Floquet theory of laser-induced phase transitions and applications to quantum magnets

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new group "nonequilibrium quantum matter")

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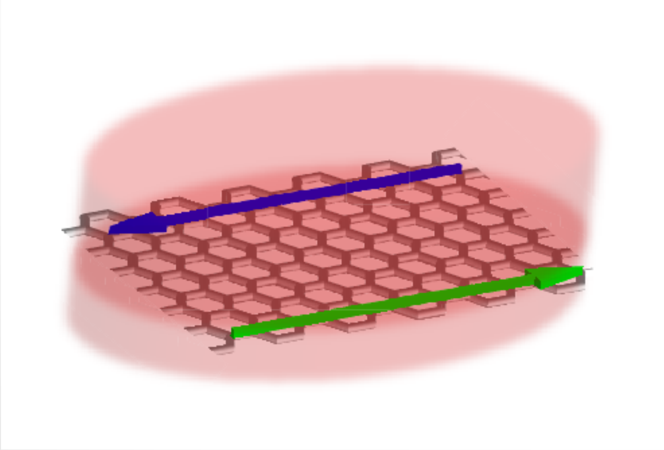
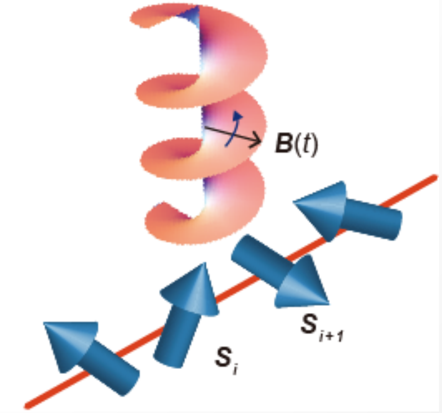


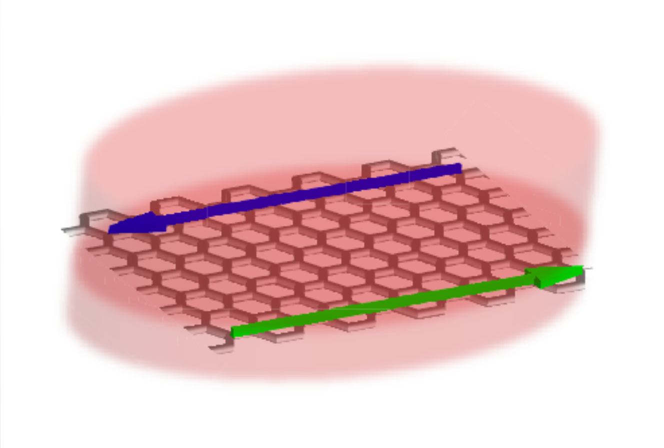


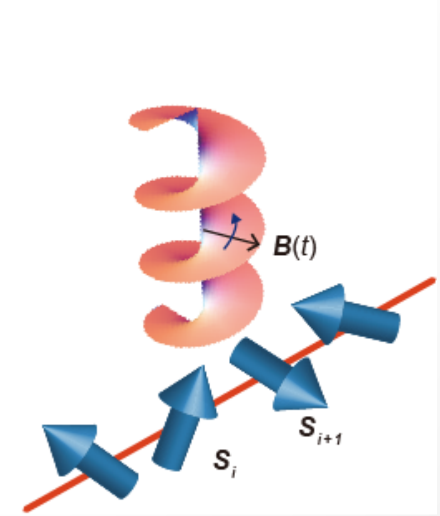
Take home message of my talk

For time-periodic system,

$$H(t) = \sum_m H_m e^{-im\Omega t}$$

$$H_{\text{eff}} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$$





Take home message 2

Some experiments with High magnetic field facilities
might be replacable by effective magnetic fields induced by laser

High magnetic field facility

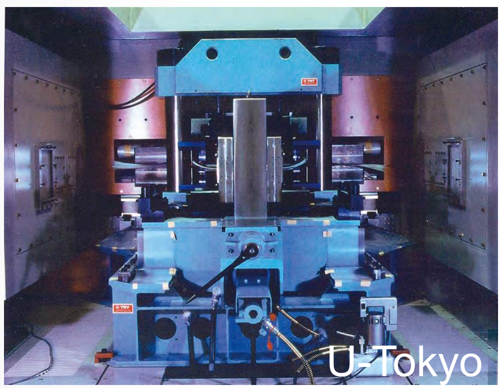
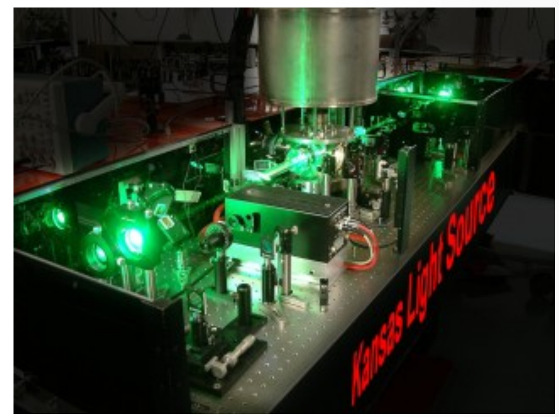


Table top laser experiments



Quantum Hall effect with Landau levels

Magnetization process in magnets

B<80 Tesla

Quantum Hall effect **without** Landau levels

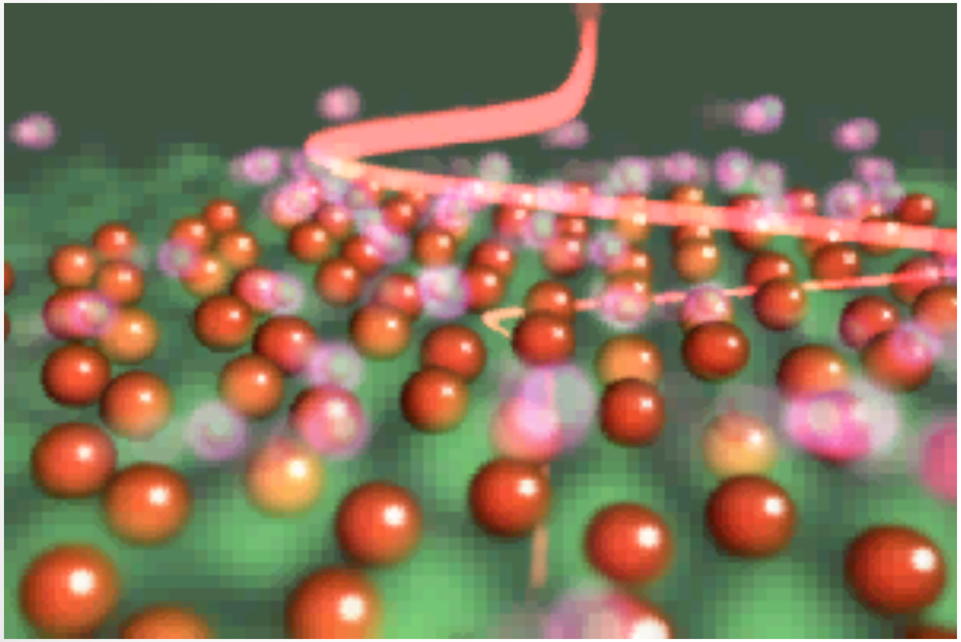
“Berry curvature effect”

Magnetization process in magnets

“effective magnetic field”

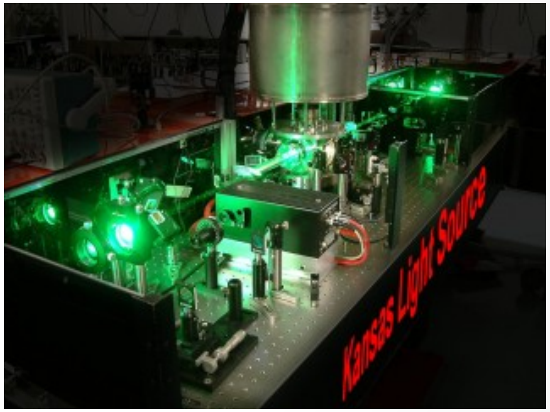
> 1000 Tesla!

“Nonequilibrium control of quantum matter”



animation by K. Tanaka (Kyoto)

Table top laser experiments



Quantum Hall effect **without** Landau levels

“Berry curvature effect”

Magnetization process in magnets

“effective magnetic field”

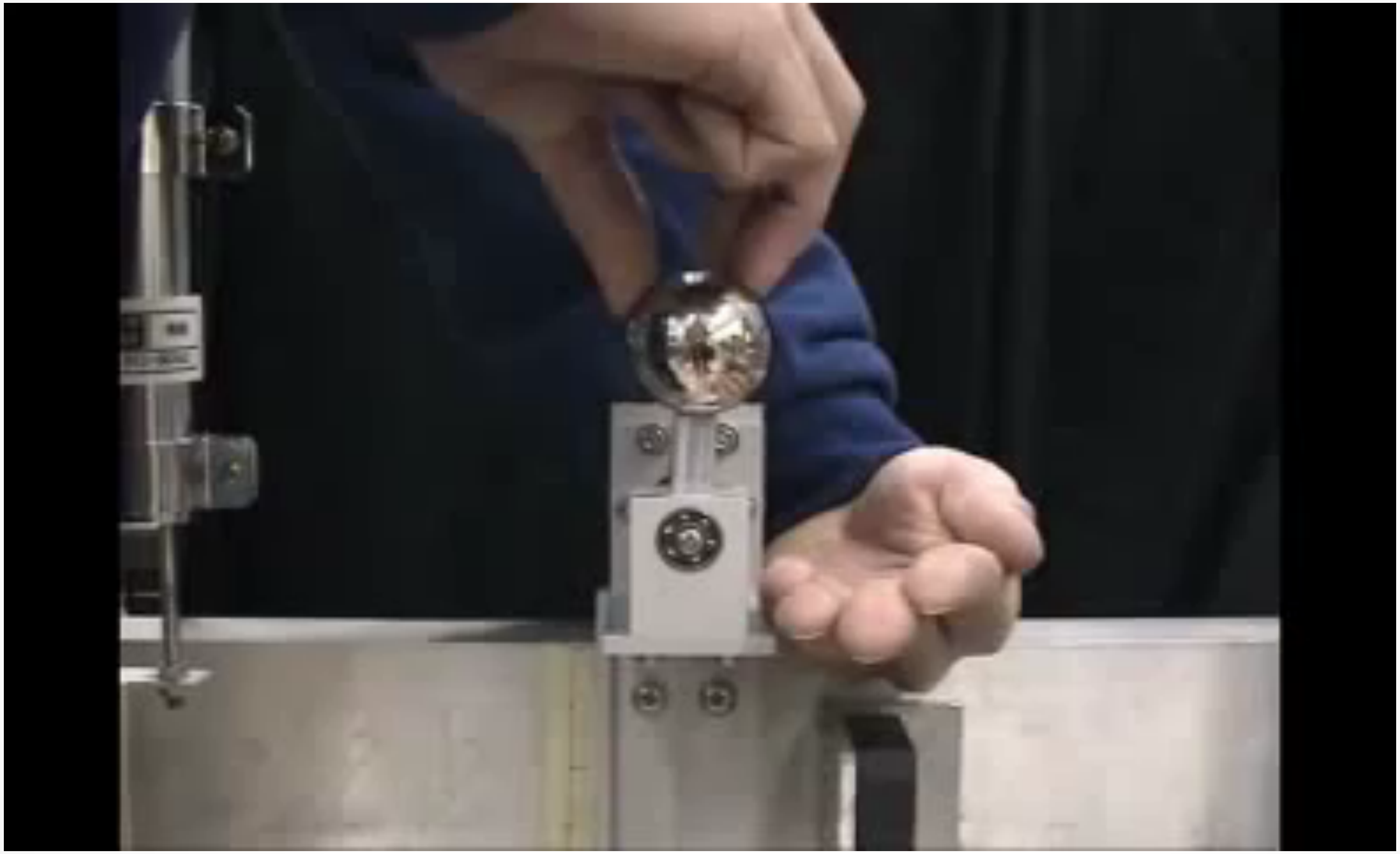
> 1000 Tesla!

What is a Floquet state?

Lets see some examples

ex.1: Kapitza's inverted pendulum

$$H(t) = \frac{p_\theta^2}{2m} - (g_0 + g_1 \cos(\Omega t)) \cos(\theta)$$



youtube

extension to quantum many-body systems (sine-Gordon model)

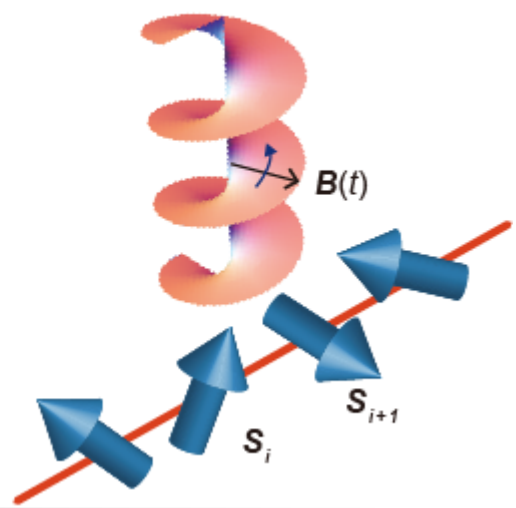
R. Citro, E. G. Dalla Torre, L. D'Alessio, A. Polkovnikov, M. Babadi, TO, and E. Demler, *'15*

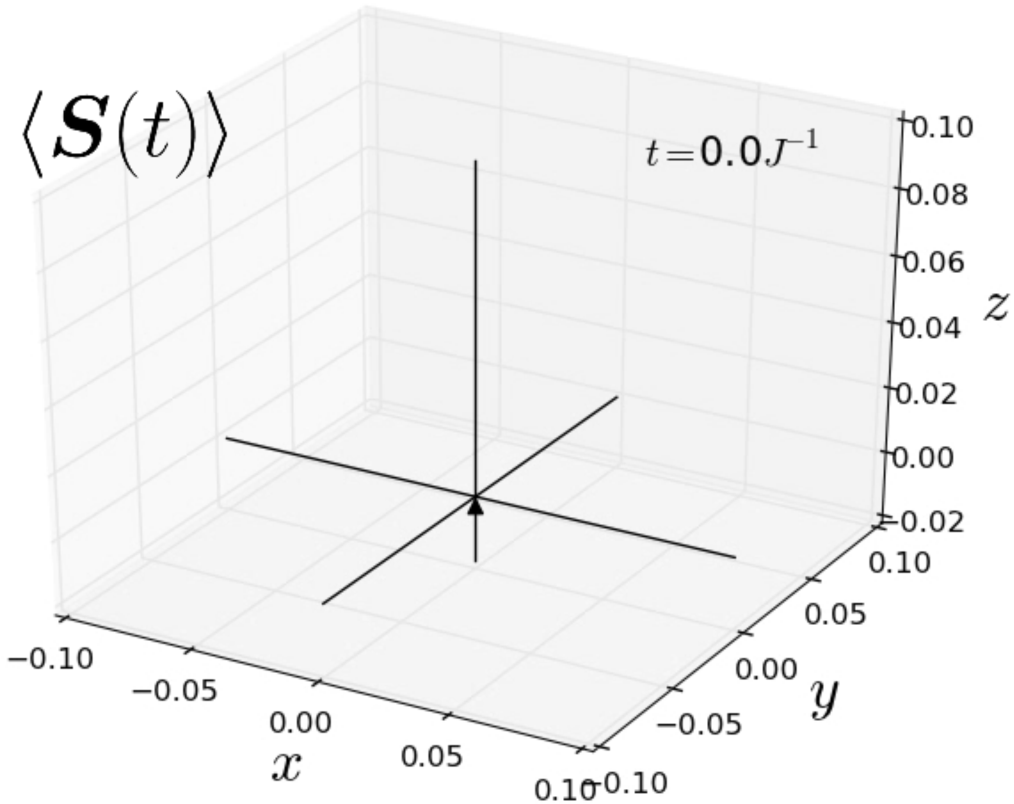
ex. 2: Magnets in rotating magnetic fields

$$H(t) = \sum_i \left(J\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + D(S_i)^2 + \boxed{\boldsymbol{B}(t)\boldsymbol{S}_i} \right)$$

Laser induced magnetization

$$\boldsymbol{B}(t) = (B\cos\Omega t, B\sin\Omega t)$$





Takayoshi, Aoki, TO, PRB2014
Takayoshi, Sato, TO, PRB2014

THz ultrafast memory

Light matter coupling for lattice electrons

$$H(t) = \sum_{ij} t_{ij} e^{-i\phi_{ij}(t)} c_i^\dagger c_j$$

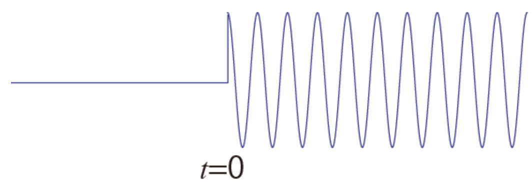
$$\phi_{ij} = \boldsymbol{e}_{ij} \cdot \boldsymbol{A}(t)$$

ex. 3: Dynamical localization

Dunlap Kenkre '86

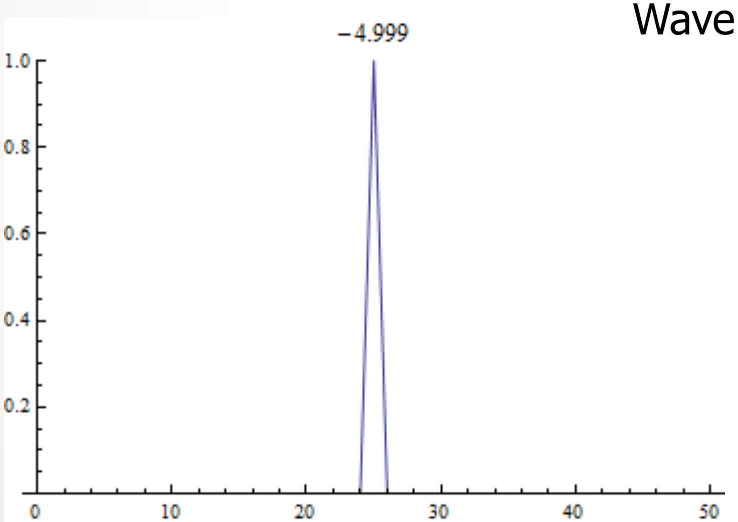
$$H(t) = \sum_{ij} t_{ij} e^{-i\phi_{ij}(t)} c_i^\dagger c_j$$

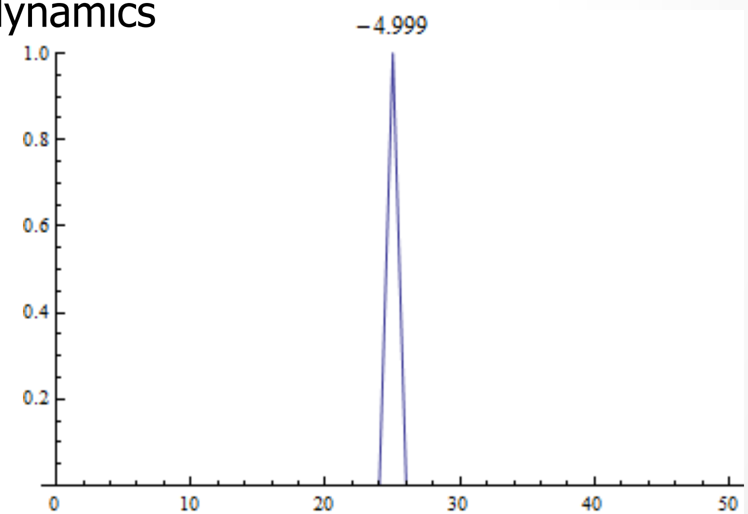
$$E(t) = E_0 \cos \Omega t \quad (t > 0)$$



$\Omega = 8, E_0/\Omega = 2.41$ $\Omega = 8, E_0/\Omega = 4$

Wave packet dynamics

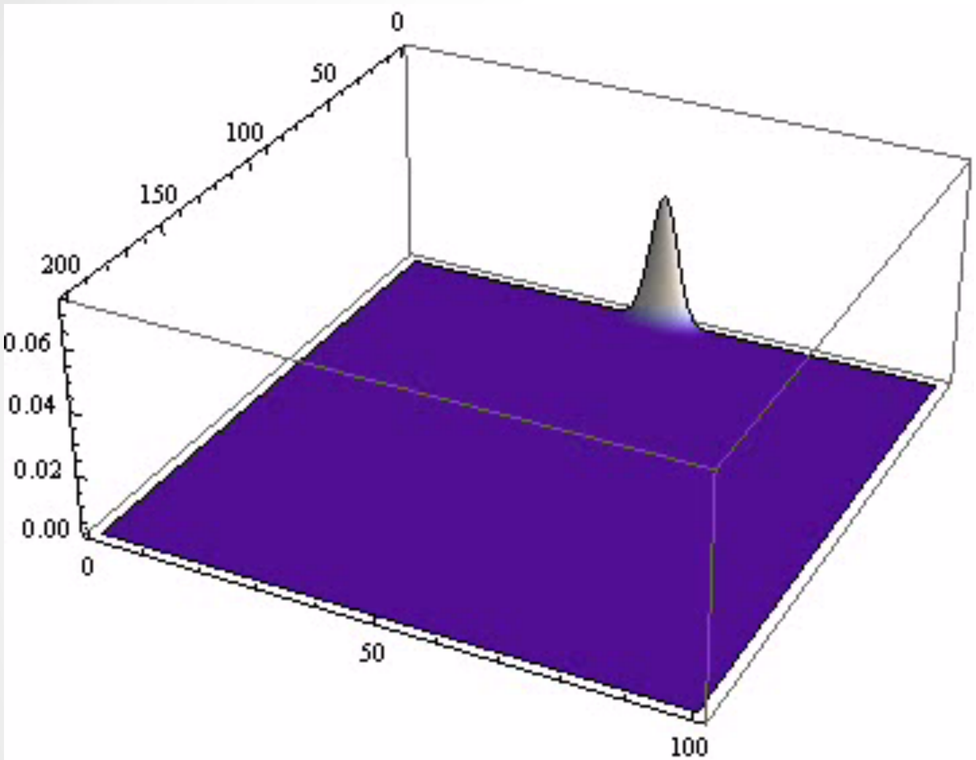




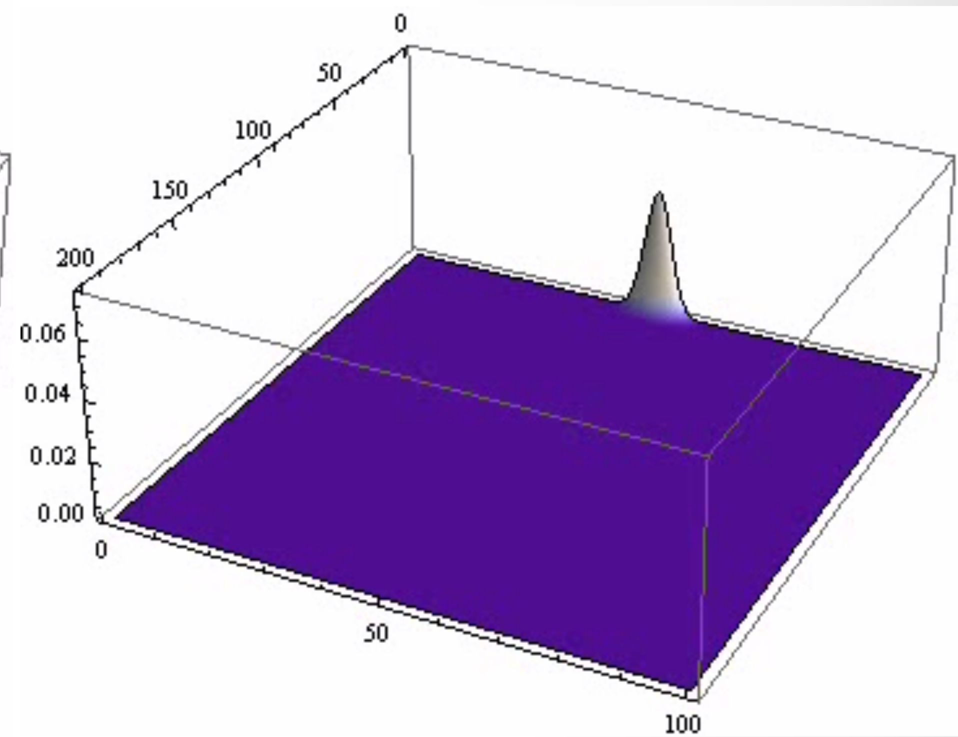
ex. 4: Floquet topological insulator

TO, Aoki ’09
Kitagawa, TO, Fu, Brataas, Demler ‘11

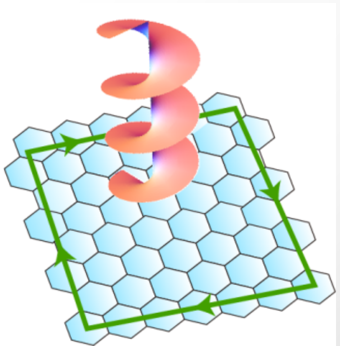
Wave packet dynamics in a honeycomb lattice



Without field



With circularly polarized laser



How can we understand these phenomena?

Floquet theory basics (1/4)

Choose the best frame

unitary transformation

$$|\Psi(t)\rangle = U^\dagger(t)|\Psi'(t)\rangle$$

$$H'(t) = U(t)H(t)U^\dagger(t) + i\partial_t U(t)U^\dagger(t)$$

e.g. 1d lattice fermion in ac-electric field

$$H(t) = -\sum_i \left[e^{i\frac{E}{\Omega}\sin\Omega t} c_{i+1}^\dagger c_i + \mathrm{h.c.}\right] \qquad H'(t) = -\sum_i \left[c_{i+1}^\dagger c_i + \mathrm{h.c.}\right] + \sum_j n_j j E \cos\Omega t$$

physically equivalent

Floquet theory basics (2/4)

Use Fourier transformation

time periodic system

cf) P. Hanggi "Driven quantum systems" '98
a chapter in a book "Quantum transport and dissipation".

$$i\partial_t \psi = H(t)\psi$$

$$H(t) = H(t+T) \qquad \Omega = 2\pi/T$$

$$\Psi(t) = e^{-i\varepsilon t} \sum_m \phi^m e^{-im\Omega t}$$

Floquet Hamiltonian (static eigenvalue problem)

$$\sum_{m=-\infty}^{\infty} \mathcal{H}^{mn} \phi_\alpha^m = \varepsilon_\alpha \phi_\alpha^n$$

ε: Floquet quasi-energy

$$(\mathcal{H})^{mn} = \frac{1}{T}\int_0^T dt H(t) e^{i(m-n)\Omega t} \boxed{+ m\delta_{mn}\Omega I}$$

comes from the $i\partial_t$ term

$$H_m = \mathcal{H}^{m0}$$

~ absorption of m "photons"

Floquet theory basics (3/4)

Time-periodic quantum system = Floquet theory (exact) ~ effective theory

$$i\partial_t\psi = H(t)\psi$$

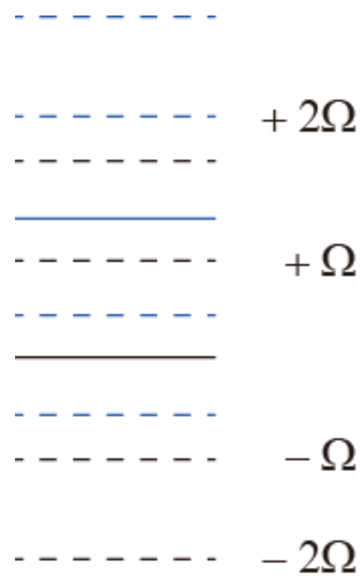
$$H(t) = H(t+T)$$

$$\mathcal{H}\phi = \varepsilon\phi$$

$$H_{\text{eff}} = H_0 + \sum_m \frac{[H_{-m}, H_m]}{m\Omega} + \mathcal{O}(\Omega^{-2})$$

two states + periodic driving

Floquet theory



n-photon dressed state

Floquet side bands

projection to the original Hilbert space

Hilbert sp. size
= original system

Floquet theory basics (4/4)

various expansion schemes

Floquet-Magnus expansion

$$
\begin{aligned}
H_{\rm eff} = H_0 &+ \frac{1}{2}\sum_{m\neq 0}\frac{[H_{-m}, H_m]}{m\Omega} \\
&+ \frac{1}{3}\sum_{m,n\neq 0}\frac{[H_{-m},[H_{m-n},H_n]]}{nm\Omega^2} + \frac{1}{2}\sum_{m,n\neq 0}\frac{[H_m,[H_0,H_{-m}]]}{m^2\Omega^2} + \cdots
\end{aligned}
$$

The series expansion may diverge,
especially when there is resonance

ex. 3: Dynamical localization

Dunlap Kenkre '86

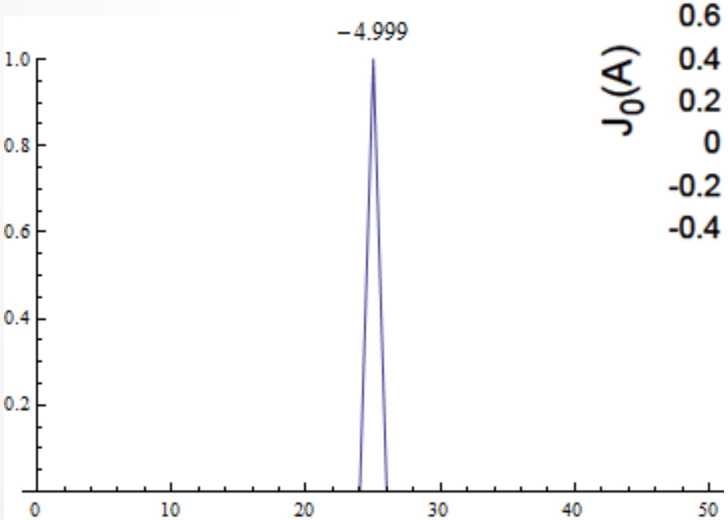
Fourier trans.

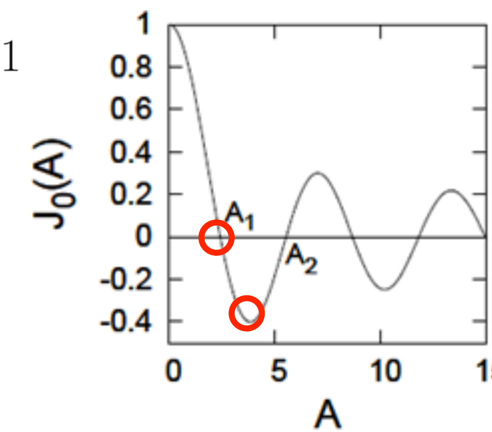
$$H(t) = -\sum_i \left[e^{i\frac{E}{\Omega}\sin\Omega t} c_{i+1}^{\dagger} c_i + \text{h.c.} \right] \longrightarrow H_m = -J_m(E/\Omega)\sum_i c_{i+1}^{\dagger} c_i - J_{-m}(E/\Omega)\sum_i c_i^{\dagger} c_{i+1}$$

0-th order

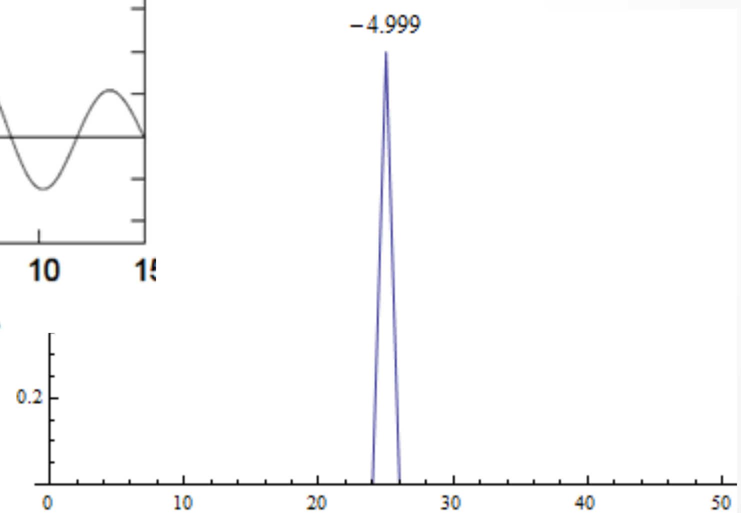
$$\longrightarrow \quad H_{\text{eff}} = -\boxed{J_0(E/\Omega)}\sum_i \left[c_{i+1}^{\dagger} c_i + \text{h.c.} \right] + \dots$$

$\Omega = 8,\ E_0/\Omega = 2.41$





$\Omega = 8,\ E_0/\Omega = 4$



evolution stops at the zero of the Bessel function

ex. 4: Floquet topological insulator

TO, Aoki ’09
Kitagawa, TO, Fu, Brataas, Demler ‘11

1st order

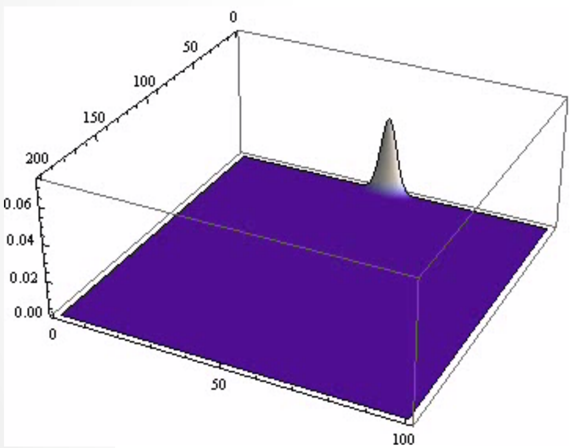
$$H_{\text{eff}} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$$

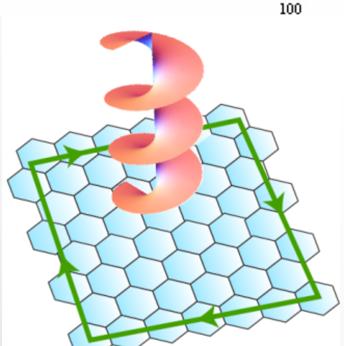
$$A = F/\Omega$$

applied to honeycomb lattice

n. hopping + n. hopping = n.n. hopping with phase $\pi/2$

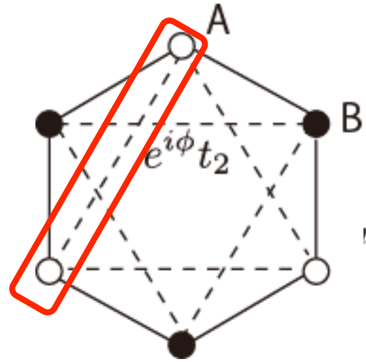
honeycomb + circularly polarized light

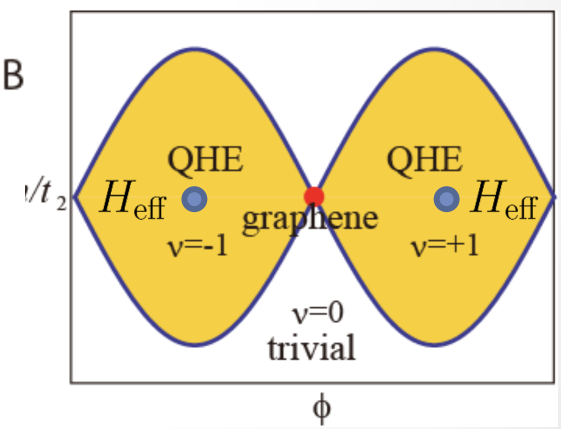




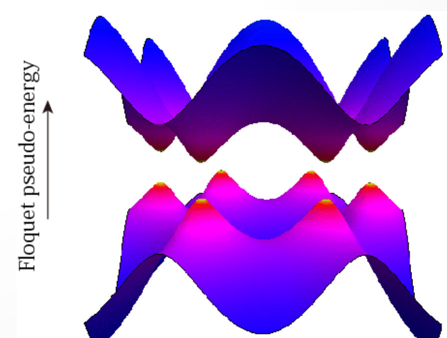
1-st order

Haldane’s Model of QHE without LL (1988)





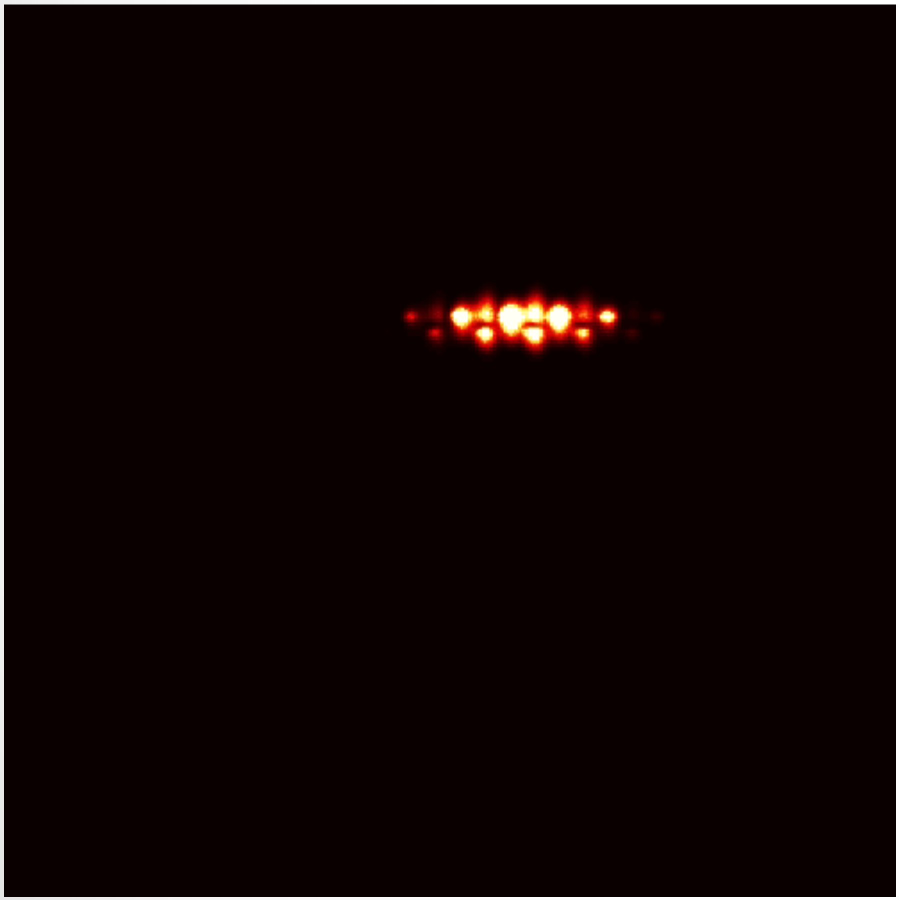
local magnetic field ϕ
AB-level offset m



Experiments of Floquet topological ins.

Rechtsman *et al.* Nature ‘13

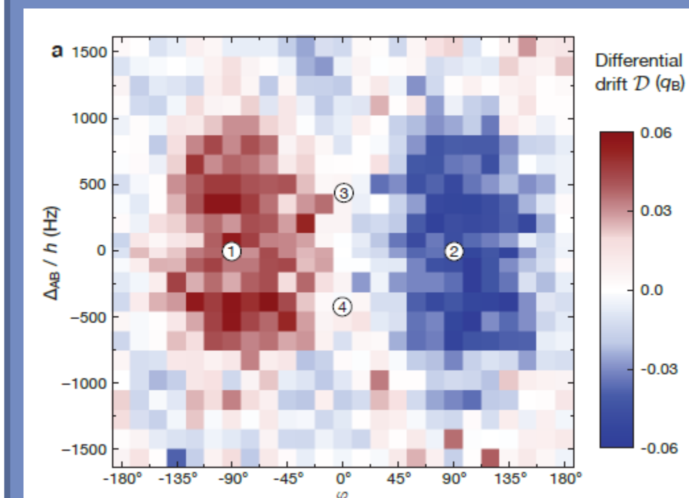
Realization in photonic crystals

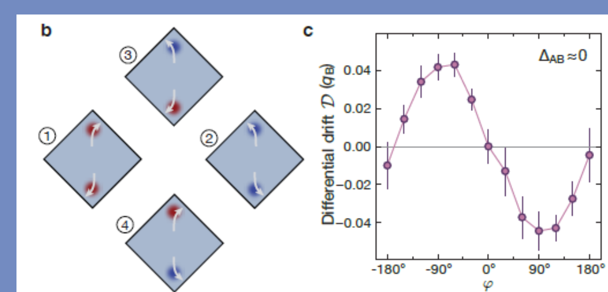


Jotze, Esslinger *et al.* Nature ‘14

shaken cold atom

Drift measurement ~ conductivity





Ex. 4: Floquet topological insulator

TO, Aoki '09
Kitagawa, TO, Fu, Brataas, Demler '11

2D Dirac electron in circularly polarized laser

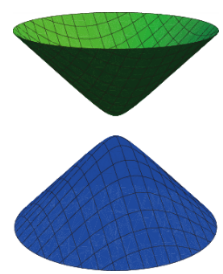
$$H(t) = k_x\sigma_x + k_y\sigma_y + Ae^{i\Omega t}\sigma_- + Ae^{-i\Omega t}\sigma_+$$

1st order

$$H_{\text{eff}} = H_0 + \frac{[\overset{\sim A\sigma_-}{\boxed{H_{-1}}}, \overset{\sim A\sigma_+}{\boxed{H_1}}]}{\Omega} + \mathcal{O}(A^4)$$

$$= k_x\sigma_x + k_y\sigma_y + \frac{A^2}{\Omega}\sigma_z + \ldots$$

Dirac point



2κ

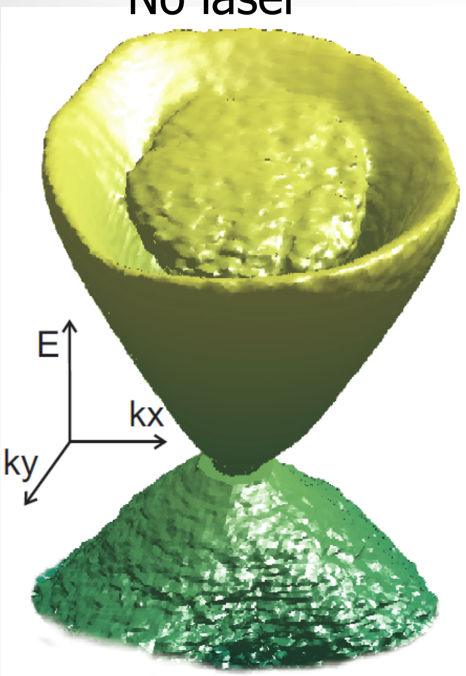
Dynamical gap

$$\kappa = \frac{\sqrt{4A^2+\Omega^2}-\Omega}{2} \sim A^2/\Omega$$

Experiment using time resolved ARPES

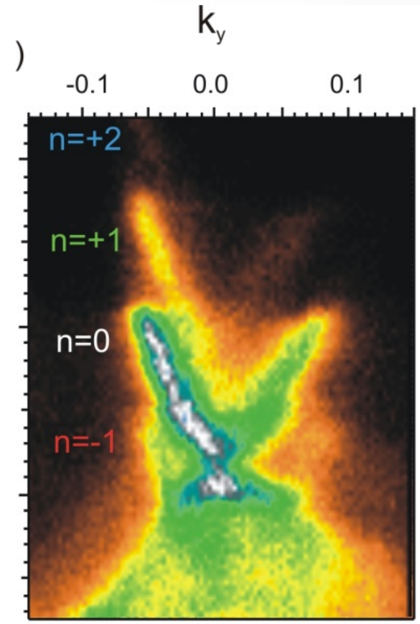
surface Dirac state of a TI

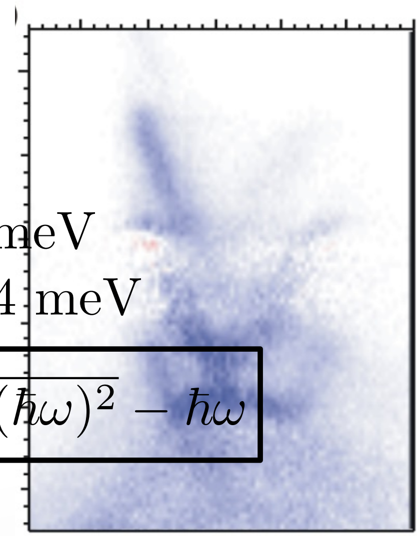
No laser

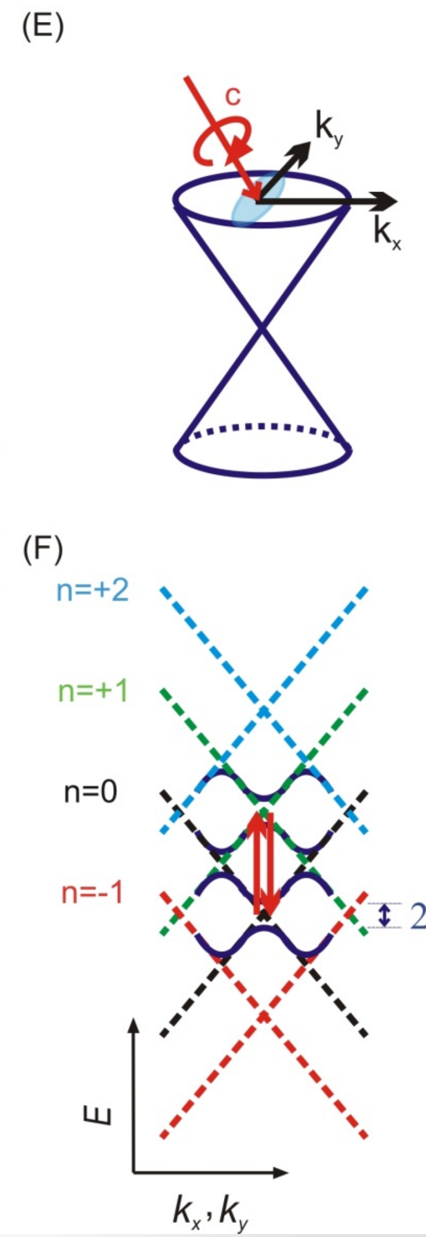


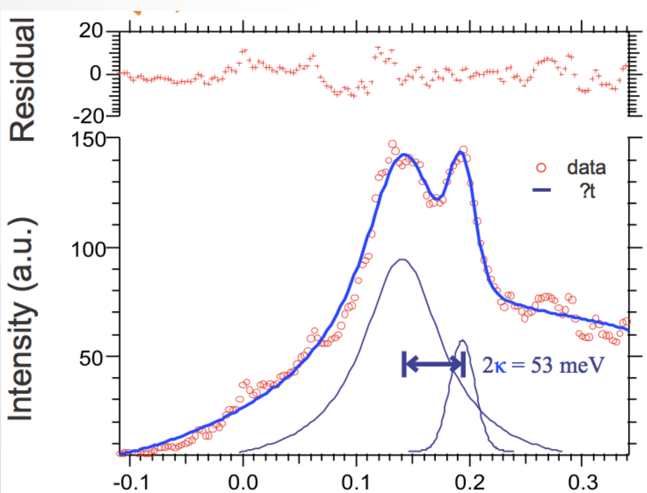
circularly polarized laser

Gedik (MIT) Science'13









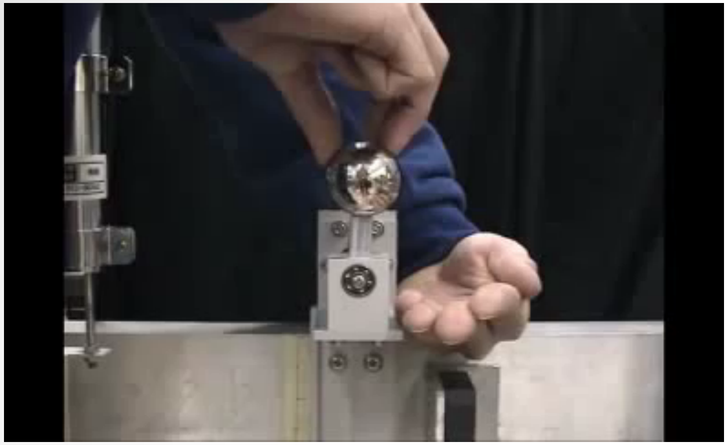
Exp.: $2\kappa = 53$ meV

Theory: $2\kappa = 54$ meV

$$\boxed{2\kappa = \sqrt{4V^2 + (\hbar\omega)^2} - \hbar\omega}$$

TO, Aoki '09

ex. 1: Kapitza's inverted pendulum



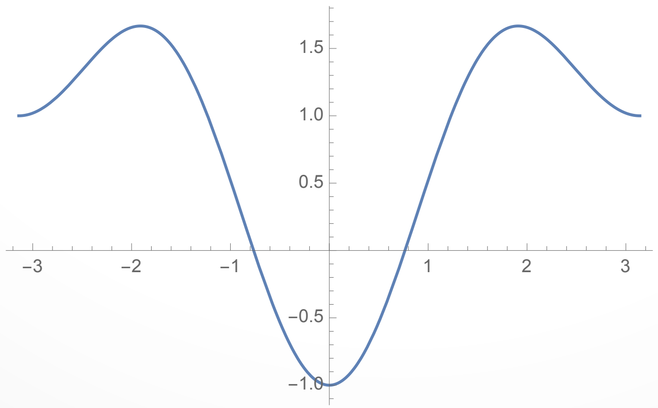
$$H(t) = \frac{p_\theta^2}{2m} - (g_0 + g_1\cos(\Omega t))\cos(\theta)$$

$$H_{\text{eff}} = H_0 + \frac{1}{2}\sum_{m\neq 0}\frac{[H_{-m}, H_m]}{m\Omega} + \boxed{\frac{1}{3}\sum_{m,n\neq 0}\frac{[H_{-m},[H_{m-n},H_n]]}{nm\Omega^2}} + \frac{1}{2}\sum_{m,n\neq 0}\frac{[H_m,[H_0,H_{-m}]]}{m^2\Omega^2} + \dots$$

2nd order

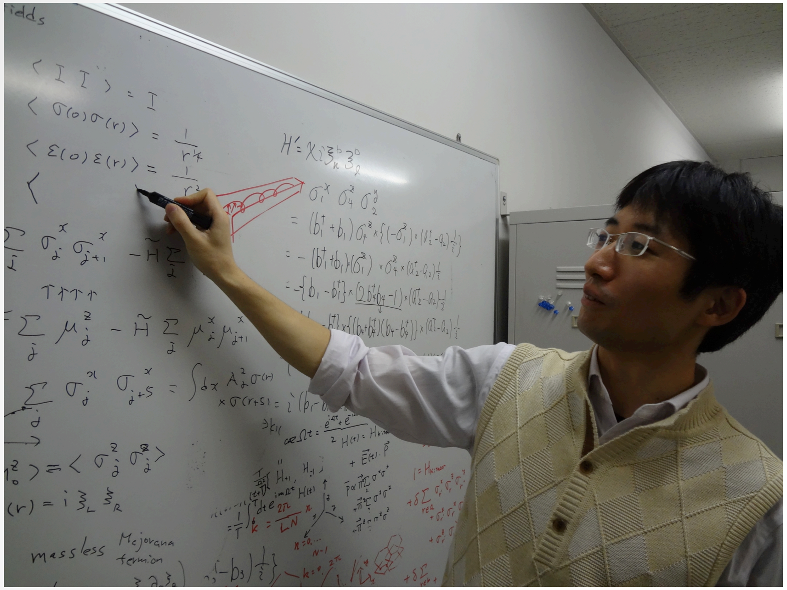
$$[\cos\theta, [\cos\theta, p_\theta^2]] \propto \sin^2\theta$$

$$V(\theta) = -\cos\theta + \alpha\sin^2\theta$$

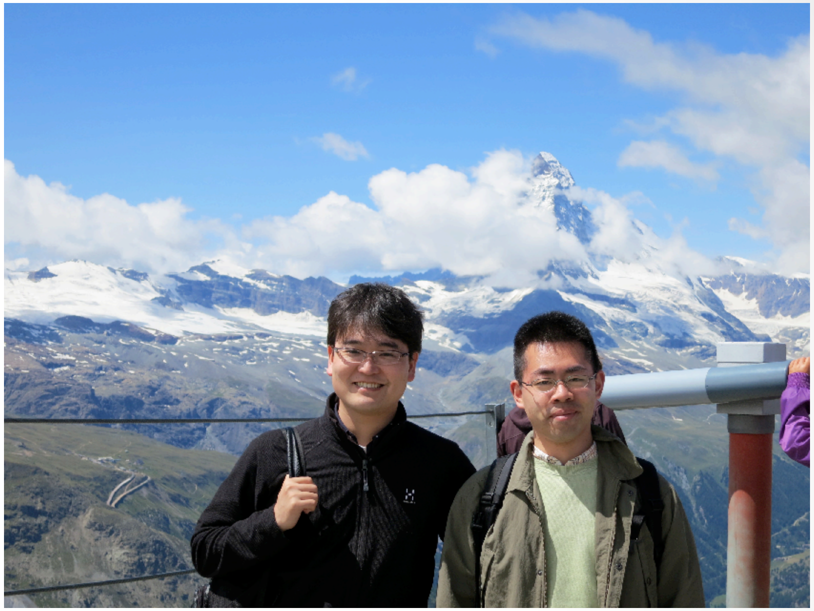


extra minima
in the effective potential

Application to quantum magnets



M. Sato (Aoyama Gakuin U.
→Japan Atomic Energy Agency)



S. Takayoshi
(U-Tokyo→NIMS→U-Tokyo→U-Geneva)

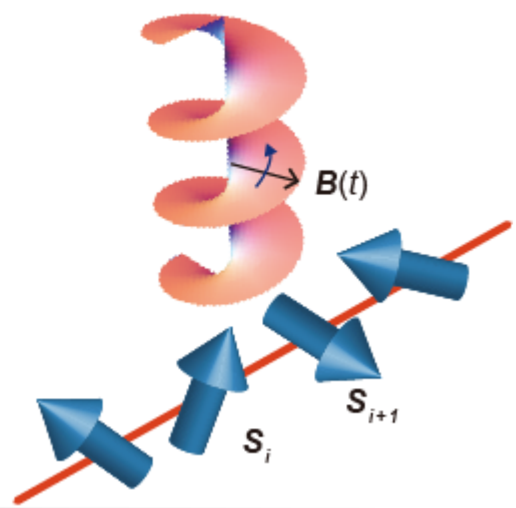
ex. 2: Magnets in rotating magnetic fields

S=1 anisotropic Heisenberg model in rotating magnetic fields

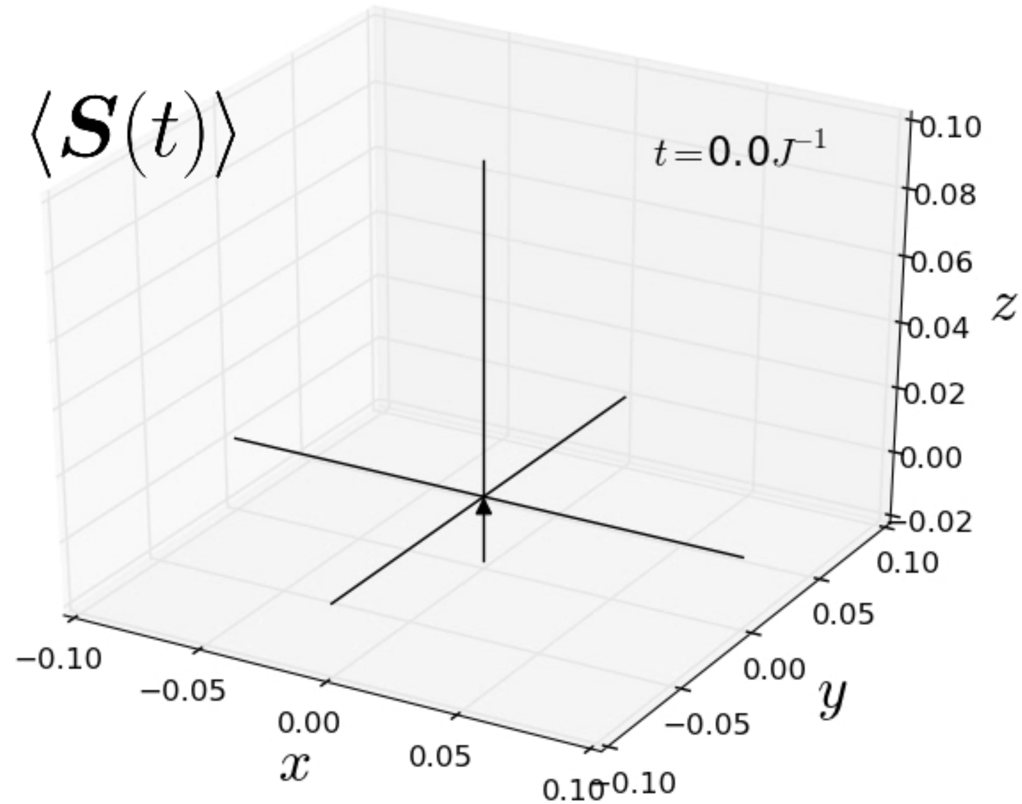
$$H(t) = \sum_i \left(J\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + D(S_i)^2 + \boxed{\boldsymbol{B}(t)\boldsymbol{S}_i} \right)$$

Laser induced magnetization

$$\boldsymbol{B}(t) = (B\cos\Omega t, B\sin\Omega t)$$



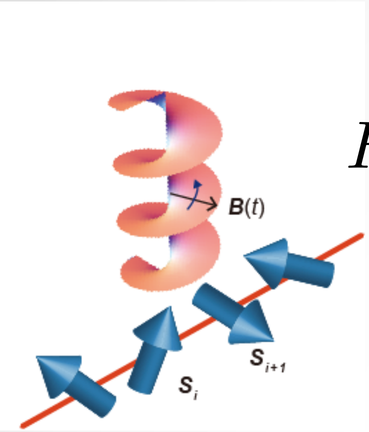
$\langle \boldsymbol{S}(t) \rangle$



Takayoshi, Aoki, TO, PRB2014
Takayoshi, Sato, TO, PRB2014

iTEBD by Takayoshi

THz Laser induced magnetization



S=1 anisotropic Heisenberg model in rotating magnetic fields

$$H(t) = \sum_i \left(J\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + D(S_i)^2 + \boxed{\boldsymbol{B}(t)\boldsymbol{S}_i} \right)$$

rotating magnetic field

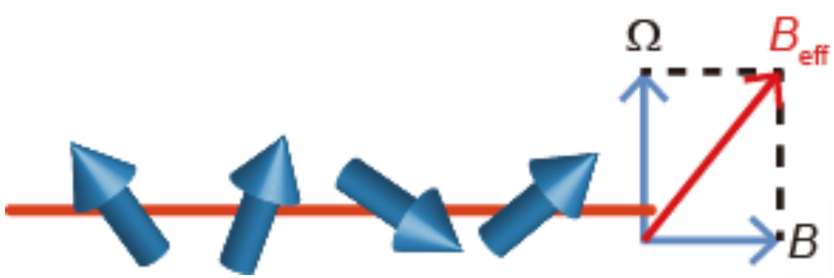
$$\boldsymbol{B}(t) = (B\cos\Omega t, B\sin\Omega t)$$

Choose the best frame

$$U(t) = e^{i\Omega t S^z_{\mathrm{tot}}}$$

$$\mathcal{H}_{\mathrm{eff}} = J\sum_{i,j}\mathbf{S}_i \cdot \mathbf{S}_j + D\sum_i (S_i^z)^2 \boxed{- \Omega S^z_{\mathrm{tot}} - B S^x_{\mathrm{tot}}}$$

"effective magnetic field"



spin model with slanted magnetic fields

Takayoshi, Aoki, TO, PRB2014

How large can the effective field be?

$$\mathcal{H}_{\mathrm{eff}} = J\sum_{i,j}\mathbf{S}_i\cdot\mathbf{S}_j + D\sum_i (S_i^z)^2 \boxed{-\Omega S^z_{\mathrm{tot}} - BS^x_{\mathrm{tot}}}$$

"effective magnetic field"

photon energy	effective field
Ω= 1 THz	50 Tesla
Ω= 10 THz	500 Tesla

assuming *J=2meV*
NDMAP

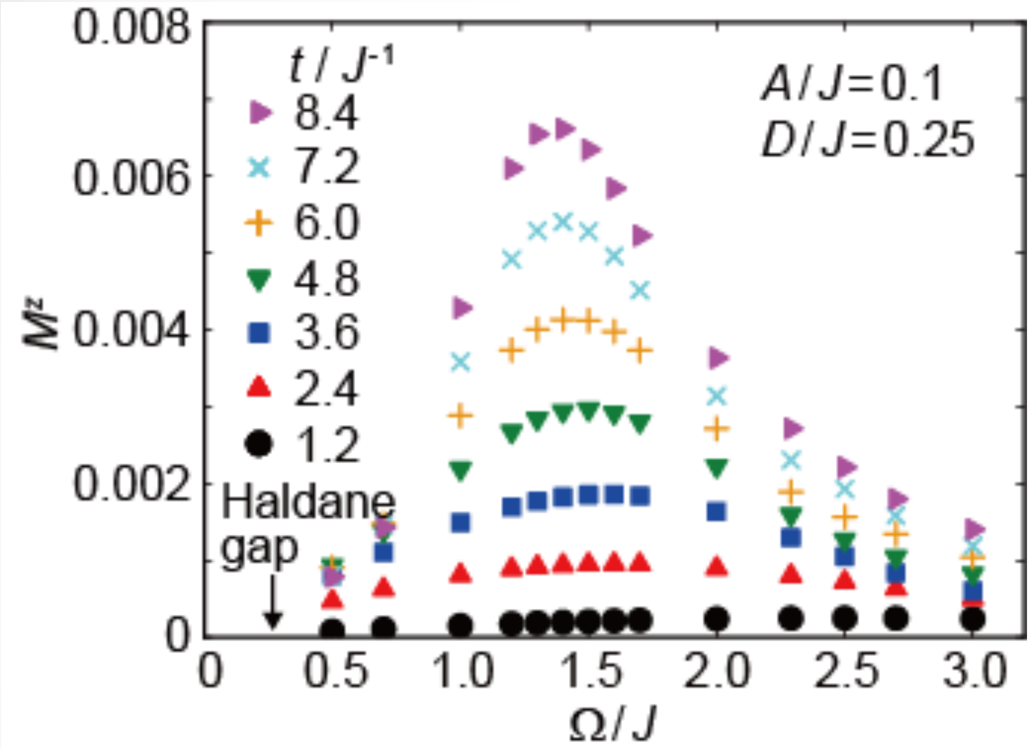
Is larger Ω always better??

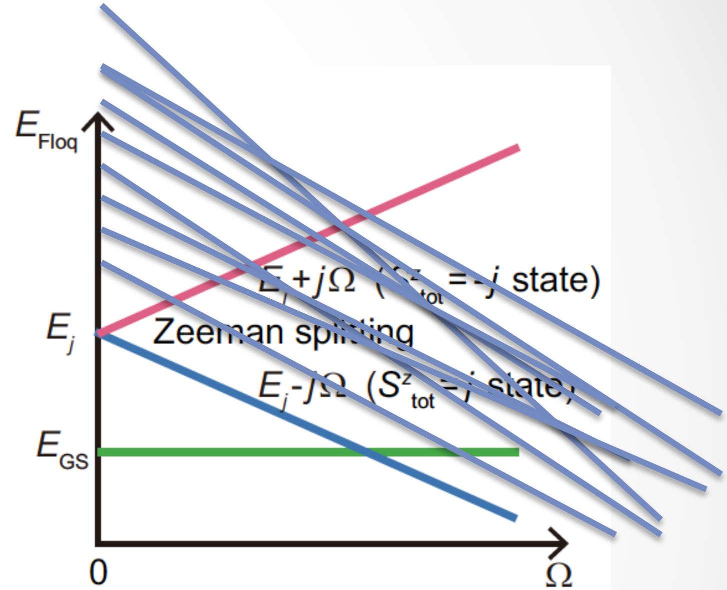
Resonance … Takayoshi, Aoki, TO, PRB2014

Chirping … Takayoshi, Sato, TO, PRB2014

Resonance

Takayoshi, Aoki, TO, PRB2014



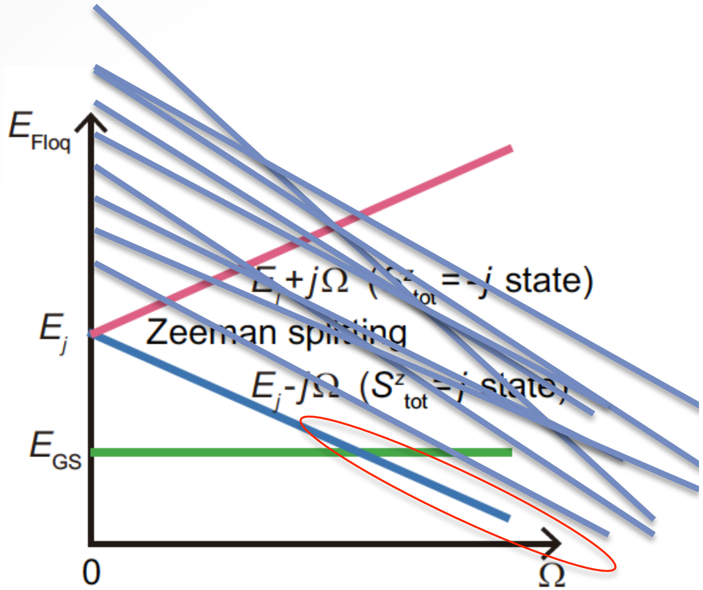


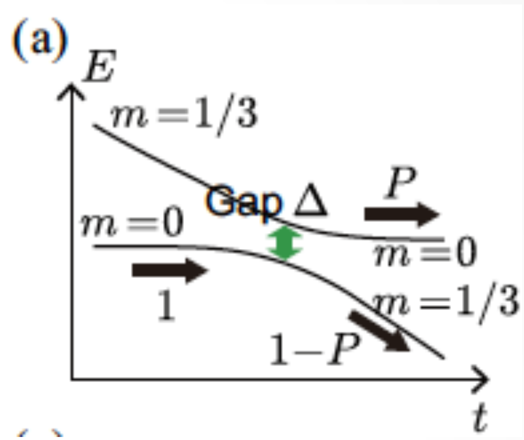
Resonant-like behavior at $\Omega = 1.4J$ (~4meV=1THz),
(above the Haldane gap)

~ peak of the “*magnetic density of states*”

Laser induced Magnetization curve by Chirping

Takayoshi, Sato, TO, PRB2014

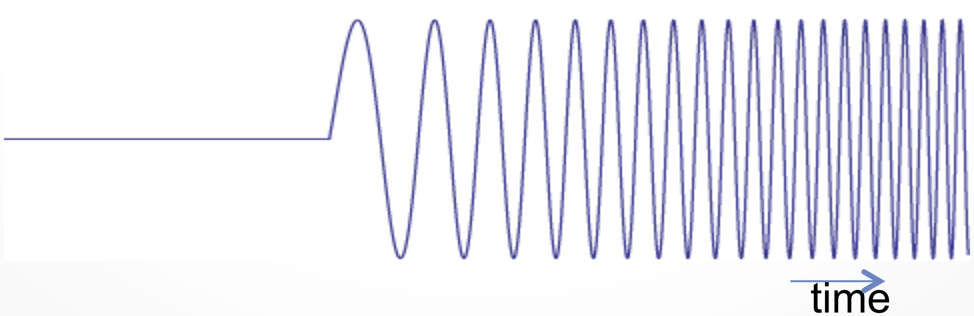




adiabatically follow the “groundstate”

increase Ω slowly

chirped laser

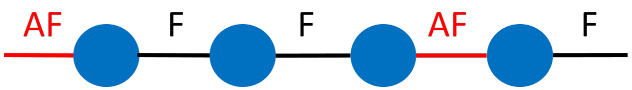


Laser induced Magnetization curve by Chirping

Takayoshi, Sato, TO, PRB2014
cf) next talk

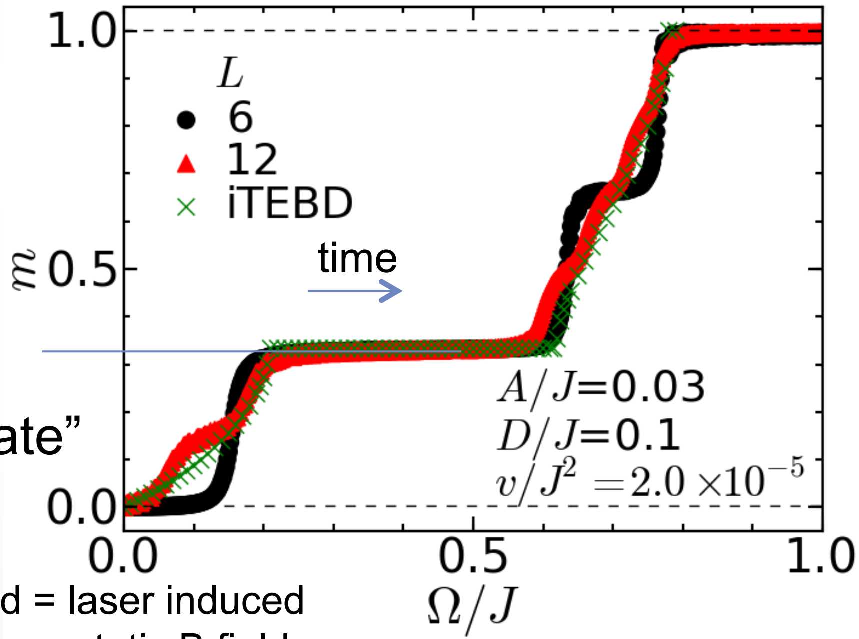
increase Ω slowly = increase B_{eff} slowly

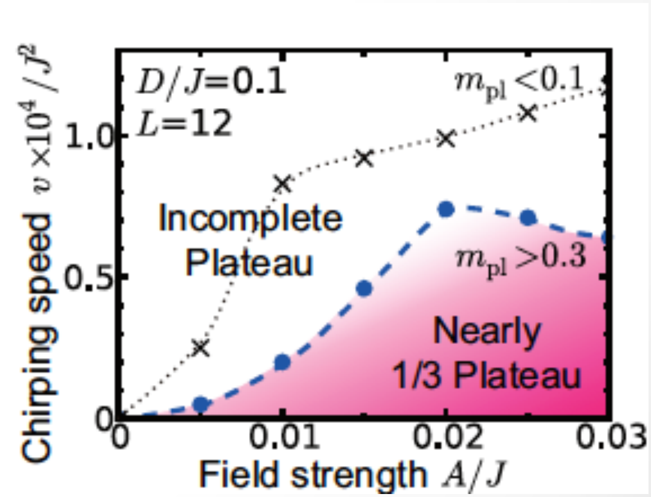
$$H_0 = \sum_j \left(-J_F \boldsymbol{B}_{3j-2}\cdot\boldsymbol{S}_{3j-1} - J_F \boldsymbol{B}_{3j-1}\cdot\boldsymbol{S}_{3j} + J_{AF}\boldsymbol{B}_{3j}\cdot\boldsymbol{S}_{3j+1}\right)$$



1/3 plateau

“topological state”



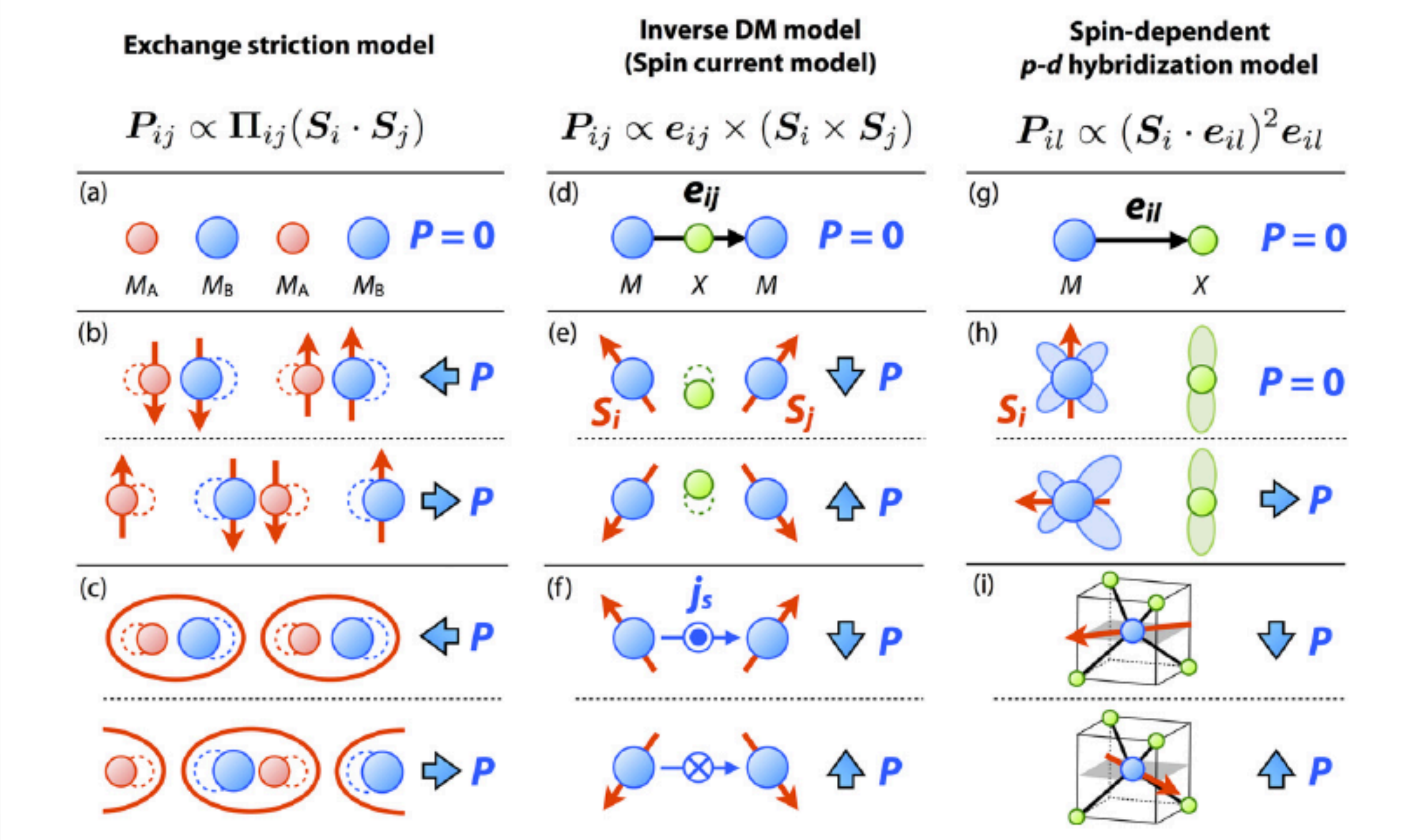


black,red = laser induced
green = static B field

Another light matter coupling for quantum magnets

$$\boldsymbol{E}(t)\boldsymbol{P}$$ coupling with polarization

This originates from spin-orbit interaction and is relevant in multiferroics (magnetisation+ferroelectricity)

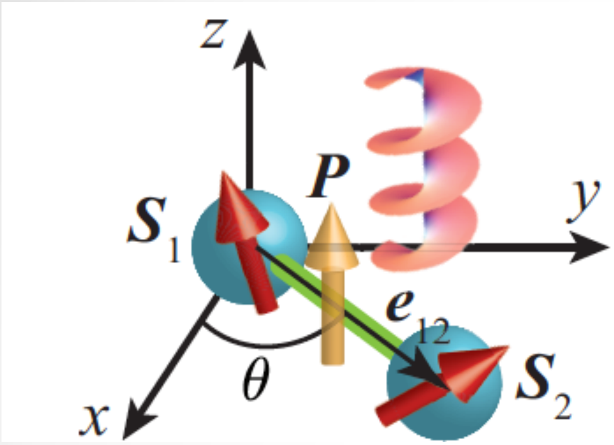


review: Tokura, Seki, Nagaosa, Rep. Prog. Phys. '14

Laser induced effective terms in multiferroic quantum magnets

Sato, Sasaki, TO, arXiv'15
Sato, Takayoshi, TO, *in prep.*

circularly polarized laser



$$\boldsymbol{E} = E_0(\cos\Omega t, \sin\Omega t),\ \boldsymbol{B} = B_0(\sin\Omega t, \cos\Omega t)$$

$$H(t) = H_0 + \boldsymbol{B}(t)\boldsymbol{S} + \boldsymbol{E}(t)\boldsymbol{P}$$

Fourier transform

$$H_1 = E_0(P_x - iP_y)/2 + B_0(S_y - iS_x)/2$$

$$H_{-1} = E_0(P_x + iP_y)/2 + B_0(S_y + iS_x)/2$$

1st order effective model

$$H_{\mathrm{eff}} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$$

effective terms

$$H_\Omega = \frac{E_0E_0}{\Omega}(3\ \text{spin term}) + \frac{E_0B_0}{\Omega}(2\ \text{spin term}) + \frac{B_0B_0}{\Omega}(1\ \text{spin term})$$

3-spin terms: Example in Kitaev's honeycomb lattice

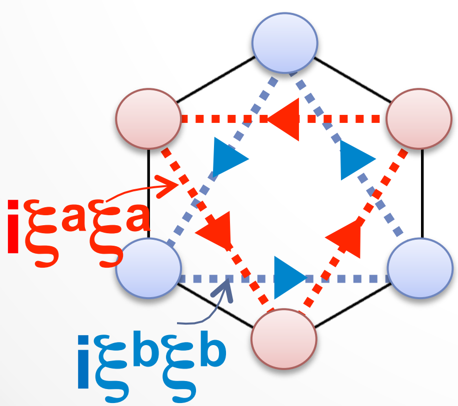
Details in Sato, Sasaki, TO, arXiv'15

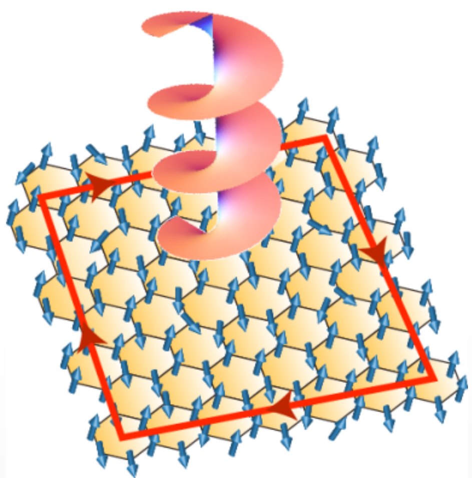
$$\vec{P}_{\rm tot} = \sum_\alpha \sum_{\langle \vec{r},\vec{r}'\rangle_\alpha} \vec{P}_{(\vec{r},\vec{r}')_\alpha} \quad \vec{P}_{(\vec{r},\vec{r}')_\alpha} = \vec{\pi}_\alpha \sigma^\alpha_{\vec{r}} \sigma^\alpha_{\vec{r}'}.$$

$$\begin{aligned}\hat{\mathcal{H}}_\Omega \;=\; & \pm\frac{1}{\Omega}E^2\cos\delta\Big[G_{12}\Big(\sum_{\vec{r}\in A}\sigma^x_{\vec{r}_1}\sigma^z_{\vec{r}}\sigma^y_{\vec{r}_2}+\sum_{\vec{r}\in B}\sigma^x_{\vec{r}_1}\sigma^z_{\vec{r}}\sigma^y_{\vec{r}_2}\Big)\\ & +G_{23}\Big(\sum_{\vec{r}\in A}\sigma^y_{\vec{r}_2}\sigma^x_{\vec{r}}\sigma^z_{\vec{r}_3}+\sum_{\vec{r}\in B}\sigma^y_{\vec{r}_2}\sigma^x_{\vec{r}}\sigma^z_{\vec{r}_3}\Big)\\ & +G_{31}\Big(\sum_{\vec{r}\in A}\sigma^z_{\vec{r}_3}\sigma^y_{\vec{r}}\sigma^x_{\vec{r}_1}+\sum_{\vec{r}\in B}\sigma^z_{\vec{r}_3}\sigma^y_{\vec{r}}\sigma^x_{\vec{r}_1}\Big)\Big]\end{aligned}$$

$$G_{\alpha\beta} = \hat{z}\cdot(\vec{\pi}_\alpha \times \vec{\pi}_\beta)$$

After Jordan-Wigner transformation, the effective Majorana fermion model becomes a quantum Hall model (topological Chern insulator)

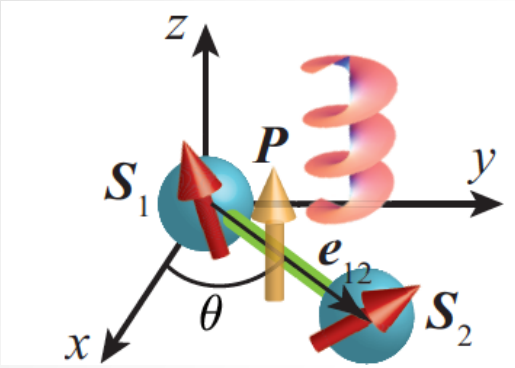




Gapped topological spin liquid
with Majorana edge mode
and nonabelian anyon excitation

2-spin terms: Laser controlled Dzyaloshinskii-Moriya coupling

Sato, Takayoshi, TO, *in prep.*



Assume “inverse DM coupling” in a 1D chain

$$\boldsymbol{P}_{\mathrm{tot}} = g_{\mathrm{me}} \sum_j \boldsymbol{e}_{j,j+1} \times \boldsymbol{\mathcal{V}}_{j,j+1}$$

$$\boldsymbol{\mathcal{V}}_{j,j+1} \equiv \boldsymbol{S}_j \times \boldsymbol{S}_{j+1}$$

in the chain direction

↓ high frequency circularly polarized laser

$$H_\Omega = \cancel{\frac{E_0E_0}{\Omega}(3 \text{ spin term})} + \frac{E_0B_0}{\Omega}(2 \text{ spin term}) + \frac{B_0B_0}{\Omega}(1 \text{ spin term})$$

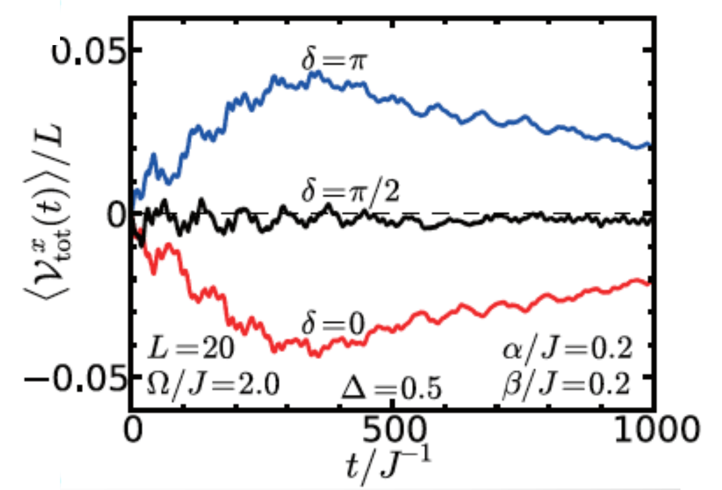
$$= \pm \sum_j \frac{\alpha\beta}{2\Omega}\mathcal{V}^x_{j,j+1} \pm \sum_j \frac{\beta^2}{2\Omega} S^z_j$$

$$\alpha = E_0 g_{\mathrm{me}}/2$$

$$\beta = g\mu_{\mathrm{B}} E_0 c^{-1}$$

= “Laser controlled DM coupling”
\+ effective Zeeman term

Numerical test in 1D XXZ chain (L=20)



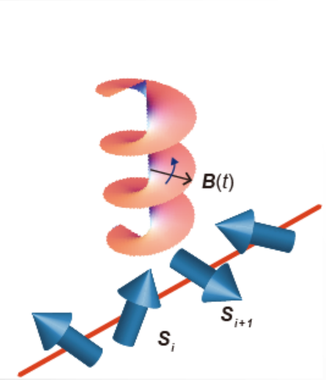
Conclusion

Floquet theory

1. Choose the best frame
2. Fourier transform
3. Effective Hamiltonian

$$H(t) = \sum_m H_m e^{-im\Omega t}$$

$$H_{\text{eff}} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$$



Application to quantum magnets

1. Huge effective magnetic field

$$\mathcal{H}_{\text{eff}} = J \sum_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j + D \sum_i (S_i^z)^2 - \Omega S^z_{\text{tot}} - B S^x_{\text{tot}}$$

2. Laser induced 3-spin and DM terms

$$H_\Omega = \frac{E_0 E_0}{\Omega}(3 \text{ spin term}) + \frac{E_0 B_0}{\Omega}(2 \text{ spin term}) + \frac{B_0 B_0}{\Omega}(1 \text{ spin term})$$