Spin Superfluidity, magnon BEC and Majorana

Yury Bunkov
Institute Neel, CNRS, Grenoble, France.

Coherent states and BEC

The magnon BEC

1. In bulk 3He-B
2. 3He-A in aerogel
3. In the antiferromagnets with Suhl-Nakamura
4. In YIG
5. Excited states of BEC

Spin Supercurrent
Heike Kamerlingh Onnes
Superconactivity, 1908

Piotr Kapitsa
Superfluidity, 1938
Atomic BEC

BEC states: Neutron stars, Universe,

Quasiparticles BEC states:

Exitons and Magnons BEC.
Discovery of Spin Superfluidity and Magnon BEC
In 1984

Russian Federation State Prize
1993

Fritz London Memorial Prize
Duke University, USA
2008

Cooling Flight out atoms

$\psi(\vec{r}) = \phi(\rho, z)e^{i\ell \theta}$

Bose-Einstein Condensation of Rb 87
Total number of atoms \[ N = \int d\vec{r} |\psi(\vec{r})|^2 = \text{constant} \]

Distribution of states \[ n_k = \left[ \exp \left( \frac{\hbar \omega_k - \mu}{k_B T} \right) - 1 \right]^{-1} \]

Temperature of BEC formation ($\mu=0$) \[ T_c = \left( \frac{n}{\zeta(3/2)} \right)^{2/3} \frac{2\pi \hbar^2}{mk_B} \approx 3.3125 \frac{\hbar^2 n^{2/3}}{mk_B} \]

**Gross–Pitaevskii equations**

The energy ($E$) associated with the state \[ E = \int d\vec{r} \left[ \frac{\hbar^2}{2m} |\nabla \psi(\vec{r})|^2 + V(\vec{r}) |\psi(\vec{r})|^2 + \frac{1}{2} U_0 |\psi(\vec{r})|^4 \right] \]

Minimizing this energy with respect to infinitesimal variations \[ i\hbar \frac{\partial \psi(\vec{r})}{\partial t} = \left( -\frac{\hbar^2 \nabla^2}{2m} + V(\vec{r}) + U_0 |\psi(\vec{r})|^2 \right) \psi(\vec{r}) \]
Thermal magnons \((\mu=0)\)

\[
N(\mu, T_{\text{eff}}) = \sum_k \left[ \exp \left( \frac{\hbar \omega_k - \mu}{k_B T_{\text{eff}}} \right) - 1 \right]^{-1}
\]

\(N \to 0\) at \(T \to 0\)
Thermal magnons

Excited magnons

\[ N(\mu, T_{\text{eff}}) = \sum_k \left[ \exp \left( \frac{\hbar \omega_k - \mu}{k_B T_{\text{eff}}} \right) - 1 \right]^{-1} \]

\( N \to 0 \) at \( T \to 0 \)
\[ M_x + iM_y = M_\perp e^{i\omega t + i\alpha}, \quad M_\perp = \chi HV \sin \beta. \]

\[ \langle \hat{a}_0 \rangle = N_0^{1/2} e^{i\mu t + i\alpha}, \quad \langle \hat{S}_+ \rangle = S_x + iS_y = \frac{M_\perp}{\gamma} e^{i\omega t + i\alpha}, \]

\[ N_{k=0} = \frac{M_0 - M_\parallel}{\mu} = \frac{M_0}{\mu} (1 - \cos \beta) \]

\[ N_c = \left( \frac{k_BT m^*}{3.31 \hbar^2} \right)^{3/2} \]
Preparing Magnons

HPD in 3He-B
\[ |\Psi|^2 = m(1 - \cos \beta). \]  

**Gross-Pitaevskii equation**  

\[ \frac{\delta F}{\delta \Psi^*} = 0 \]

\[ F = \int d^3r \left( \frac{|\nabla \Psi|^2}{2m_M} + (\omega_L(z) - \omega)|\Psi|^2 + F_D \right) \]

\[ k = 0 \text{ Spin Supercurrent} \]
\[ k \neq 0 \text{ Spin waves} \]

\[ F = A + (B+\mu) |\Psi|^2 + C|\Psi|^4 \]

- **Dipole-dipole spin-orbit energy**  
  \[ BI\Psi^2 + C|\Psi|^4 \]

- **Gradient energy**

- **Spectroscopic energy**

- **True energy at the rotating frame**
  \[ \mu = \omega_L(z) - \omega \]

- **Convex**  
  \[ C < 0 \]

- **Concave**  
  \[ C > 0 \]
Gross-Pitaevskii equation

\[ \frac{\delta F}{\delta \Psi^*} = 0 \]

\[ F = \int d^3r \left( \frac{|\nabla \Psi|^2}{2m_M} + (\omega_L(z) - \omega)|\Psi|^2 + F_D \right) \]

\[ \text{He}_3, \text{He}_3 - \text{A, MnCO}_3 \]

\[ \text{He}_3 - \text{B, Q-ball} \]

YIG

BEC-condensates

Group of NonLinear

Frequency shift (kHz)

Amplitude (V)

0.08V
0.1V
0.5V
1.0V
1.5V
2V
2.5V
3V
4V

0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08

-1.5
-1
-0.5
0
0.5
1
Spin-Orbit interaction energy in $^3$He-B

Gross-Pitaevskii equation

\[ F = \int d^3r \left( \frac{|\nabla \Psi|^2}{2m_M} + (\omega_L(z) - \omega)|\Psi|^2 + F_D \right) \]
Magnons BEC in the gradient of magnetic field

For $\beta<104^\circ$

$$F_D = 0$$

For $\beta>104^\circ$

$$F_D = \frac{8}{15} \chi \Omega_L^2 \left( \frac{25}{16} - \frac{5}{2} \frac{|\Psi|^2}{S} + \frac{|\Psi|^4}{S^2} \right)$$

$$\Delta \omega = \Omega_L^2 / 2 \omega_L$$

$$\mu = (\omega - \omega_L) / \Delta \omega$$

Minimization of energy in the conditions of magnetization
Conservation and the gradient of chemical potential

The magnetic relaxation leads to decrease of BEC region

It can be compensated by a small resonance RF pumping

Yu.M. Bunkov and G.E. Volovik
Bose-Einstein condensation of magnons in superfluid 3He
\[ \Psi(\vec{k}) = \Psi_{\uparrow\uparrow}(\vec{k}) | \uparrow\uparrow \rangle + \Psi_{\downarrow\downarrow}(\vec{k}) | \downarrow\downarrow \rangle + \sqrt{2} \Psi_{\uparrow\downarrow}(\vec{k}) | \uparrow\downarrow + \downarrow\uparrow \rangle \]
Magnetization transport by Spin Supercurrent

\[ J_{M_z}^\perp = -\left(\frac{\chi}{\gamma}\right)\left[(1 - \cos\beta)^2c_\parallel^2 + (1 - \cos^2\beta)c_\perp^2\right]\nabla \alpha \]
Critical Spin Current

\[ \nabla \alpha_c = \sqrt{\frac{4\omega_L (\omega_{RF} - \omega_L)}{5c_1^2 - c_2^2}} \]

\[ \Delta \alpha_c \]

\[ (\omega_{rf}, \gamma B_{ch}) \text{kHz} \]

Josephson effect

\[ \xi = \frac{1.088 c_{||}}{\sqrt{\omega (\omega - \gamma H)}} \]


Spin Supercurrent vortex

\[ \Delta \varphi \]

\[ J_p \]

\[ \text{Vortex axis} \]

\[ \text{Vortex axis} \]

\[ \cos \beta \]

\[ J_p \]

\[ \eta \kappa a \]

\[ \eta \kappa a \]
Spin Supercurrent vortex

Frequency and amplitude of the signal from pick-up coil after

Homogeneous excitation  Quadrupole excitation

Q-ball in Magnon trap

\[ f - f_0 = a_r(\Omega)(2n_r + 1) + a_z(2n_z + 1/2) \]

\[ U(r, z) \propto \kappa^2 r^2 z^2 / (r^2 + z^2) \]

\[ F_D = \chi \Omega_L^2 \left[ \frac{4 \sin^2(\beta_L/2)}{5 S} |\Psi|^2 - \frac{\sin^4(\beta_L/2)}{S^2} |\Psi|^4 \right] \]
$P = 0.5 \text{ bar}, \ T = 0.14 \ T_C, \ f_L = 0.827 \text{ MHz}$

$\Omega = 0.9 \text{ rad/s (vortices)}$

- ground level, $\tau = 5.36 \text{ s}$
- (2,0) level, $\tau = 0.74 \text{ s}$

$0$ \quad $0.2$ \quad $0.4$ \quad $0.6$ \quad $0.8$ \quad $1.0$ \quad $1.2$

$f - f_L, \text{ kHz}$

- rf pumping off
- rf pumping
Superfluid $^3\text{He}$-A in squeezed aerogel, ISSP, Japan

\[
F = \int d^3r \left( \frac{\left| \nabla \Psi \right|^2}{2m_M} + (\omega_L(z) - \omega) \left| \Psi \right|^2 + F_D \right)
\]

\[
F_D = \frac{\chi \Omega_L^2}{4} \left[ -\frac{2}{S} \left| \Psi \right|^2 + \frac{\left| \Psi \right|^4}{S^2} + \left( -2 + \frac{4}{S} \frac{\left| \Psi \right|^2}{S} - \frac{7}{4} \frac{\left| \Psi \right|^4}{S^2} \right) \sin^2 \beta_L \right]^+
\]

\[\Delta \omega = \Omega_L \frac{2}{2\omega_L} \]

\[\omega = \omega_L - A \Delta \omega\]

\[\Psi^2 = 1 - \cos \beta\]

\( J^z_i = A \cos^2 \beta (1 + \cos^2 \beta) \nu_i \alpha \)
BEC in MnCO₃
Coupled Nuclear-electron precession
Coupled Nuclear electron precession

\[
\omega^e = \sqrt{(\omega^{e0})^2 + (\omega_{\text{hf}}^e)^2}, \quad \omega^{e0} = \gamma_e \sqrt{H_0 (H_0 + H_D)},
\]

\[
\omega_{\text{hf}}^e = \gamma_e H_{\text{hf}}^e, \quad H_{\text{hf}}^e = A \gamma_e m_z.
\]

\[
\omega^n_0(\beta) = \omega^{n0} - \frac{\omega_p(\beta)}{1 + (kr_0)^2}, \quad \omega^{n0} = \gamma_n (H_0 + H_{\text{hf}}^n),
\]

\[
\omega_p(\beta) = \omega^{n0} \frac{H_{\text{ex}}^e H_{\text{hf}}^e}{2H^2} \frac{m \cos \beta}{M}, \quad H_{\text{hf}}^n = A \gamma_n M.
\]

\[
\mathcal{F} = \int d^3 r \left\{ \frac{\left| \nabla \Psi \right|^2}{2m_M} + \left[ \omega^n_0 - \omega_{\text{rf}} \right] |\Psi|^2 + \frac{\omega_p}{2} |\Psi|^4 \right\}.
\]

\[
|\Psi|^2 = \tilde{m} (1 - \cos \beta)
\]

\[
\omega_{\text{rf}} = \omega^n_0(\beta) = \omega^{n0} - \omega_p(0) \cos \beta.
\]
CW NMR experiments, Kazan 2013

\[ \beta \] vs. \( t_{\text{PULSE}} \) (ms)

\[ \beta, \text{deg.} \] vs. \( \Delta f', \text{MHz} \)

\[ B_0, \text{mT} \] vs. \( \beta \)

Amplitude, V vs. Time, \( \mu \text{s} \) and \( \Delta f', \text{MHz} \)
$T = 1.5 \text{ K}$

$f_{RF} = 562.55 \text{ MHz}$

(a) FID in normal state

(b) Echo

(c) FID in BEC state
\[ N_c = \left( \frac{k_B T m^*}{3.31 \hbar^2} \right)^{\frac{1}{2}} \]

\[ M_0 \cos \beta = M_0 - \mu N_{k=0}, \]

\[ m^* = \frac{\hbar H (H^2 + H^2_\Delta)^{\frac{3}{2}}}{\omega_{n_0} \alpha^2 H^2_\Delta}, \]

\[ \mu = -\frac{\hbar \omega_{n_0} H^2_\Delta}{(H^2 + H^2_\Delta)^{\frac{3}{2}}}. \]
We showed that external pumping at some $k_{in}$ increase the number of magnons for $k < k_{in}$ while energy of magnon reservoir increases only for magnon $k > k_{in}$. This result is based only on the analysis of two integrals of motion and is independent of the details of interactions in the system (provided that total number of magnons and their energy is conserved at least approximately). Therefore it valid for ALL cases of magnon condensation on the bottom of their frequency spectrum: in bulk [1] and film [2] IYG, in Q-ball of $^3$He-B [3] and in Suhl-Nakamura AFMs[4].

Energy in rotating frame
Conclusion

For 30 years from its discovery the Spin Superfluidity was discovered and widely studied in Moscow, Grenoble, Lancaster, Ithaca, Helsinki, Kosice, Kyoto, Tokyo, Los-Alamos, Okinawa, Kazan and by many theoreticians.

There was discovered and observed:

1. The coherent transport of magnetization in superfluid 3He-B.
2. Its based only on an antiferromagnetic properties of 3He and now was found in antiferromagnets with Suhl-Nakamura interaction
3. The formation of domain with coherent precession of magnetization.
4. This excited state correspond to a Bose-Einstein Condensation of spin waves.
5. Josephson phenomena, critical current and phase slippage in a channel.
5. Different modes of HPD oscillations.
6. Horizontal and vertical Spin vortex.
7. HPD techniques was applied for studies of counterflow and mass vortices in 3He, new types of vortex - spin-mass vortex was observed
8. 6 different states with coherent precession in A and B phases of 3He
9. And new coherent states in CsMnF3 and MnCO3 antiferromagnets with dinamical frequency shift as well in YIG.
Direct observation of Majorana in 3He-B

Yuriy M. Bunkov, Rasul R. Gazizulin

Institut NEEL & Univ. Grenoble Alpes, Grenoble, France

ANR MajoranaPRO project.
Superfluid $^3$He

Quantum vacuum, characterized by

\[ \Psi \text{ (phase)} \quad S \text{ (magnetization)} \quad L \text{ (orbital momentum)} \]

Particles:
- Quasiparticles
- Magnons
- Majorana
- Acoustic modes

Fields:
- Texture of orbital momentum

Topological defects:
- Boojum
- Vortex
- Brane
In Search of Majorana

1937: Majorana publishes his modification of the Dirac equation of relativistic quantum theory

Spin ½ particle = antiparticle: $\psi = \psi^\dagger$

1938: Majorana mysteriously disappears at sea

Observation of a Majorana fermion is among the great challenges of physics today.

Fundamental Particles might be Majorana fermions.

- e.g. neutrino
- Allows neutrinoless double $\beta$-decay.
- Candidate for Dark matter
\[ C_{\text{bulk}} \sim V P_F^2 \left( \frac{\Delta}{kT} \right)^{3/2} \exp\left( -\frac{\Delta}{kT} \right) \]

\[ C_{\text{maj}} \sim A \xi P_F^2 \left( \frac{\Delta}{kT} \right)^{-2} \]

\[ \frac{V}{A \xi} = 10^4 \]
Cell A \( S/V = 0.9 \text{ mm}^{-1} \)
Cell B \( S/V = 18 \text{ mm}^{-1} \)

We can draw the conclusion that the existence of Majorana fermions is confirmed in topological insulator $^{3}$He-B by the very direct method of measuring the Majorana heat capacity.

We have made a direct measurements of heat capacity of $^{3}$He-B inside the bolometer. We have found the additional heat capacity, which corresponds well to the Majorana quasiparticles on the surface of $^{3}$He.

We have test this phenomena by increasing of the surface on 22 times and by changing the pressure. Also we have measured the time constant of bolometer cooling. All data correspond well to the density of Majorana without any fitting parameter!

This work was supported by the “Agence Nationale de la Recherche (France) within MajoranaPRO project (ANR-13-BS04-0009-01);