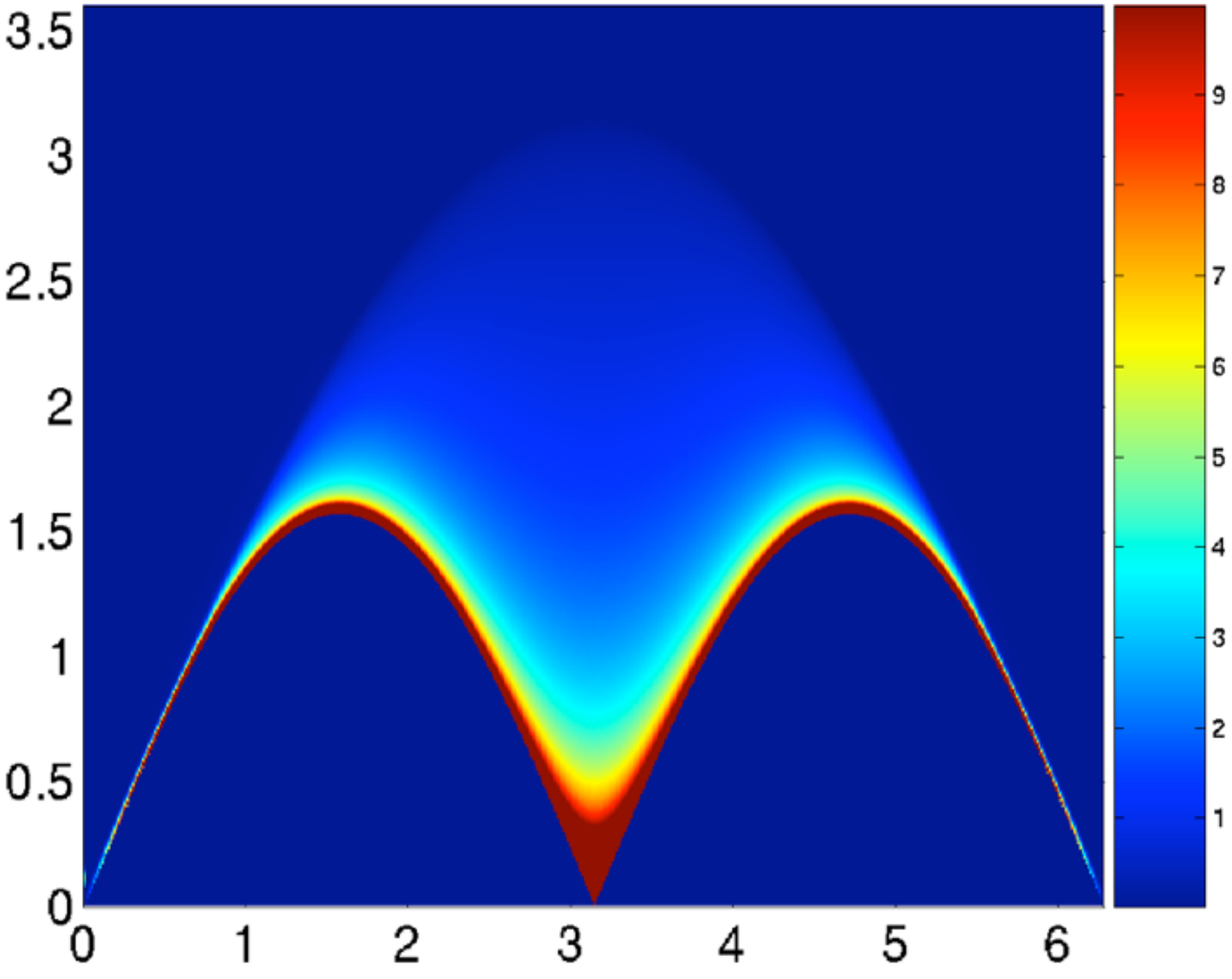
Equilibrium dynamics using integrability

Cases which we've handled:

- Heisenberg model
- XXZ gapless antiferromagnets
- XXZ gapped antiferromagnets
- Spin-1 chain (Babujan-Takhtajan)
- Lieb-Liniger (repulsive interactions)
- Lieb-Liniger (attractive interactions)
- Richardson model
- Central spin model
- Perturbed integrable models: NRG from BA

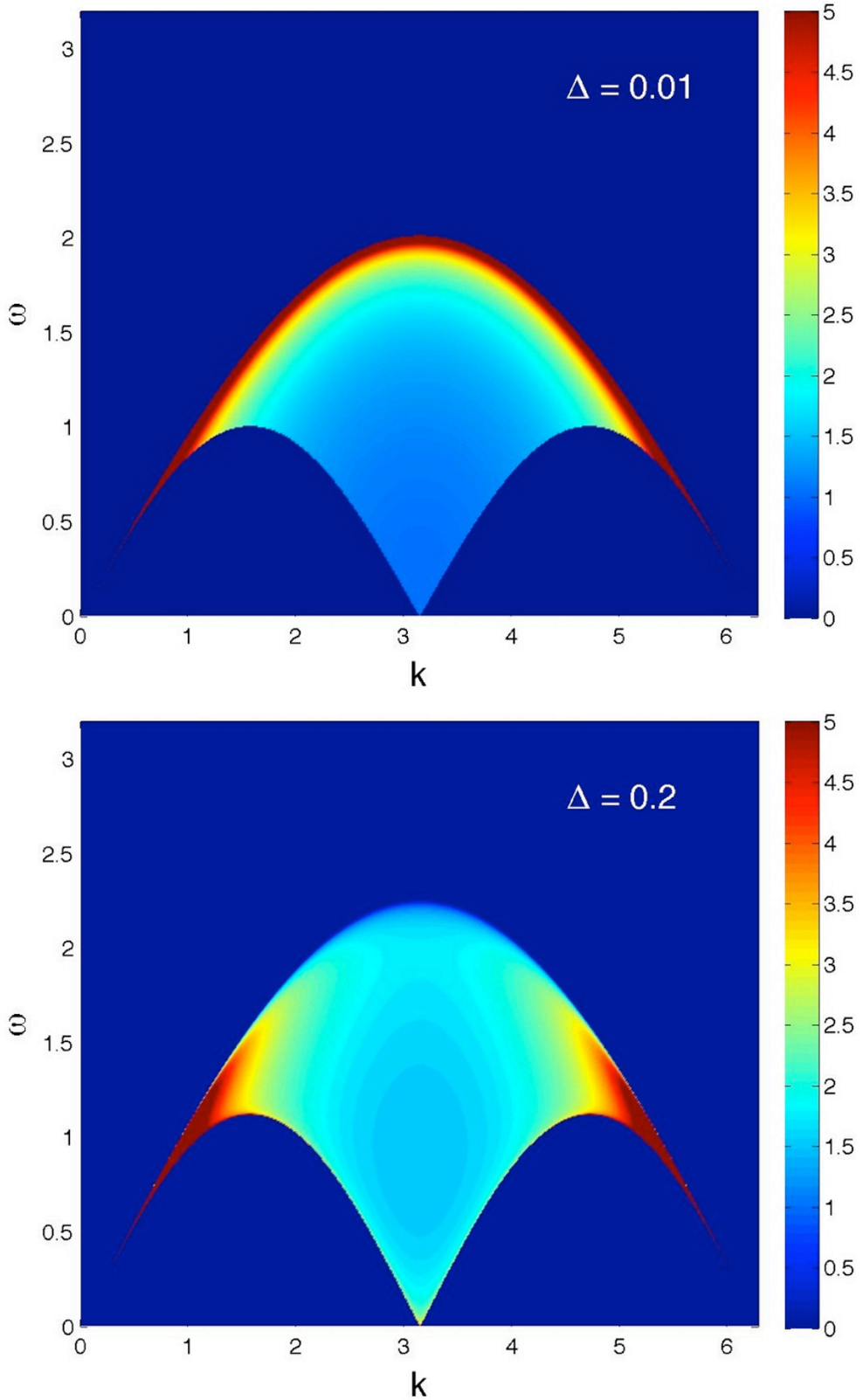
Heisenberg spin chain

$$S(k,\omega), \quad \Delta = 1, \quad h = 0$$



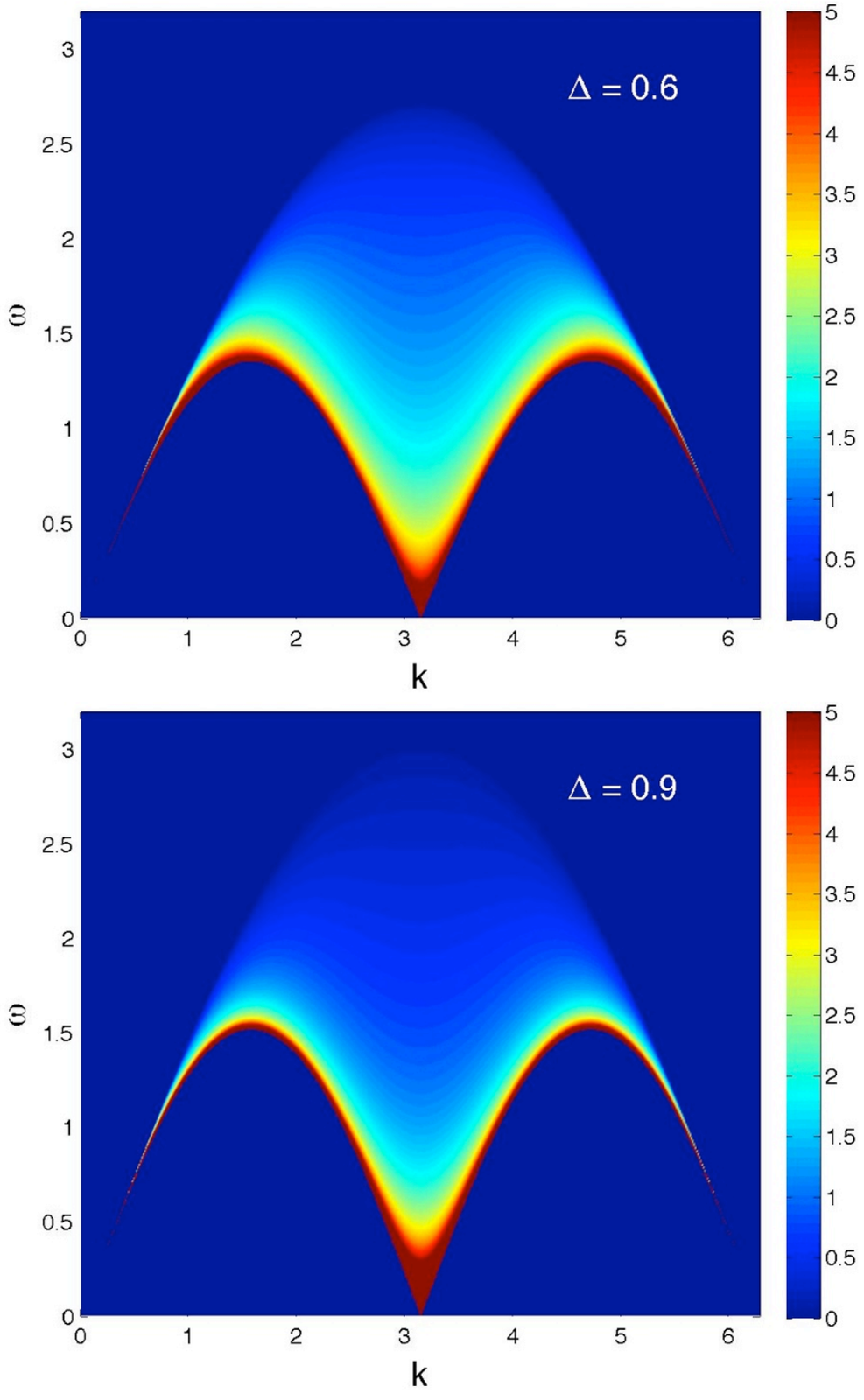
XXZ spin chain at zero field

J.-S. C., H. Konno, M. Sorrell and R. Weston, PRL 2011; ibid, JSTAT 2012



XXZ spin chain at zero field

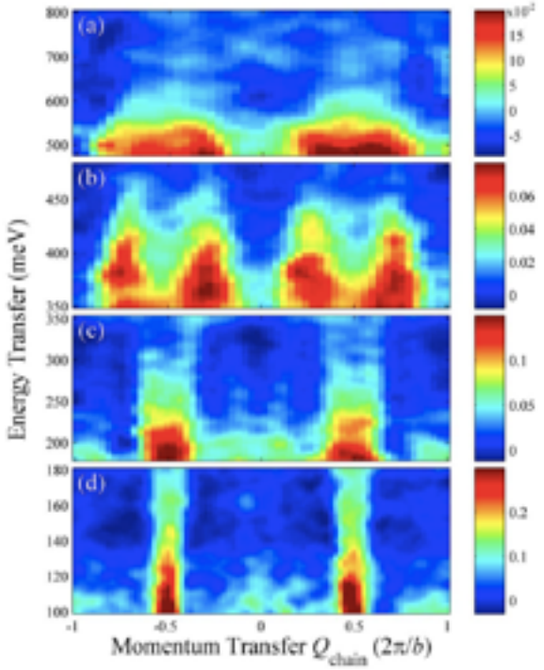
J.-S. C., H. Konno, M. Sorrell and R. Weston, PRL 2011; ibid, JSTAT 2012



Quantum spin chains

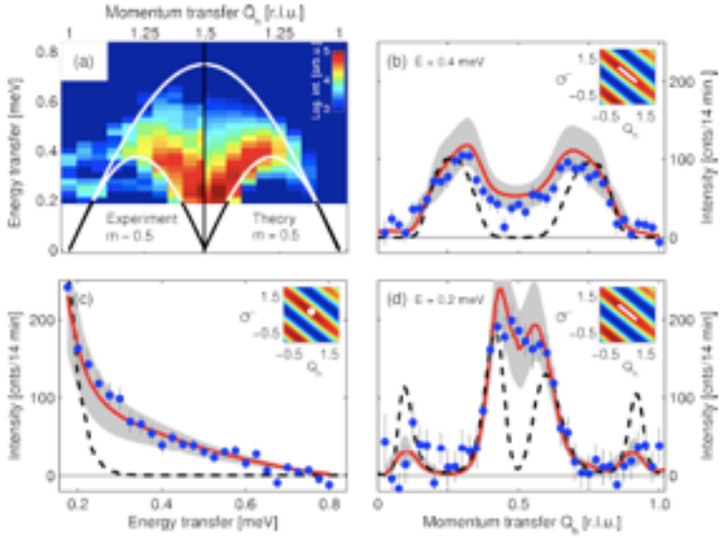
Correlations, experiments (INS, RIXS), prefactors, ...

Sr_2CuO_3



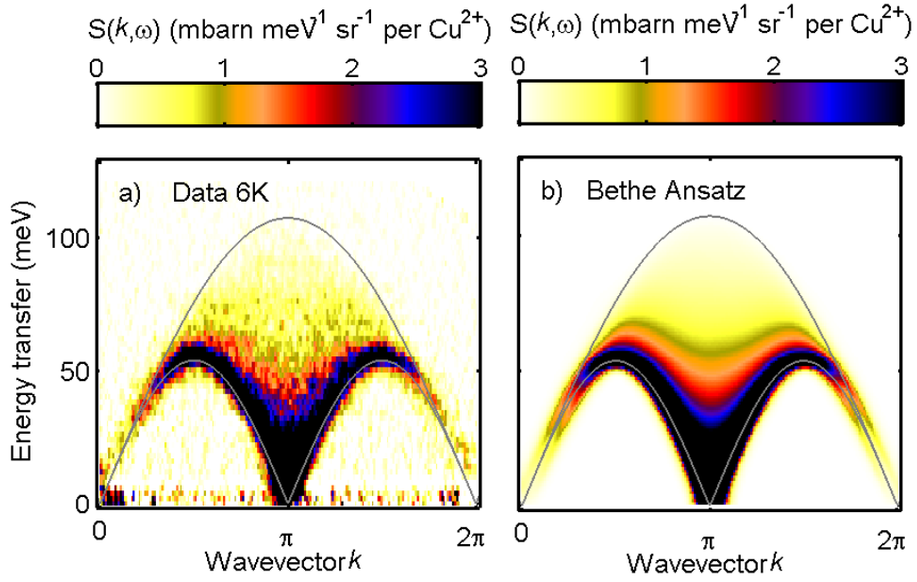
Walters, Perring, Caux, Savici, Gu, Lee, Ku, Zaliznyak, NATURE PHYSICS 2009

$(C_5D_{12}N)_2CuBr_4$

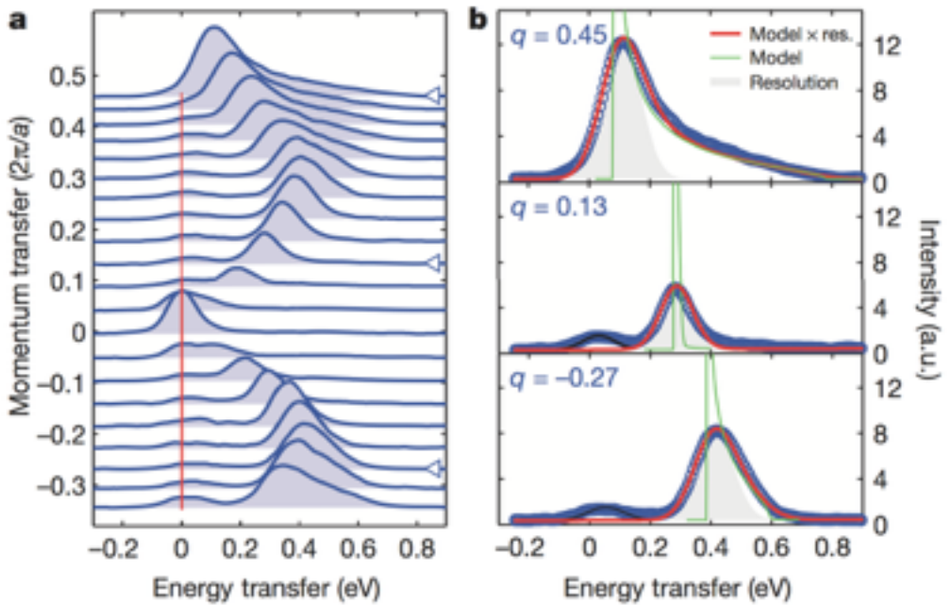


Thielemann, Rüegg, Rønnow, Läuchli, Caux, Normand, Biner, Krämer, Güdel, Stahn, Habicht, Kiefer, Boehm, McMorrow, Mesot, PRL 2009

$KCuF_3$



Lake, Tennant, Caux, Barthel, Schollwöck, Nagler, Frost, PRL 2013



Sr_2CuO_3

(RIXS)

Schlappa, Wohlfeld, Zho, Mourigal, Haverkort, Strocov, Hozoi, Monney, Nishimoto, Singh, Revcolevschi, Caux, Patthey Rønnow, van den Brink, Schmitt, NATURE 2012

Counting fractional spinon excitations in the quantum Heisenberg antiferromagnetic chain

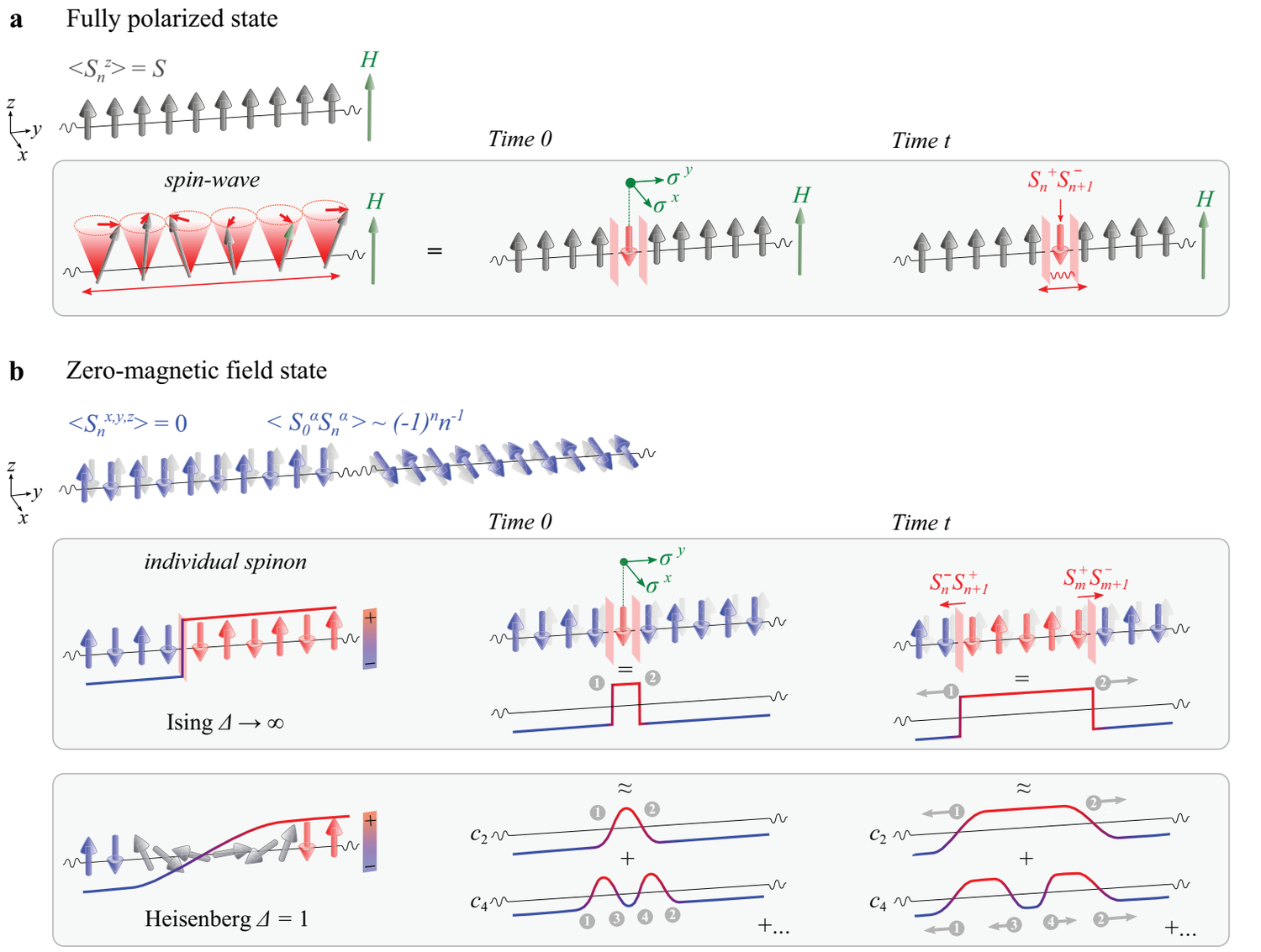
Martin Mourigal,[1,2,3,*] Mechthild Enderle,[1] Axel Klöpperpieper,[4]
Jean-Sébastien Caux,[5] Anne Stunault,[1] and Henrik M. Rønnow[2]

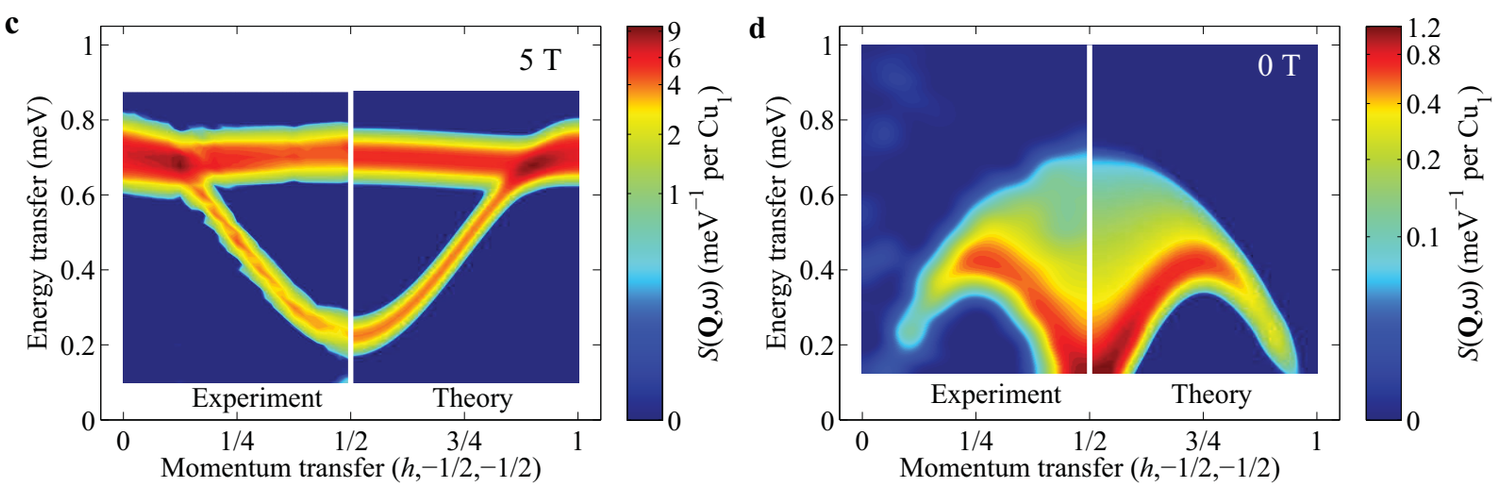
(Nature Physics 2013)

$CuSO_4 \cdot 5D_2O$

Roger Hiorns, *Seizure*

Multi-spinon processes: intuitive picture

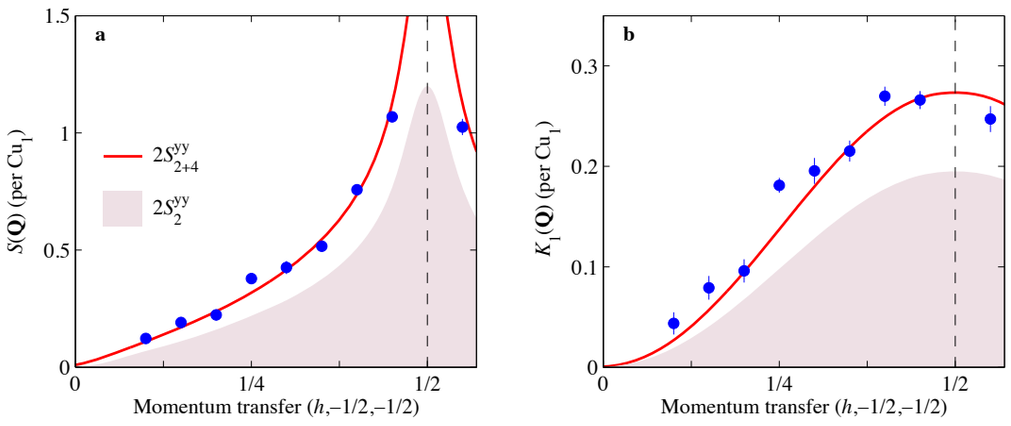


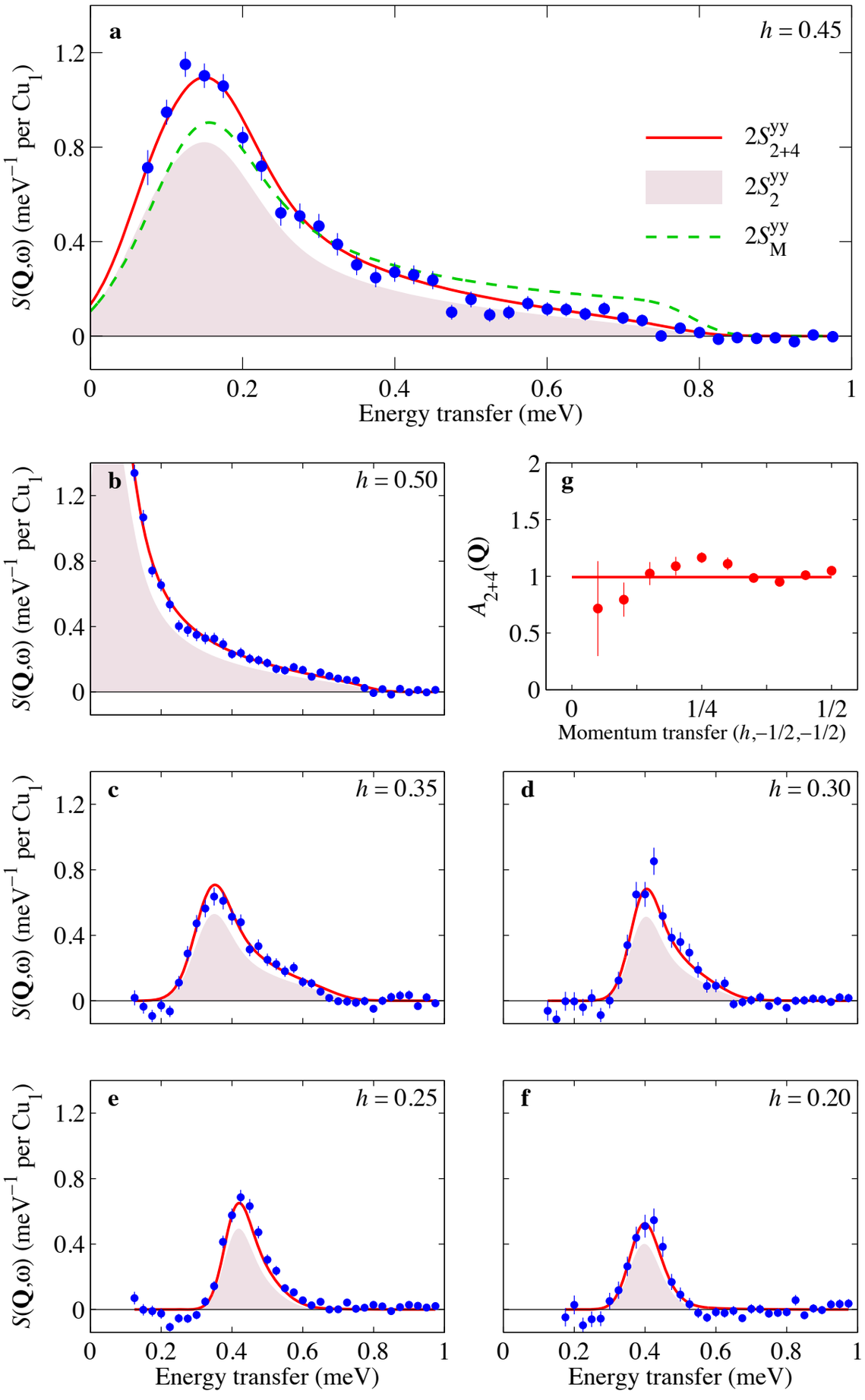


Experimental data is accurate enough to discriminate between:

- Müller ansatz (off)
- BA: 2 spinons (off)
- BA: 2 + 4 spinons (on!)

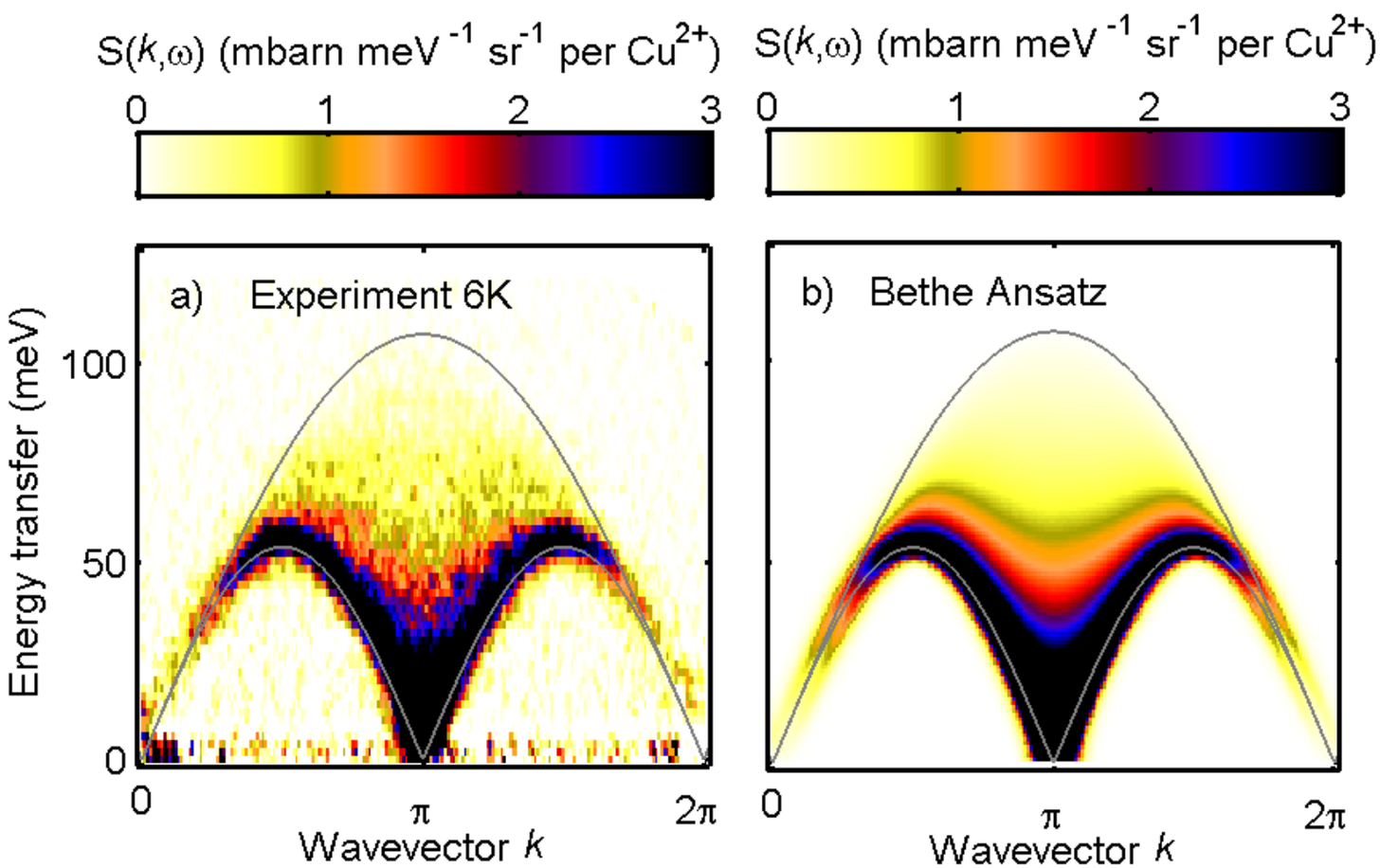
Frequency-integrated:

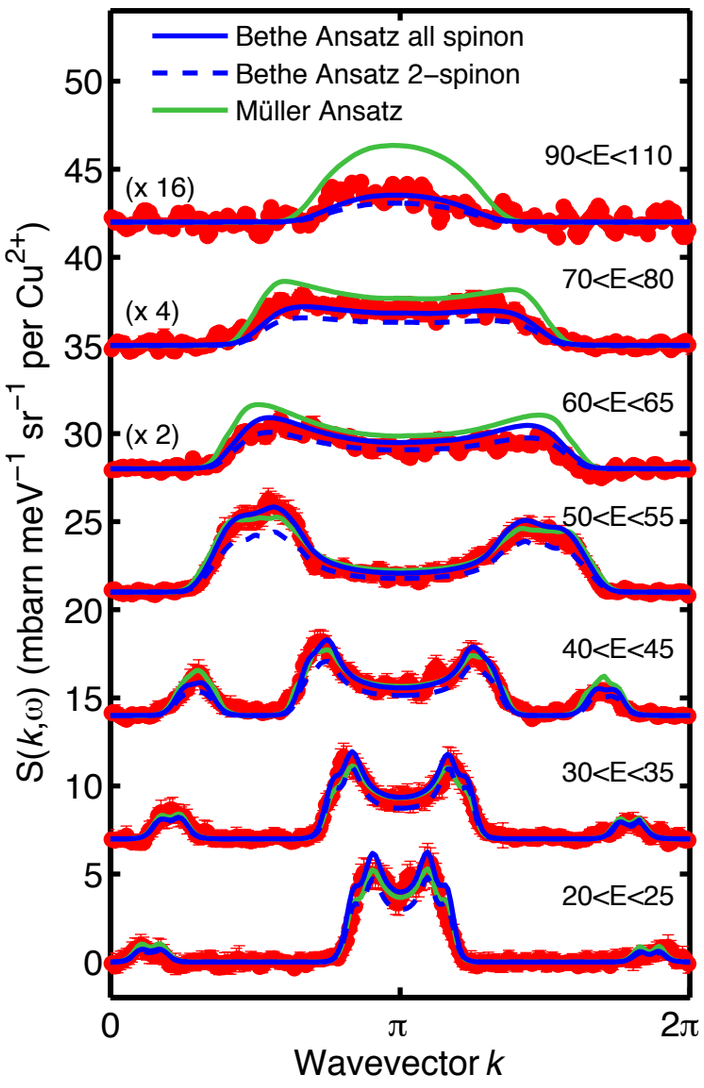


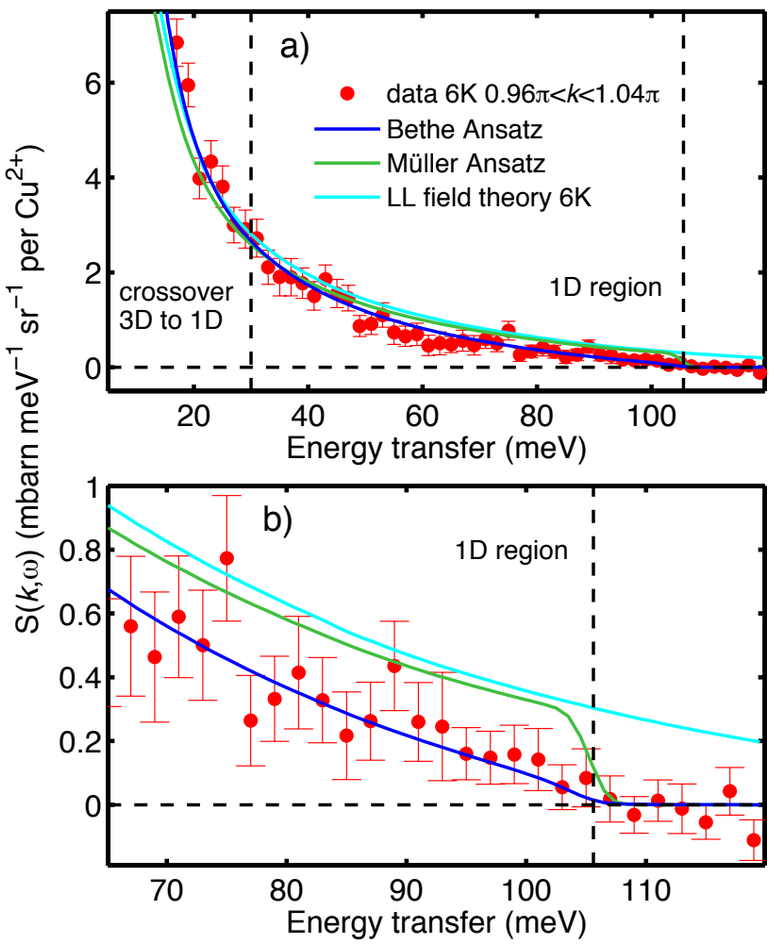


$KCuF_3$

Lake, Tennant, Caux, Barthel, Schollwöck, Nagler, Frost PRL 2013







Experiment again accurate enough to discriminate between:

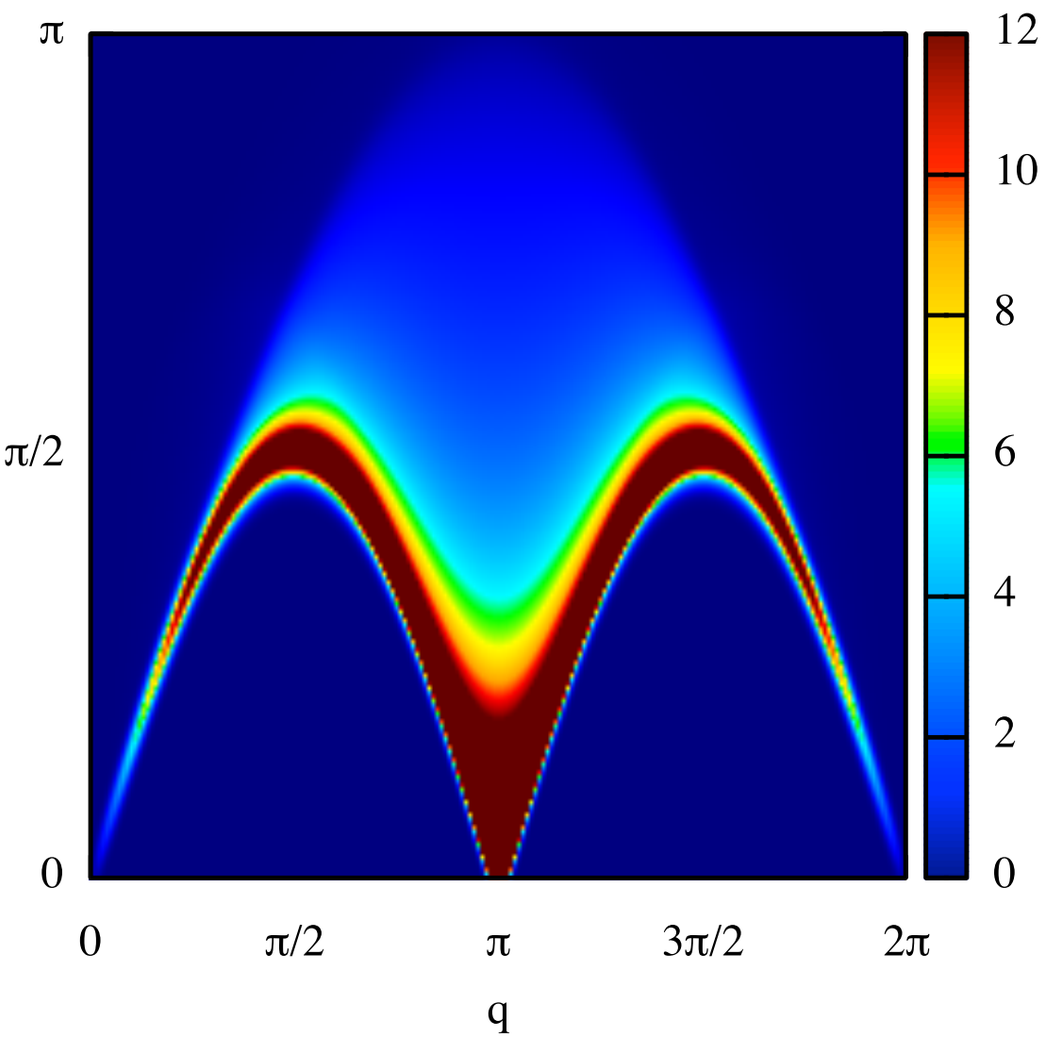
- Müller Ansatz
- Field theory
- Bethe Ansatz (best)

Babujan-Takhtajan spin-1 chain

$$H = J \sum_{j=1}^{N} \left[\mathbf{S}_j \cdot \mathbf{S}_{j+1} - (\mathbf{S}_j \cdot \mathbf{S}_{j+1})^2 - H_z S_j^z \right]$$

R. P. Vlijm and J.-S. C., JSTAT 2014

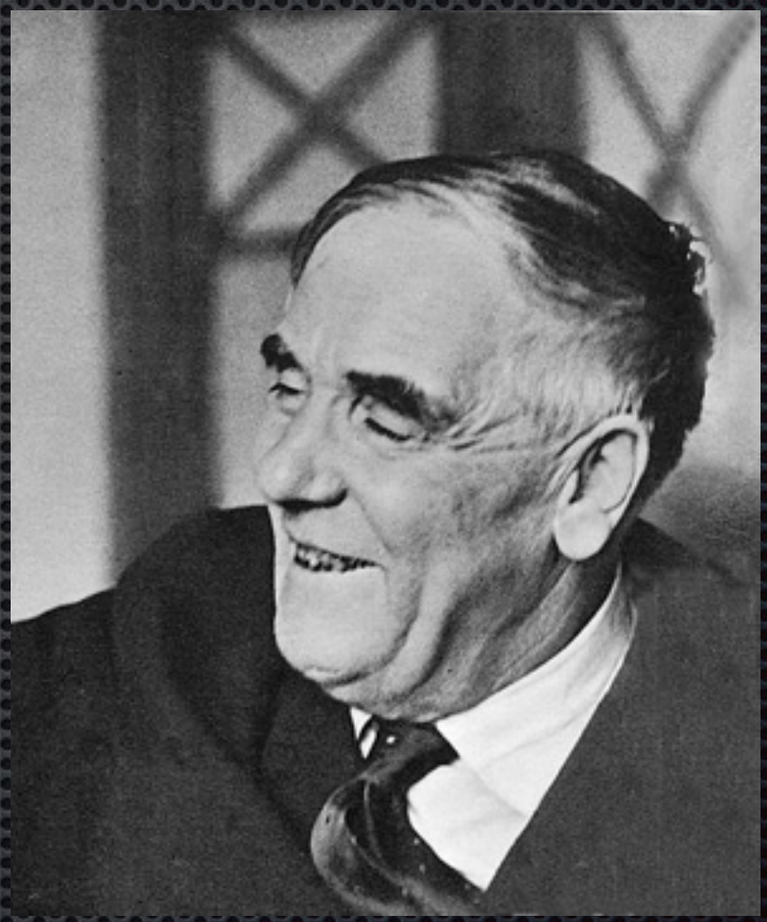
- Ground state: correlated gas of 2-strings
- String deformations: must be taken into account
- Correlations: skewed to higher energies (as compared to S=1/2)



Out-of-equilibrium dynamics from integrability

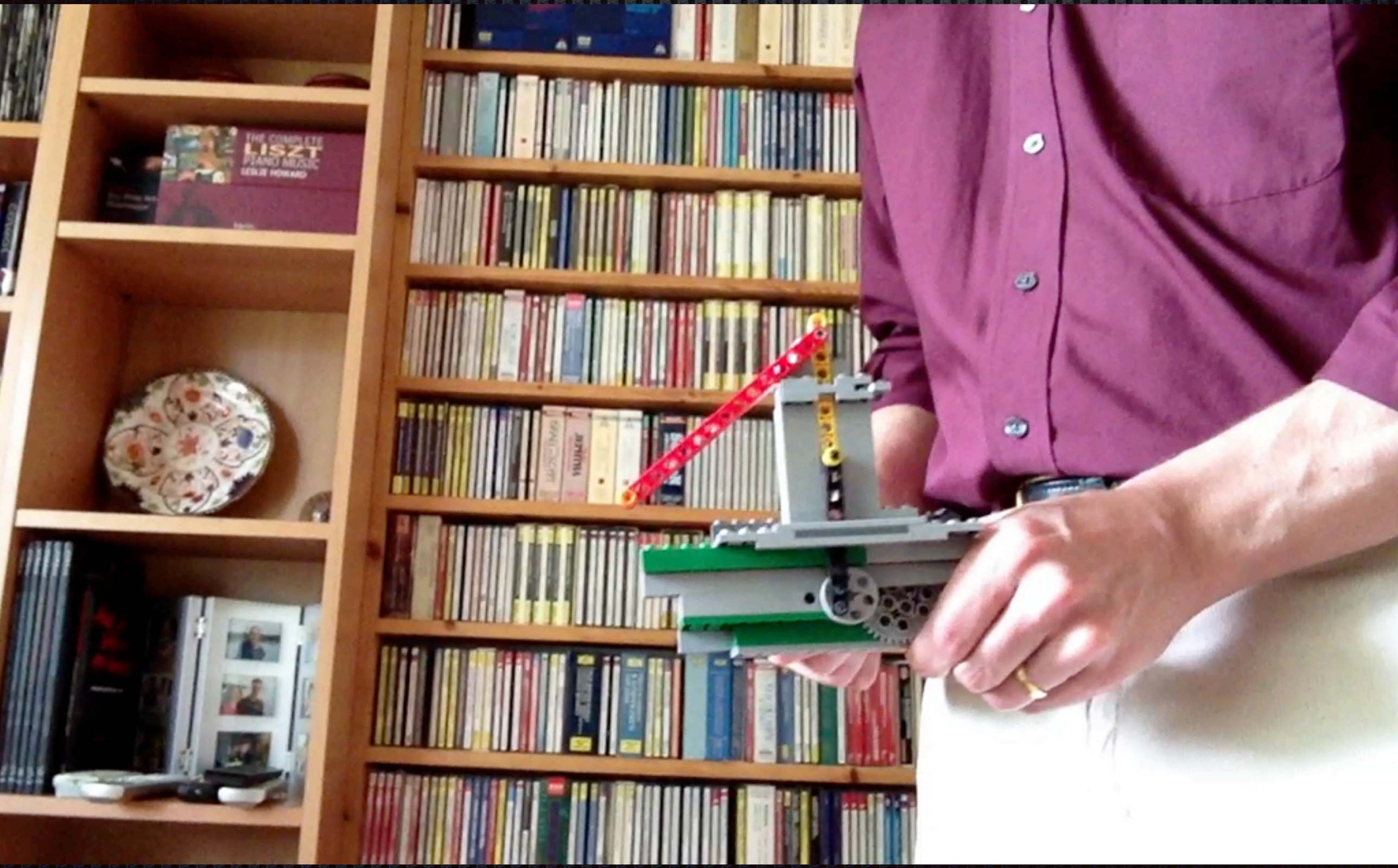
The simple pendulum on its head

Kapitsa pendulum, 1951



Pyotr L. Kapitsa
(8/7/1894-8/4/1984)

The Kapitza pendulum



Out-of-equilibrium using integrability

Highly excited initial (eigen)states:

- The super Tonks-Girardeau gas
- Split Fermi sea in Lieb-Liniger
- Quasisolitons

Quenched states:

- Interaction quench in Richardson
- Domain wall release in Heisenberg
- Geometric quench
- Interaction turnoff in Lieb-Liniger
- Release of trapped Lieb-Liniger
- BEC to Lieb-Liniger quench
- Néel to XXZ quench

Driven systems:

- Spin echo in quantum dots

Quasisoliton dynamics in spin chains

Solitons (classical)

John Scott Russell:
wave of translation (1834)

(Herriot-Watt University)

Solitons (classical)

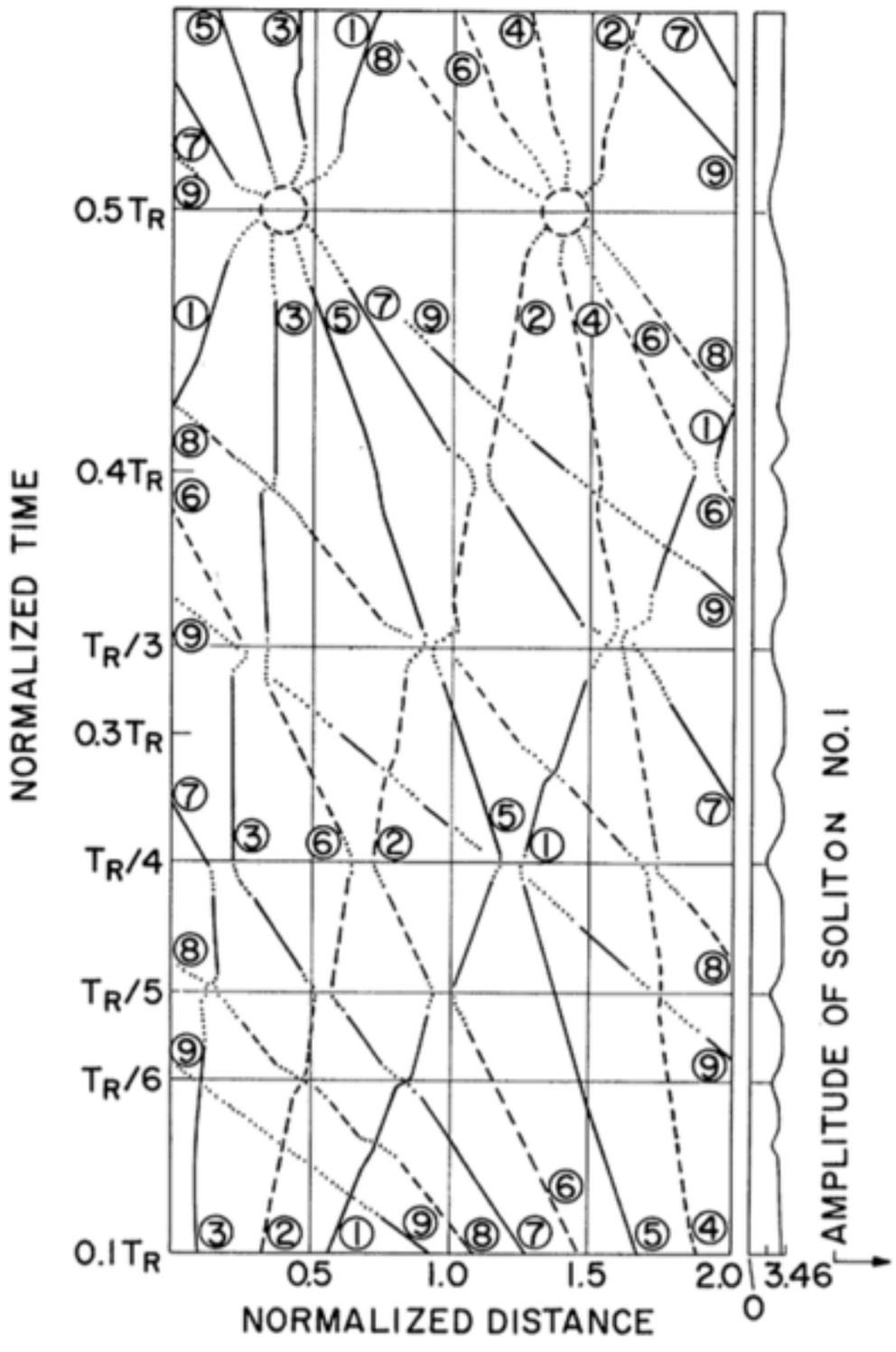
(Boussinesq)
Korteweg-de Vries equation

$$\partial_t u + u\partial_x u + \delta^2 \partial_x^3 u = 0$$

First simulations:
Fermi-Pasta-Ulam-(Tsingou)
absence of ergodicity

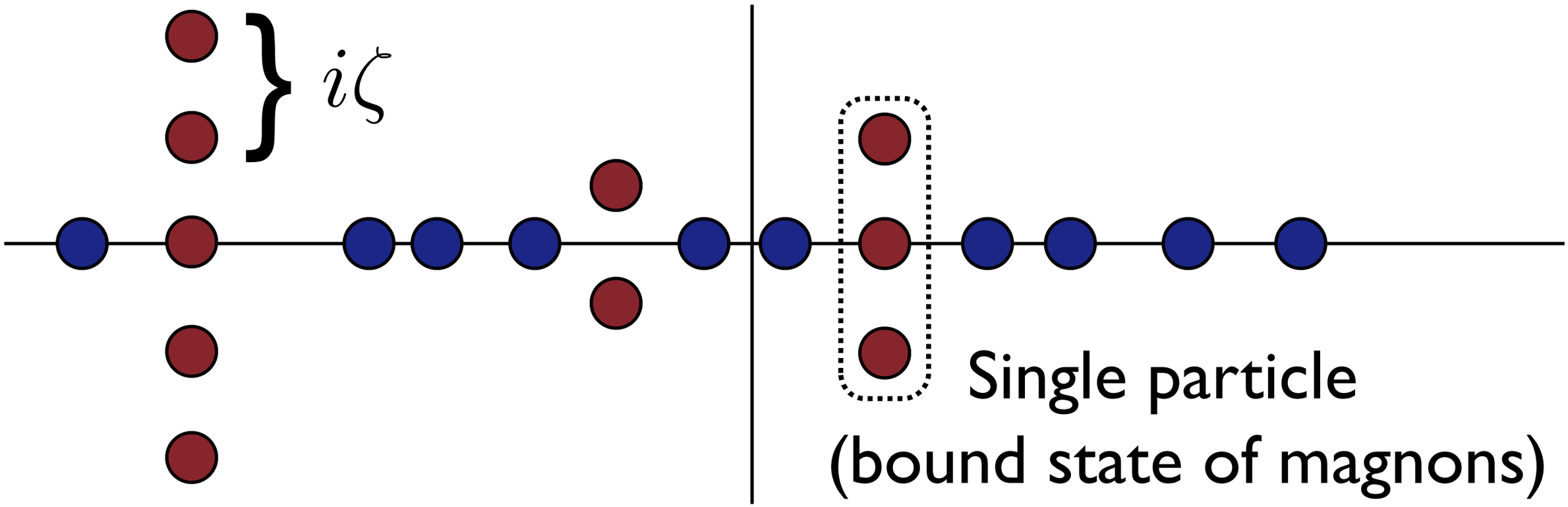
Further simulations:
Zabusky & Kruskal 1965
concept of a soliton

<u>Classical inverse scattering</u>



"Particle content" of XXZ: nontrivial

Solution of Bethe equations: rapidities + strings



$$\lambda_\alpha^{j,a} = \lambda_\alpha^j + i\frac{\zeta}{2}(n_j + 1 - 2a) + i\delta_\alpha^{j,a} \qquad \mathrm{O}(e^{-(cst)N})$$

Classification of strings: Bethe, Takahashi, Suzuki, ...

String wavepackets

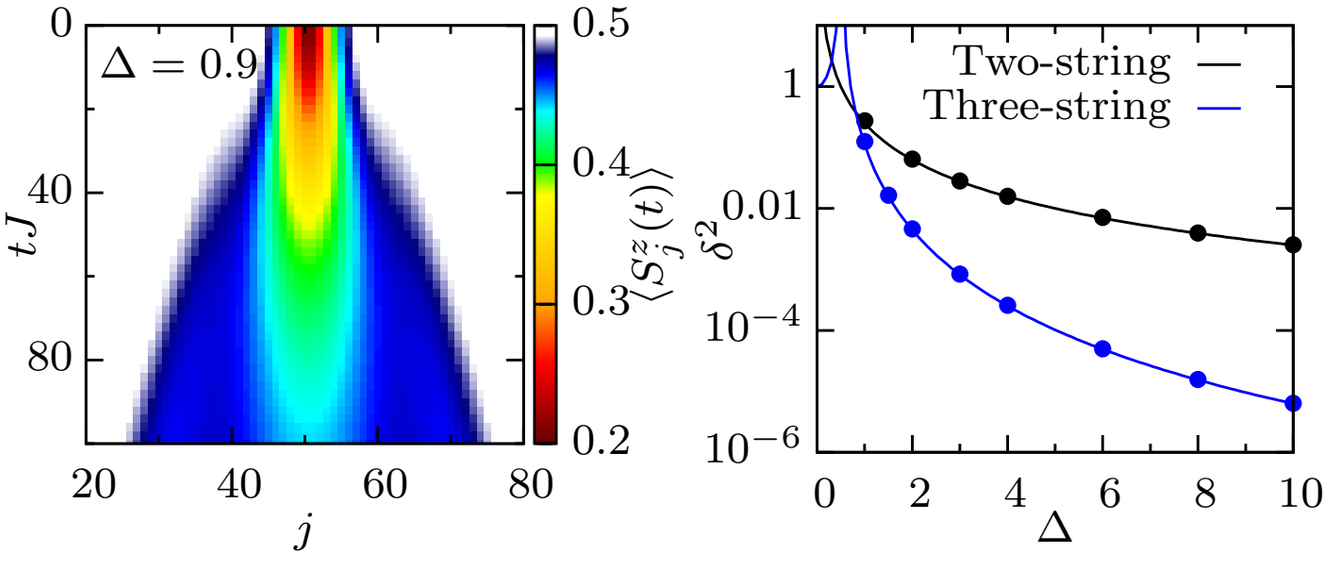
Vlijm, Ganahl, Fioretto, Brockmann, Haque, Evertz and Caux, 2015

In the eigenbasis, time evolution of a generic state: simple!

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle = \sum_{\{\lambda\}} e^{-iE_{\{\lambda\}}t} C_{\{\lambda\}} |\{\lambda\}\rangle$$

Localized wavepacket: $$|\Psi(0)\rangle = \mathcal{N}_0 \sum_p e^{-ip\overline{x} - \frac{\alpha^2}{4}(p-\overline{p})^2} |\lambda^{(n)}(p)\rangle$$

Dispersion:



width ~ t:

$$\Delta x(t) = \sqrt{\frac{\alpha^2}{4} + \frac{\delta_n^2 t^2}{\alpha^2}}$$

function of anisotropy:

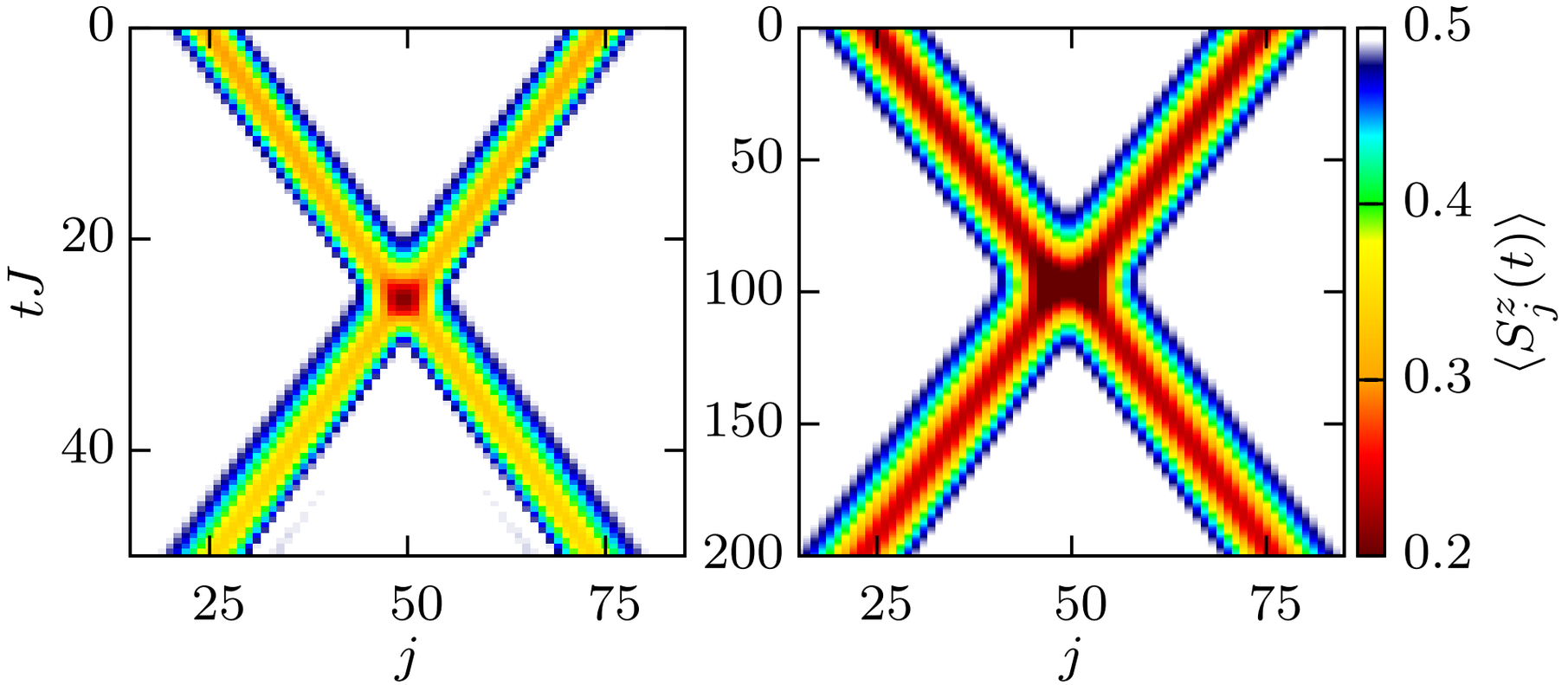
$$\delta_n^2 = J^2 \left(\frac{\phi_2(0)}{\phi_{2n}(0)}\right)^2 \cos^2(\overline{p})$$

Quasisoliton scattering (quantum)

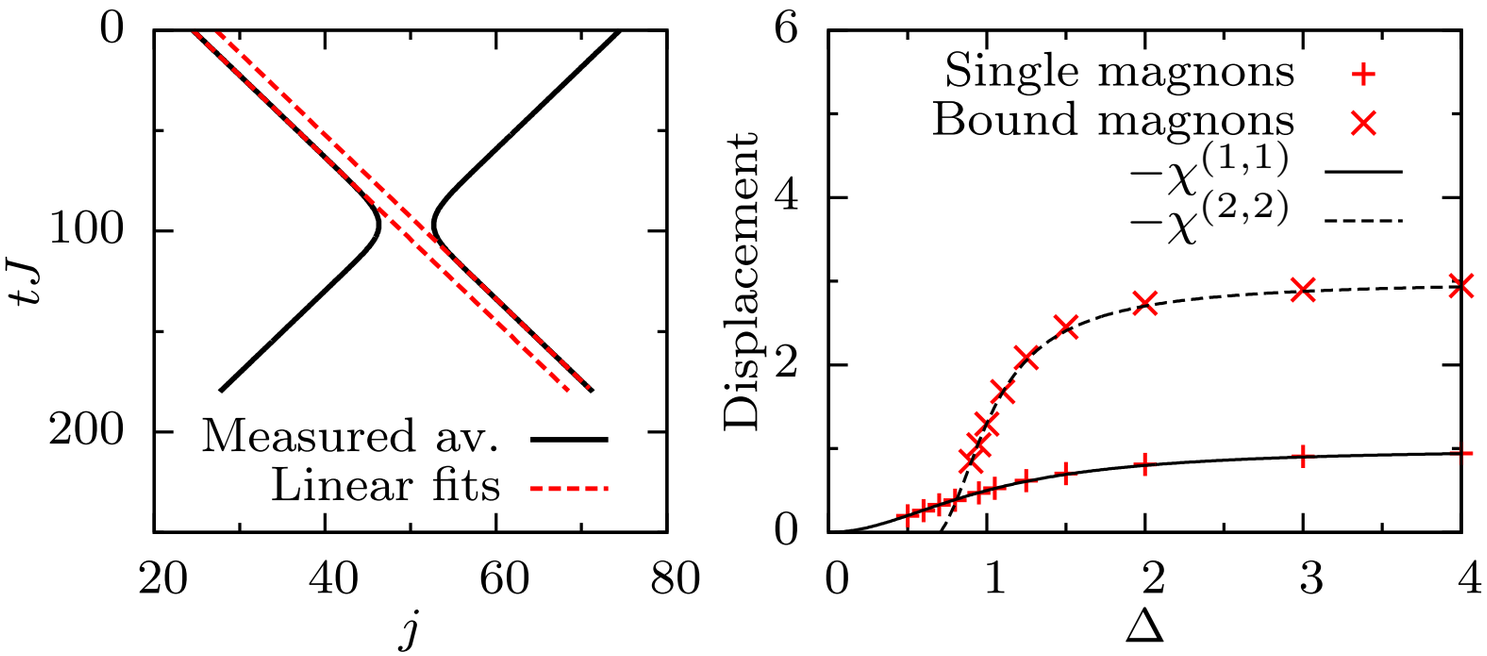
Vlijm, Ganahl, Fioretto, Brockmann, Haque, Evertz and Caux, 2015

'Worldlines' of colliding wavepackets:

$\Delta = 2$



Displacement as a function of anisotropy (fixed incoming momenta)



Quantum quenches

Quenches
(more generally: 'prepare and release')

The problem: considering a generic initial state,
what is the time evolution of the system?

Hamiltonian driving time evolution

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(t=0)\rangle$$

initial state is NOT eigenstate of H

Observable expectation values depend on time:

$$\bar{\mathcal{O}}(t) \equiv \frac{\langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle}{\langle\Psi(t)|\Psi(t)\rangle}$$

First question: what is the steady state long after the quench?

Fundamental issue: does the system relax? thermalize?

Crucial point: *time evolution in the presence of myriads of constraints (due to integrability) is special*

Conjecture: *steady state is described by a generalized Gibbs ensemble (GGE)*

$$\lim_{t\to\infty} \bar{\mathcal{O}}(t) = \langle \mathcal{O} \rangle_{GGE} = \frac{\mathrm{Tr}\{\mathcal{O} e^{-\sum_n \beta_n Q_n}\}}{\mathrm{Tr}\{e^{-\sum_n \beta_n Q_n}\}}$$

Rigol, Dunjko, Yurovsky, Olshanii, PRL 2007 see also Jaynes, Phys. Rev. 1957

GGE implementation

Generalized inverse temperatures to be set using the initial conditions on conserved charges

$$\langle \hat{Q}_m \rangle = \mathrm{Tr}\left\{ \hat{Q}_m e^{-\sum_n \beta_n \hat{Q}_n} \right\} / \mathcal{Z}_{GGE} \quad m = 0, 1, 2, \ldots$$

where $\mathcal{Z}_{GGE} = \mathrm{Tr} e^{-\sum_n \beta_n \hat{Q}_n}$

In reality, three major difficulties:

- Conserved charges are generically nontrivial
- Finding a complete set of charges is an issue
- Self-consistency problem difficult to solve

In practice: implementable for free theories only (charges: momentum occupation modes)

For interacting cases: not understood in general.

Quantum quenches:

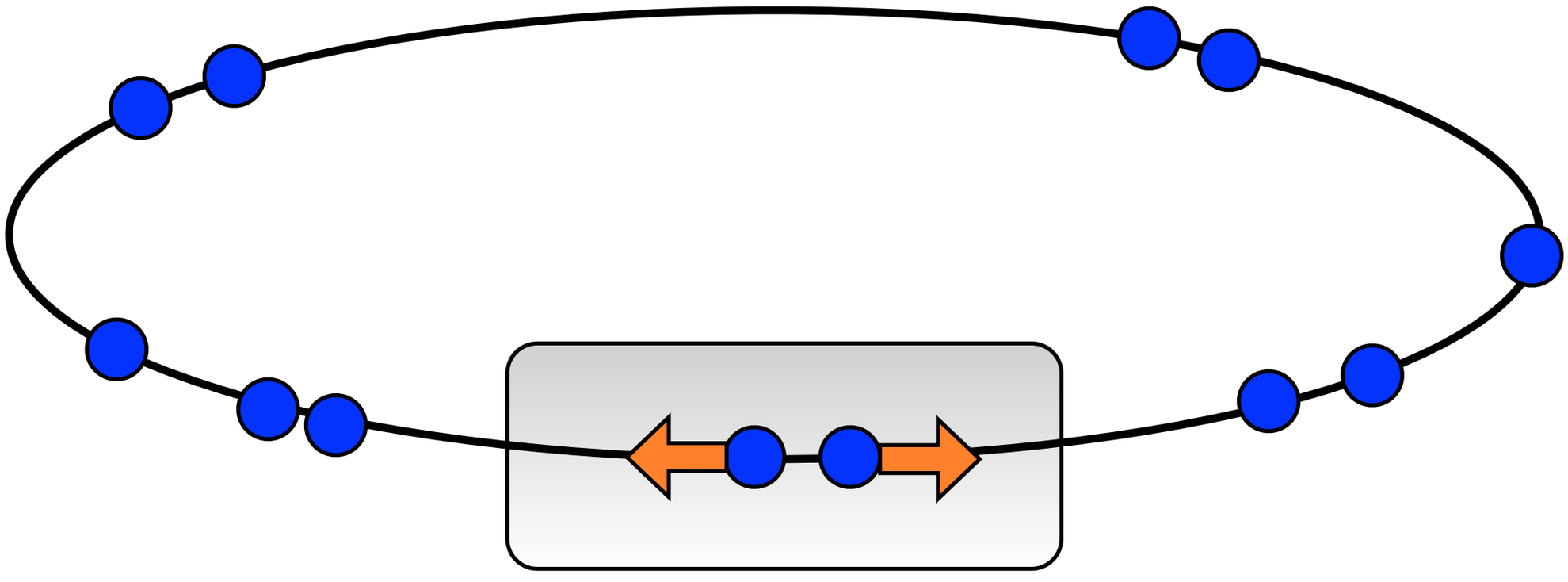
BEC to repulsive Lieb-Liniger quench

Quench from BEC to repulsive gas

Start from GS of noninteracting theory,

$$|0_N\rangle \equiv \frac{1}{\sqrt{L^N N!}} \left(\psi^\dagger_{k=0}\right)^N |0\rangle$$

Turn repulsive interactions on from t=0 onwards:



particles ‘repel away’ from each other,
system heats up, momentum distribution broadens, ...

This is a difficult problem to treat...

1) Generalized Gibbs ensemble logic

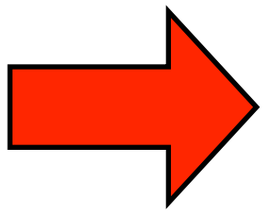
Kormos, Shashi, Chou and Imambekov, arxiv:1204.3889

Conserved charges:

$$\hat{Q}_n : \qquad \hat{Q}_n|\{\lambda\}_N\rangle = Q_n|\{\lambda\}_N\rangle$$

$$Q_n(\{\lambda\}_N) = \sum_{j=1}^{N} \lambda_j^n$$

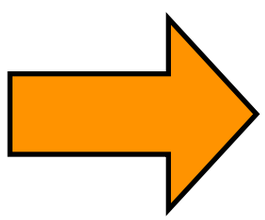
Davies 1990; Davies and Korepin



GGE inapplicable, charges take infinite values!

J-S C + J. Mossel, unpublished

2) GGE on lattice, q-deformed model



Works, partial results only (using a few charges)

Kormos, Shashi, Chou, JSC, Imambekov, PRA 2014

The ‘quench action’ approach

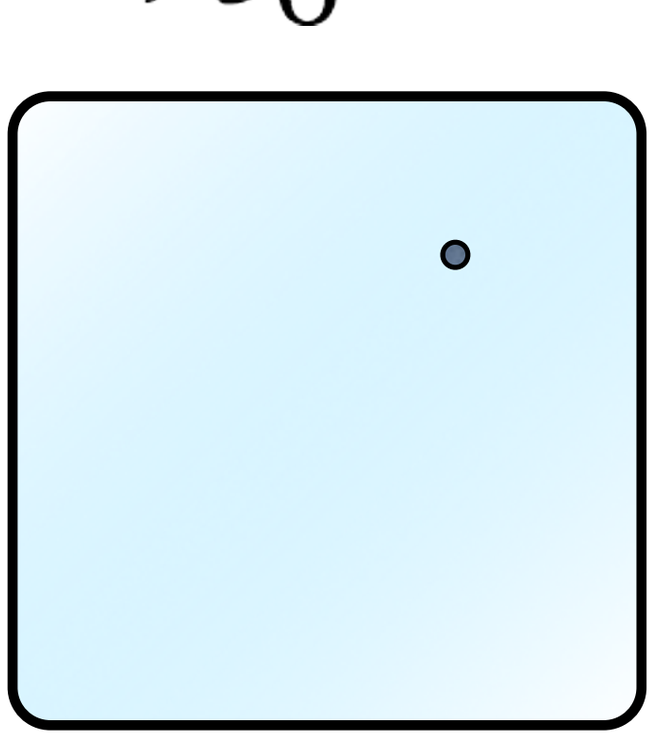
J-SC & F.H.L. Essler, PRL 2013

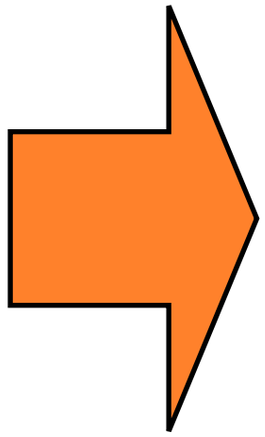
in pictures...

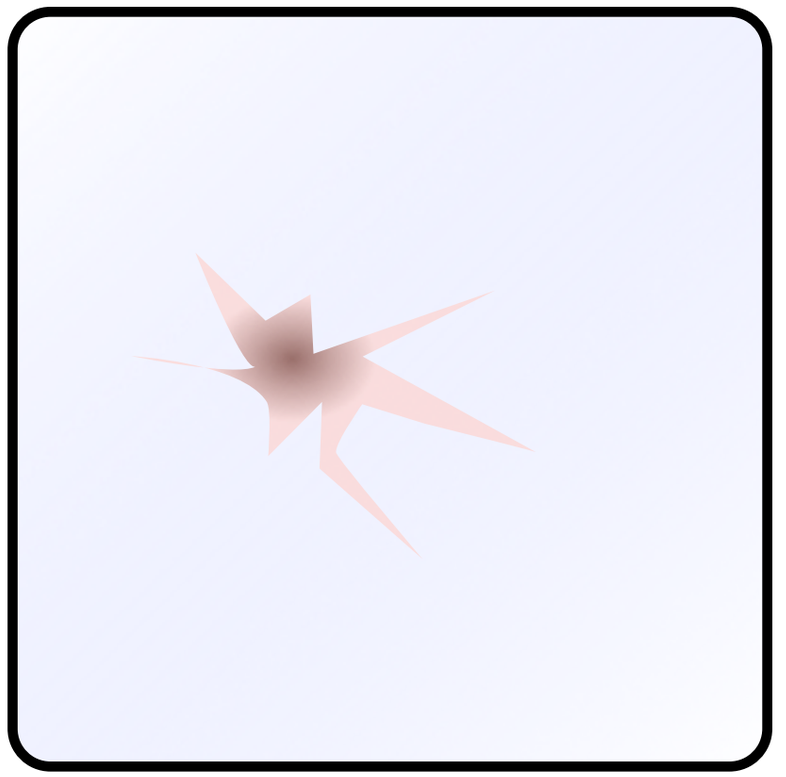
$\mathcal{H}_0$

Initial state:

$\mathcal{H}$







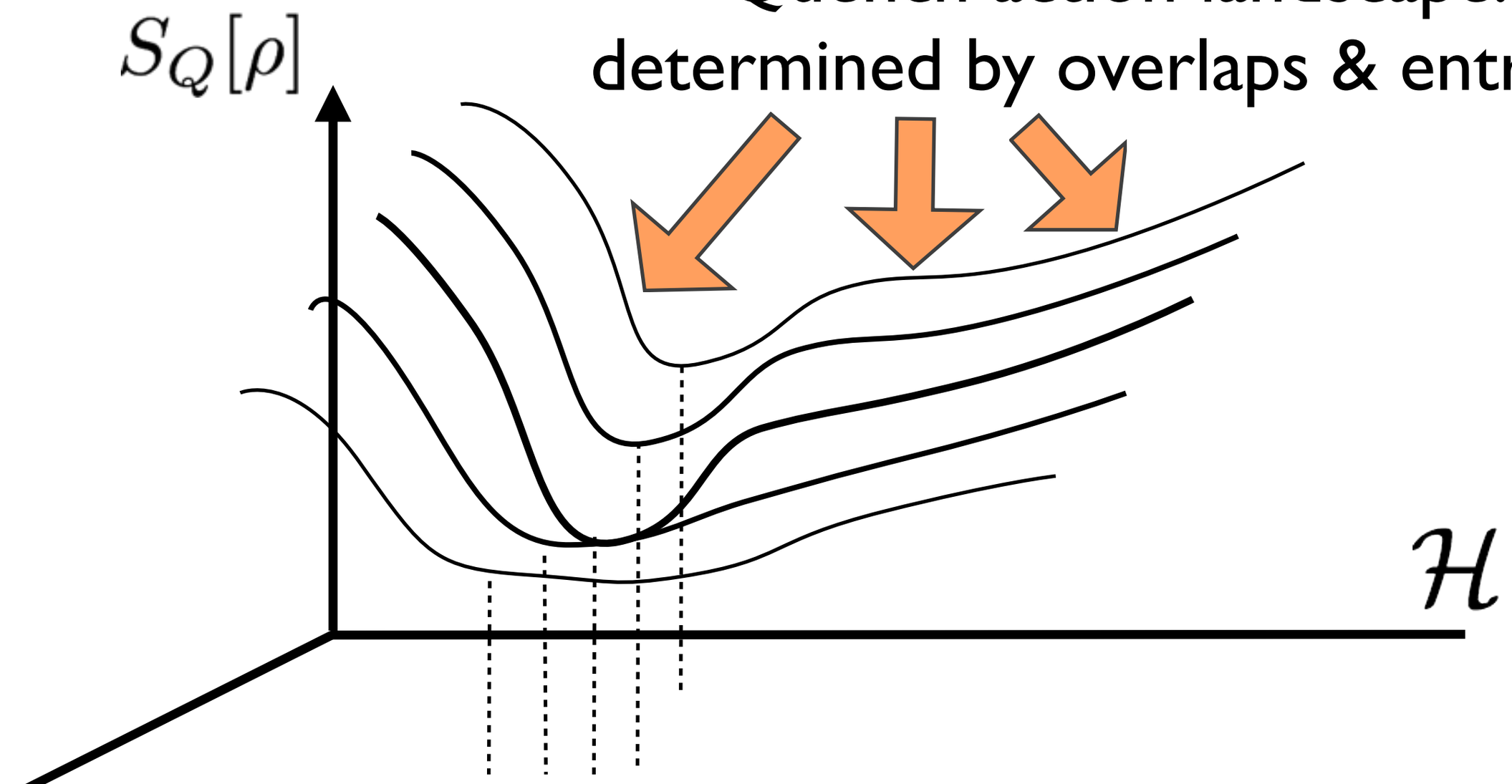
in pre-quench
Hilbert space basis

in post-quench
Hilbert space basis

The ‘quench action’ approach

J-SC & F.H.L. Essler, PRL 2013

Quench action landscape:
determined by overlaps & entropy

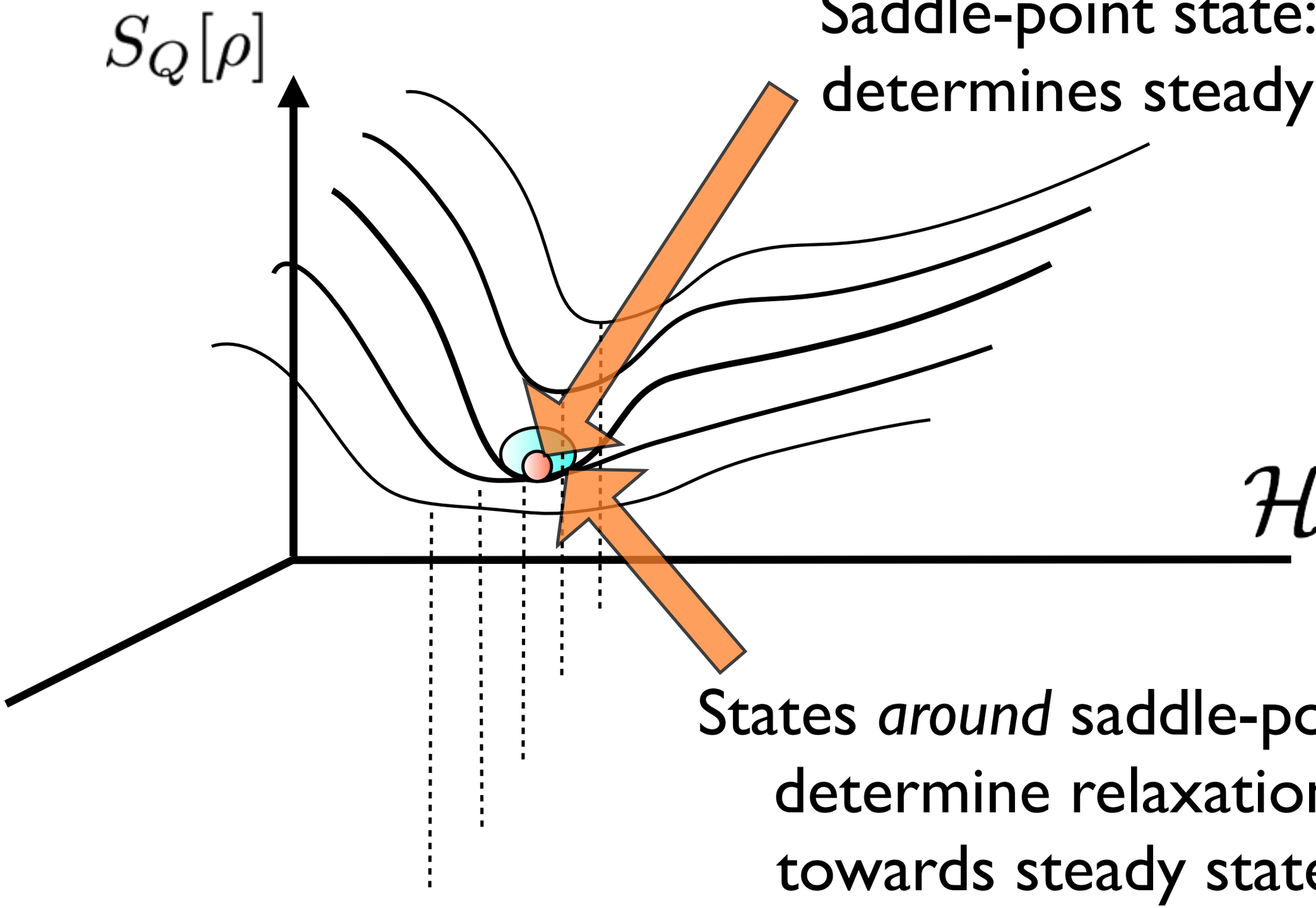


Variational approach, implemented by a
‘Generalized thermodynamic Bethe Ansatz’

J. Mossel and J-SC, JPA 2012; J-SC & R. Konik, PRL 2012,
see also Fioretto & Mussardo NJP 2010, Pozsgay JSTAT 2011

The ‘quench action’ approach

J-SC & F.H.L. Essler, PRL 2013



Saddle-point state: ρ_{sp}
determines steady state

States *around* saddle-point:
determine relaxation
towards steady state

Back to BEC-LL quench

Explicit result:

J. De Nardis, B. Wouters, M. Brockmann & J-SC, PRA 89, 2014

M. Brockmann JPA 2014

$$\langle \{\lambda_j\}_{j=1}^{N/2}, \{-\lambda_j\}_{j=1}^{N/2} | 0 \rangle = \sqrt{\frac{(cL)^{-N} N!}{\det_{j,k=1}^{N} G_{jk}}} \frac{\det_{j,k=1}^{N/2} G^Q_{jk}}{\prod_{j=1}^{N/2} \frac{\lambda_j}{c} \sqrt{\frac{\lambda_j^2}{c^2} + \frac{1}{4}}}$$

(reminiscent of Gaudin formula)

with matrix $$G^Q_{jk} = \delta_{jk} \Big(L + \sum_{l=1}^{N/2} K^Q(l_j, l_l) \Big) - K^Q(l_j, l_k)$$

$$K^Q(\lambda, \mu) = K(\lambda - \mu) + K(\lambda + \mu) \qquad K(\lambda) = \frac{2c}{\lambda^2 + c^2}$$

Quench action approach to BEC-LL quench

J. De Nardis, B. Wouters, M. Brockmann & J-SC, PRA 89, 2014

We are now in position to apply the quench action logic!

Need thermodynamic limit form of overlaps:

$$\lim_{Th}\langle\lambda,-\lambda|0\rangle=\exp\left(-\frac{L}{2}n\left(\log\frac{c}{n}+1\right)\right)$$
$$\times\exp\left\{-\frac{L}{2}\int_0^\infty d\lambda\rho(\lambda)\log\left[\frac{\lambda^2}{c^2}\left(\frac{\lambda^2}{c^2}+\frac{1}{4}\right)\right]+\mathcal{O}(L^0)\right\}$$

Quench action now defined, saddle-point solution via generalized thermodynamic Bethe ansatz

Quench action solution to BEC-LL quench

J. De Nardis, B. Wouters, M. Brockmann & J-SC, PRA 89, 2014

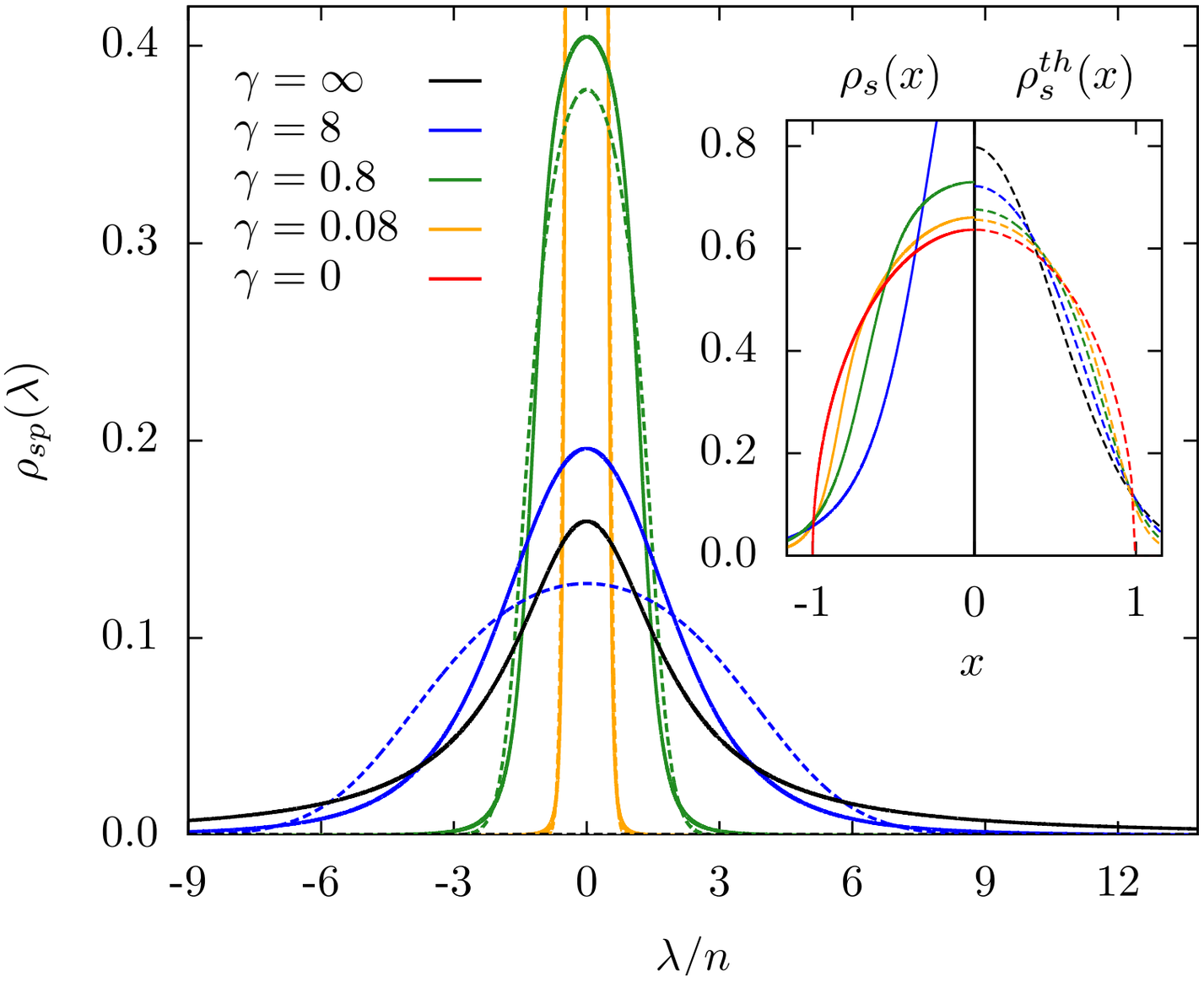
It is in fact possible to give a closed form solution of the GTBA for the saddle-point state, for any value of the interaction:

$$\rho(\lambda) = -\frac{\gamma}{2\pi}\frac{\partial a(\lambda)}{\partial \gamma}(1+a(\lambda))^{-1}$$

$$a(\lambda) = \frac{2\pi/\gamma}{\frac{\lambda}{c}\sinh\left(\frac{2\pi\lambda}{c}\right)} I_{1-2i\frac{\lambda}{c}}\left(\frac{4}{\sqrt{\gamma}}\right) I_{1+2i\frac{\lambda}{c}}\left(\frac{4}{\sqrt{\gamma}}\right)$$

Quench action solution to BEC-LL quench

J. De Nardis, B. Wouters, M. Brockmann & J-SC, PRA 89, 2014



Subplot: scaled fn

$$\rho_s(x) = \sqrt{\gamma}\rho(c\sqrt{\gamma}x/2)/2$$

Large c:

$$\rho(\lambda) = \frac{1}{2\pi}\frac{4n^2}{\lambda^2+4n^2}$$

Small c: semicircle

$$\rho(\lambda) \sim \frac{1}{\pi\sqrt{\gamma}}\sqrt{1-\frac{\lambda^2}{4\gamma n^2}}$$

Asymptotics as from q-bosons: $2\pi\rho(\lambda) \sim \frac{n^4\gamma^2}{\lambda^4} + \frac{n^6\gamma^3(24-\gamma)}{4\lambda^6} + \dots$

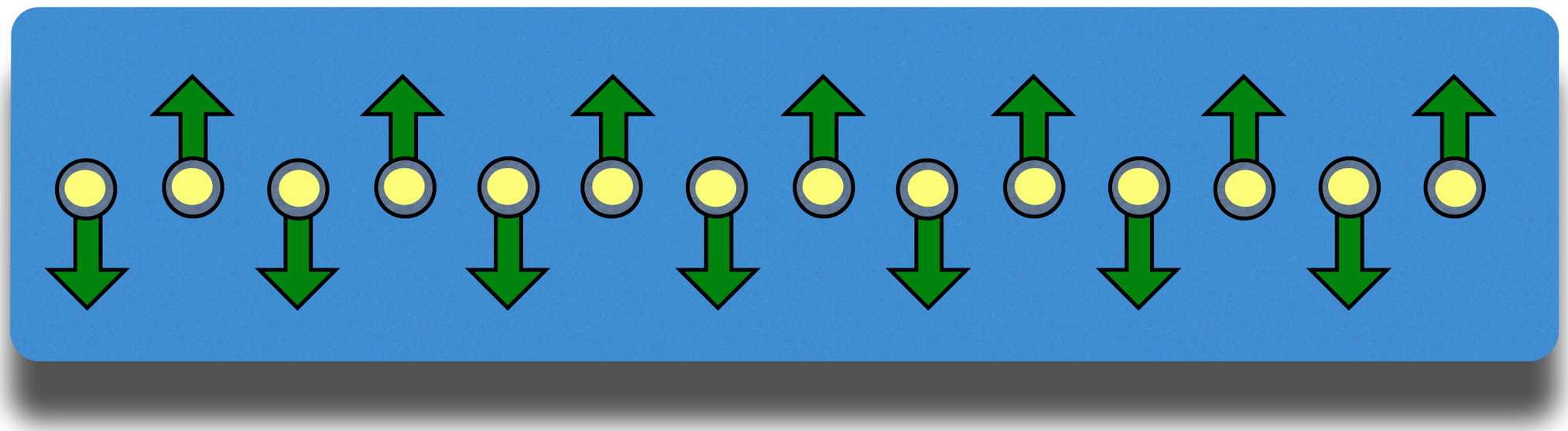
Tail explains divergences of evalues of conserved charges

Quantum quenches:

Néel to XXZ quench

Quench from Néel to XXZ

Start from Néel state:



From t=0 onwards, evolve with XXZ Hamiltonian

$$H = \sum_{j=1}^{N} \left[J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z) - H_z S_j^z \right]$$

Can one treat this problem exactly?

Quench action approach to Néel-XXZ quench

First step: exact overlaps of Néel state with XXZ eigenstates

Tsuchiya JMP1998; Kozlowski & Pozsgay JSTAT 2012

Gaudin-like form again! M. Brockmann, J. De Nardis, B. Wouters & J-SC JPA 2014

$$\frac{\langle \Psi_0 | \{\pm\lambda_j\}_{j=1}^{M/2} \rangle}{\|\{\pm\lambda_j\}_{j=1}^{M/2}\|} = \sqrt{2} \left[\prod_{j=1}^{M/2} \frac{\sqrt{\tan(\lambda_j + i\eta/2)\tan(\lambda_j - i\eta/2)}}{2\sin(2\lambda_j)} \right] \sqrt{\frac{\det_{M/2}(G^+_{jk})}{\det_{M/2}(G^-_{jk})}}$$

$$G^{\pm}_{jk} = \delta_{jk} \left(N K_{\eta/2}(\lambda_j) - \sum_{l=1}^{M/2} K^+_\eta(\lambda_j, \lambda_l) \right) + K^{\pm}_\eta(\lambda_j, \lambda_k)$$

$$K^{\pm}_\eta(\lambda, \mu) = K_\eta(\lambda - \mu) \pm K_\eta(\lambda + \mu) \qquad K_\eta(\lambda) = \frac{\sinh(2\eta)}{\sin(\lambda + i\eta)\sin(\lambda - i\eta)}$$

Quench action approach to Néel-XXZ quench

Second step: generalized TBA

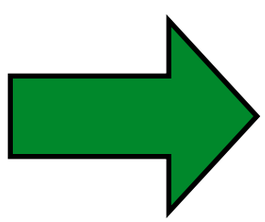
B. Wouters, J. De Nardis, M. Brockmann, D. Fioretto, M.Rigol & J-SC, PRL 2014

$$\ln \eta_n(\lambda) = -2\,h\,n - \ln W_n(\lambda) + \sum_{m=1}^{\infty} a_{nm} * \ln\left(1+\eta_m^{-1}\right)(\lambda)$$

where $\eta_n(\lambda) \equiv \rho_{n,h}(\lambda)/\rho_n(\lambda)$ $\qquad a_n(\lambda) = \dfrac{1}{\pi}\dfrac{\sin n\eta}{\cosh n\eta - \cos 2\lambda}$

and the effective driving terms (pseudo-energies) are

$$W_n(\lambda) = \begin{cases} \frac{1}{2^{n+1}\sin^2 2\lambda}\frac{\cosh n\eta - \cos 2\lambda}{\cosh n\eta + \cos 2\lambda}\prod_{j=1}^{\frac{n-1}{2}}\left(\frac{\cosh(2j-1)\eta - \cos 2\lambda}{(\cosh(2j-1)\eta + \cos 2\lambda)(\cosh 4\eta j - \cos 4\lambda)}\right)^2 & \text{if } n \text{ odd}, \\ \frac{\tan^2\lambda}{2^n}\frac{\cosh n\eta - \cos 2\lambda}{\cosh n\eta + \cos 2\lambda}\frac{1}{\prod_{j=1}^{\frac{n}{2}}(\cosh 2(2j-1)\eta - \cos 4\lambda)^2}\prod_{j=1}^{\frac{n-2}{2}}\left(\frac{\cosh 2j\eta - \cos 2\lambda}{\cosh 2j\eta + \cos 2\lambda}\right)^2 & \text{if } n \text{ even}. \end{cases}$$



Solution of this GTBA gives steady-state

(analytically!)

Quench action approach to Néel-XXZ quench

Equivalent form of generalized TBA:

$$\ln(\eta_n) = d_n + s * \left[\ln(1+\eta_{n-1}) + \ln(1+\eta_{n+1})\right]$$

with driving terms $d_n(\lambda) = \sum_{k\in\mathbb{Z}} e^{-2ik\lambda} \frac{\tanh(\eta k)}{k} \left((-1)^n - (-1)^k\right)$

GGE with local charges: same form of coupled equations, but driving term only for n=1:

$$d_1(\lambda) = -\frac{1}{\pi} \sum_{m=1}^{\infty} \beta_{2m} \sum_{k\in\mathbb{Z}} e^{-2ik\lambda} \frac{k^{2m-2}}{\cosh k\eta}$$

unknown

Néel-XXZ quench: conserved charges

Initial expectation value of local charges: Fagotti & Essler JSTAT 2013

$$\lim_{N\to\infty}\frac{1}{N}\langle\text{Néel}|Q_{n+1}|\text{Néel}\rangle = -\frac{\Delta}{2}\frac{\partial^{n-1}}{\partial x^{n-1}}\frac{1-\Delta^2}{\cosh[\sqrt{1-\Delta^2}x]-\Delta^2}\bigg|_{x=0}$$

Remarkable correspondence between 1-string hole density and local charges:

B. Wouters, J. De Nardis, M. Brockmann, D. Fioretto, M. Rigol & J-SC, PRL 2014

$$\sum_{k\in\mathbb{Z}} k^{2m-2}\left(\frac{e^{-|k|\eta}-\hat{\rho}_1^h(k)}{2\cosh k\eta}\right) = \langle Q_{2m}\rangle \quad m\in\mathbb{N}$$

which fixes

$$\rho_{1,h}^{\text{Néel}}(\lambda) = \frac{\pi^2 a_1^3(\lambda)\sin^2(2\lambda)}{\pi^2 a_1^2(\lambda)\sin^2(2\lambda)+\cosh^2(\eta)}$$

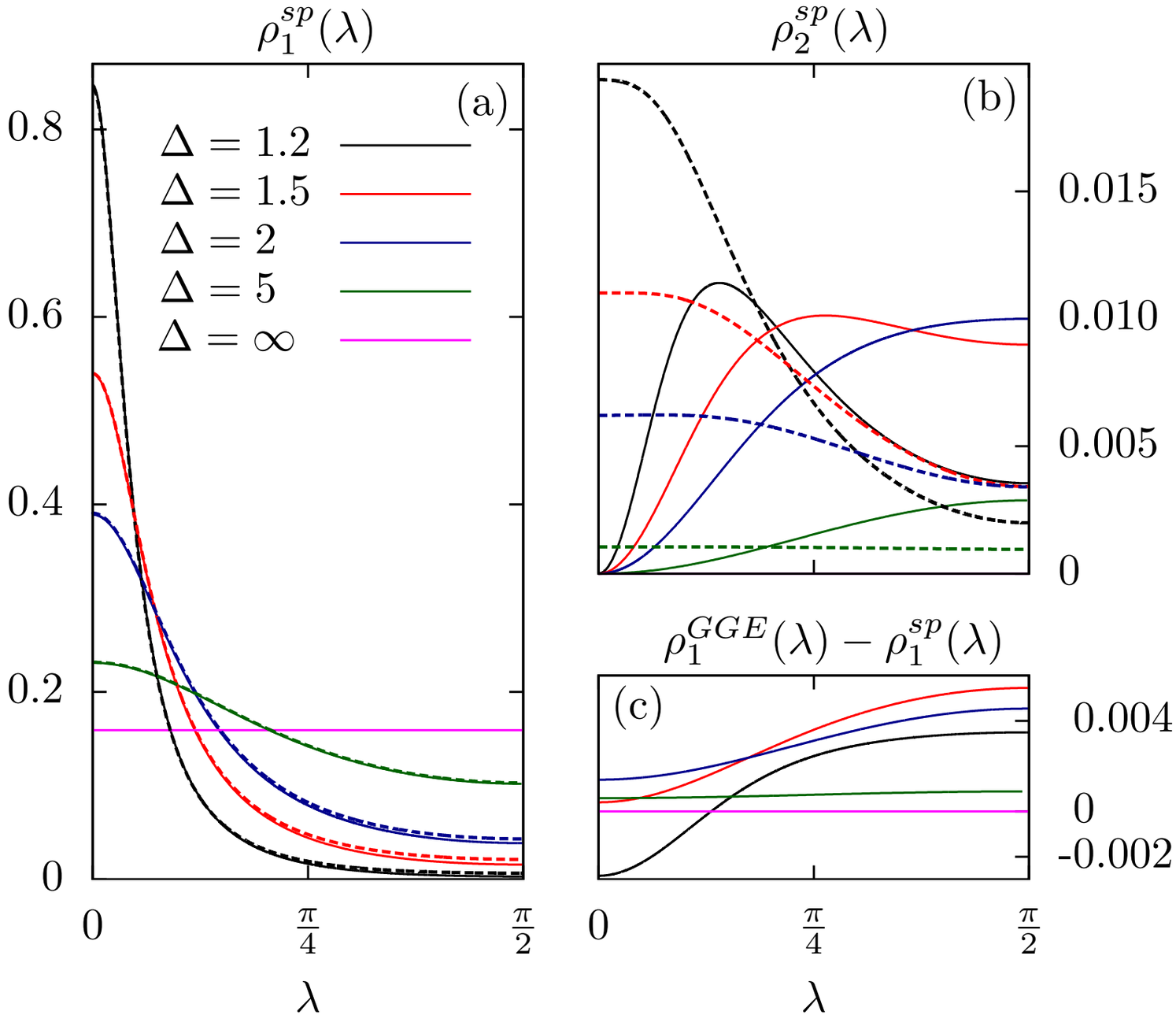
Quench action nontrivially reproduces this;
GGE also of course, but only by definition

The steady state: Néel to XXZ

Solid lines:
quench action

Dashed lines:
GGE (local charges)

QA and (local)GGE have different saddle-point densities

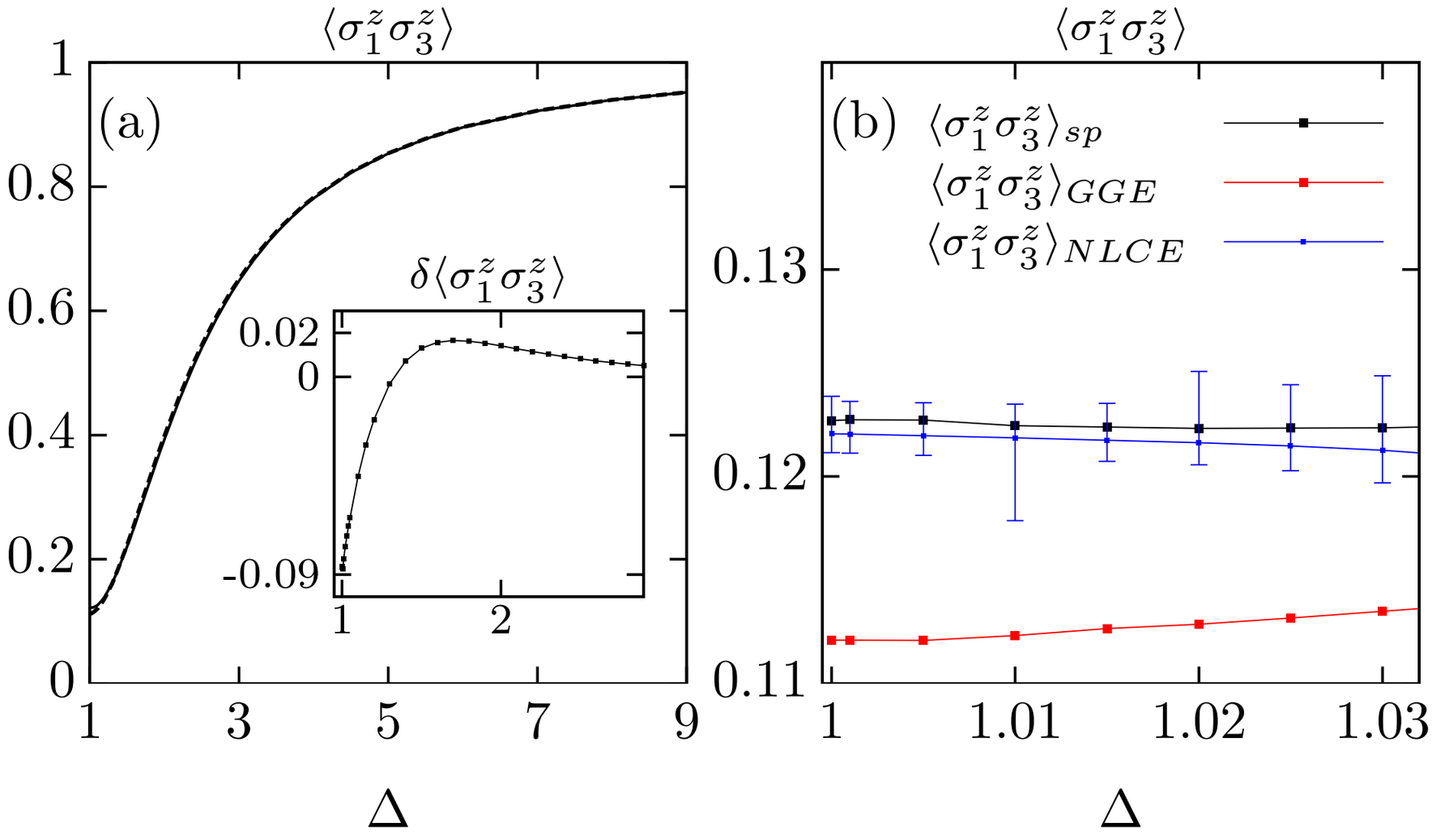


Large Delta expansion:

$$\rho_1^{GGE} - \rho_1^{sp} = \frac{1}{4\pi\Delta^2} + O(\Delta^{-3}),$$

$$\rho_2^{GGE} - \rho_2^{sp} = \frac{1 - 3\sin^2(\lambda)}{3\pi\Delta^2} + O(\Delta^{-3}).$$

Difference in distribution: impact on correlations



Numerical verification using NLCE (M. Rigol)

Large Delta expansions:

$$\langle\sigma_1^z\sigma_2^z\rangle_{QA} = -1 + \frac{2}{\Delta^2} - \frac{7}{2\Delta^4} + \frac{77}{16\Delta^6} + \dots$$

$$\langle\sigma_1^z\sigma_2^z\rangle_{GGE} = -1 + \frac{2}{\Delta^2} - \frac{7}{2\Delta^4} + \frac{43}{8\Delta^6} + \dots$$

Not convinced?

Look at other results by Budapest group

B. Pozsgay, M. Mestyán, M. A. Werner, M. Kormos, G. Zaránd, G. Takács, PRL 2014

- reobtain our Néel results
- also consider initial dimer state
- obtain numerical (iTEBD) evidence for correlations being different in dimer case

There remains no doubt about the correctness of the quench action results because…

‘Repairing’ the GGE for Néel to XXZ

Quasilocal charges in XXZ

Previously discovered in XXX, XXZ(gapless)

Prosen 2011; Prosen and Ilievski 2013; Ilievski and Prosen 2013; Prosen 2014

Pereira, Pasquier, Sirker and Affleck, JSTAT 2014

Mierzejewski, Prelovšek and Prosen 2015

Here : need generalization to XXZ(gapped)

Ilievski, Medenjak and Prosen, arXiv:1506.05049

Ilievski, De Nardis, Wouters, Caux, Essler, Prosen 2015

Starting point: q-deformed L-operator

$$L(z,s)=\frac{1}{\sinh\eta}\Big(\sinh(z)\cosh\left(\eta s_s^z\right)\otimes\sigma^0+\cos(z)\sinh\left(\eta s_s^z\right)\otimes\sigma^z+\sinh(\eta)(s_s^-\otimes\sigma^+ + s_s^+\otimes\sigma^-)\Big)$$

Auxiliary spins obey q-deformed su(2)

$$[s_s^+,s_s^-]=[2s_s^z]_q \quad [s_s^z,s_s^\pm]=\pm s_s^\pm \qquad [x]_q=\sinh\left(\eta x\right)/\sinh(\eta)$$

in 2s+1-dim irrep $s_s^z|k\rangle=k|k\rangle,\quad s_s^\pm|k\rangle=\sqrt{[s+1\pm k]_q[s\mp k]_q}|k\pm1\rangle$

Quasilocal charges in XXZ(gpd)

Higher-spin transfer matrices:

$$T_s(z) = \mathrm{Tr}_a \left[L_{a,1}(z,s) \dots L_{a,N}(z,s) \right]$$

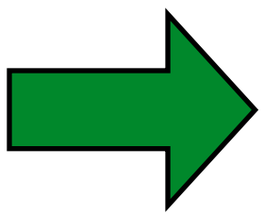
lead to spin-s conserved charges

$$X_s(\lambda) = \tau_s^{-1}(\lambda) T_s(z_\lambda^-) T_s'(z_\lambda^+), \quad z_\lambda^\pm = \pm\frac{\eta}{2} + i\lambda$$

in which $\tau_s(\lambda) = f(-(s+\frac{1}{2})\eta + i\lambda) f((s+\frac{1}{2})\eta + i\lambda)$ $\quad f(z) = (\sinh(z)/\sinh(\eta))^N$

More convenient for ThLim: $\widehat{X}_s(\lambda) := T_s^{(-)}(z_\lambda^-) T_s^{(+)\prime}(z_\lambda^+)$

built from transfer matrix with $L^{(\pm)}(z,s) = L(z,s)\sinh(\eta)/[\sinh(z \pm s\eta)]$



Families of quasilocal charges: $H_s^{(n+1)} = \frac{1}{n!}\partial_\lambda^n \widehat{X}_s(\lambda)\Big|_{\lambda=0} \quad s = \frac{1}{2}, 1, \frac{3}{2}, \dots$

A complete GGE for XXZ

Ilievski, De Nardis, Wouters, Caux, Essler, Prosen 2015

Throughout the gapped regime (including XXX limit), the GGE density matrix is given by

$$\hat{\varrho}_{\mathrm{GGE}} = \frac{1}{Z} \exp\left[- \sum_{n,s=1}^{\infty} \beta_n^s H_{s/2}^{(n)} \right]$$

Steady state: fixed by initial conditions through the generalized remarkable **'string-charge' correspondence**

$$\rho_{2s,h}^{\Psi_0}(\lambda) = a_{2s}(\lambda) + \frac{1}{2\pi} \left[\Omega_s^{\Psi_0}(\lambda + \tfrac{i\eta}{2}) + \Omega_s^{\Psi_0}(\lambda - \tfrac{i\eta}{2}) \right]$$

$$\Omega_s^{\Psi_0}(\lambda) = \lim_{\mathrm{th}} \frac{\langle \Psi_0 | \widehat{X}_s(\lambda) | \Psi_0 \rangle}{N}$$

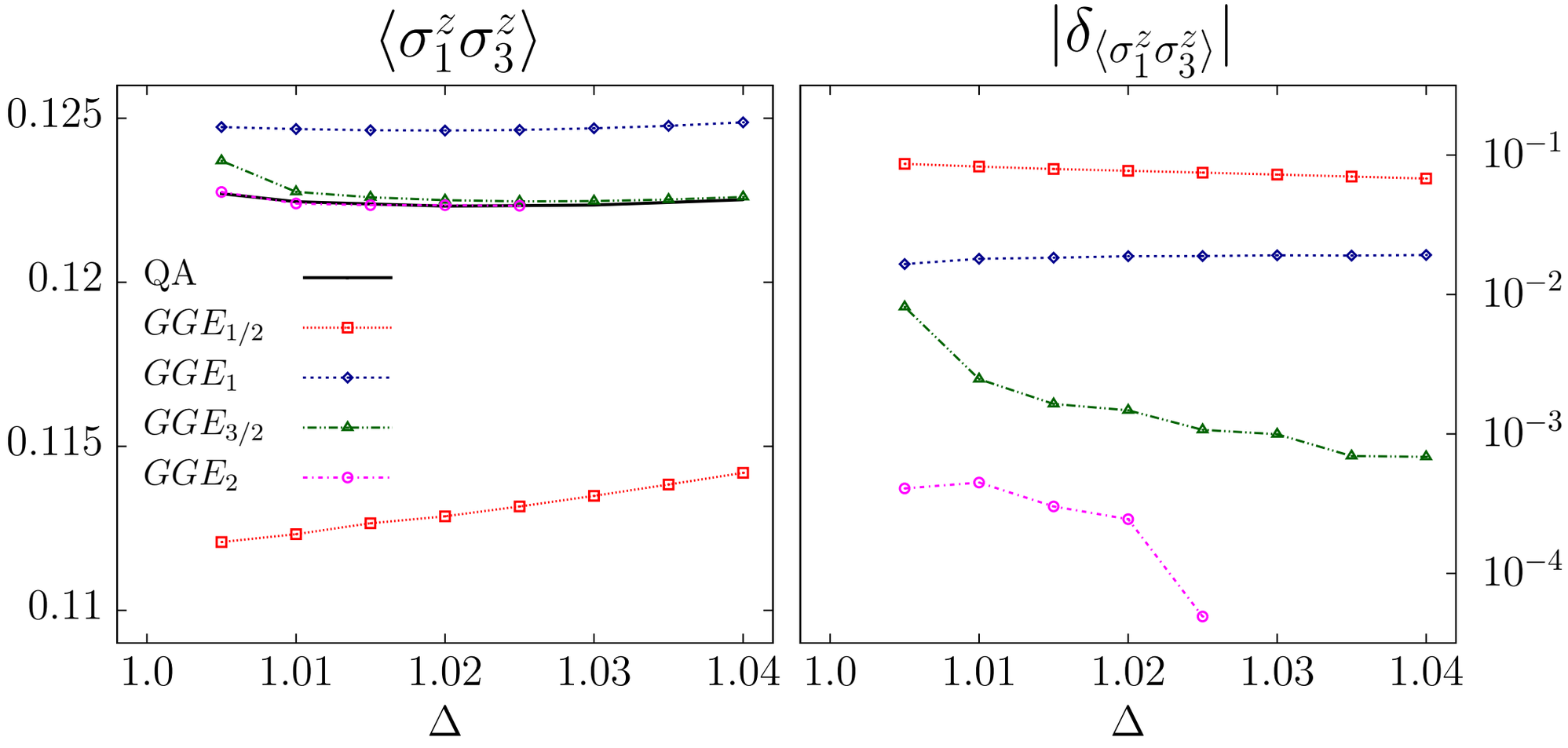
$$s = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

Fixing the Néel-to-XXZ GGE

Ilievski, De Nardis, Wouters, Caux, Essler, Prosen 2015

Implementing the construction for the Néel-to-XXZ quench makes the GGE converge to correct QA answer

Effect on some simple steady-state correlations:



Summary & perspectives

- Integrability for dynamics
  - *prototypical strong correlations under control*
  - *detailed experimental correspondence in spin chains*
  - *…and now also in cold atomic Bose gases*
  - *useful tests and lessons for field theory/numerical methods*

- Quench action logic
  - *new approach to out-of-equilibrium problems*
  - *gives access to full time evolution with minimal data*
  - *BEC to LL: exact solution (inaccessible to GGE)*
  - *Néel to XXZ: exact solution*
  - *GGE with local charges gives different steady state!*
  - *GGE needs to include quasilocal charges*